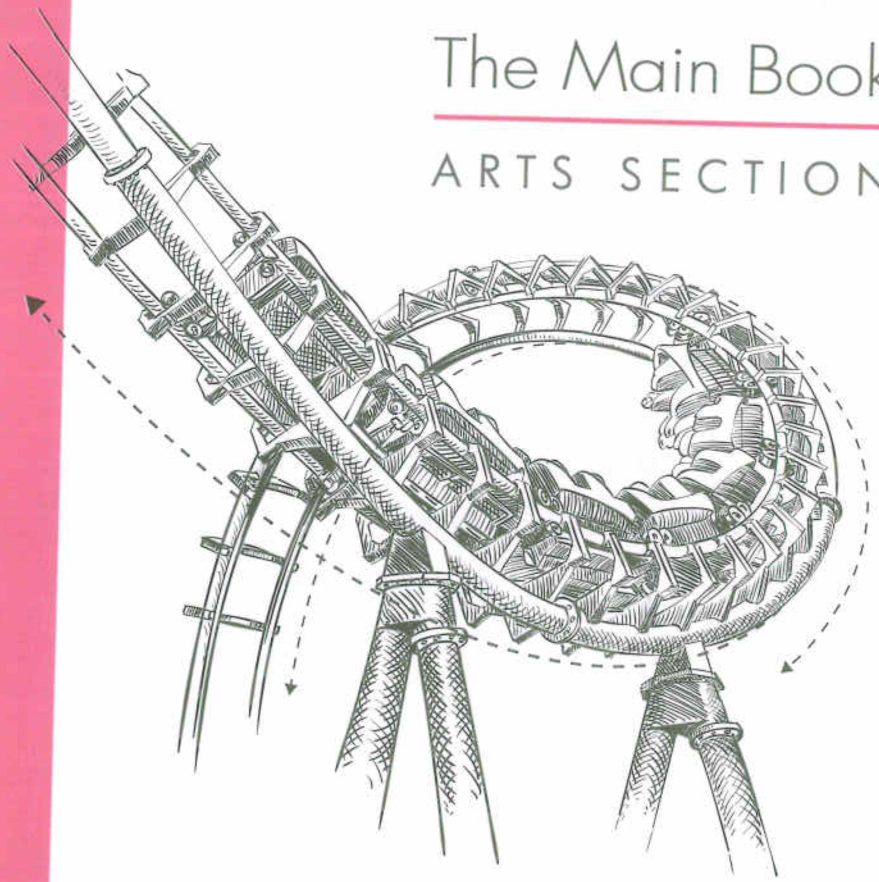
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General Mathematics

By a group of supervisors

The Main Book

ARTS SECTION



FIRST TERM

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CONTENTS

First Algebra

UNIT 1
Functions of a real variable and drawing curves.



UNIT 2
Exponents , logarithms and their applications.



Second Calculus and Trigonometry

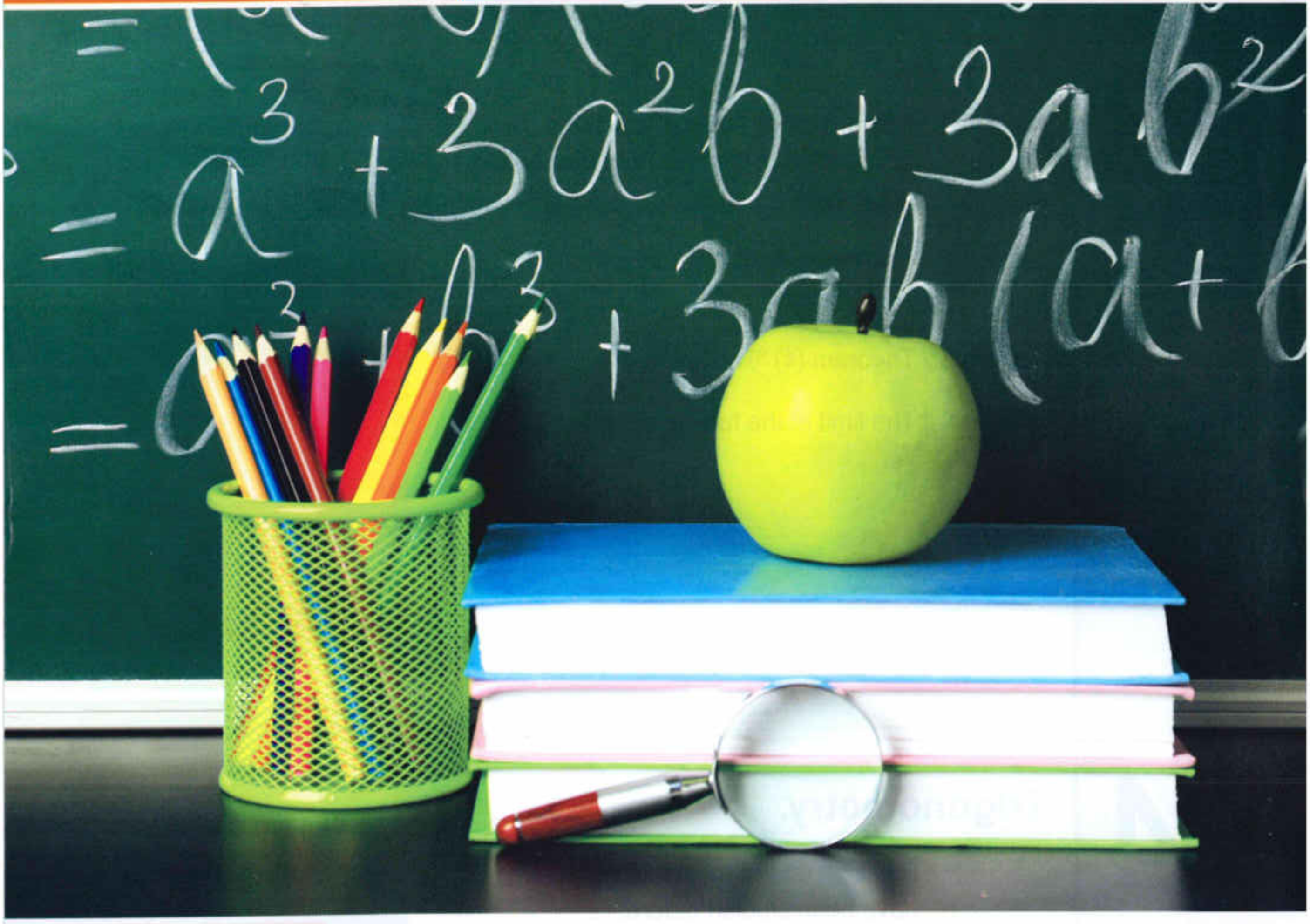
UNIT 3 Limits.



UNIT 4 Trigonometry.



First Algebra



UNIT 1 Functions of a real variable and drawing curves.

UNIT 2 Exponents , logarithms and their applications.

UNIT 1

Functions of a real variable and drawing curves.

Unit Lessons

* Pre-requirements for unit one.

Pre-requirements for unit one

* If X and Y are two non-empty sets , then :

  It is said that the relation from the set X to the set Y is a function if each element in X is related with one and only one element in Y

  The set X is called «the domain of the function».

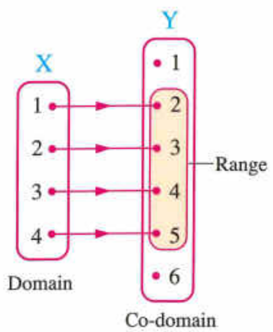
  The set Y is called «the co-domain of the function».

  The set of images of elements in the domain X is called «the range of the function» and it is subset of the co-domain Y

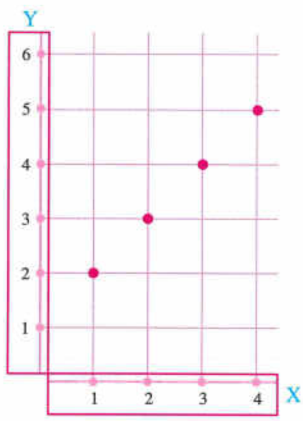
* The function f is written as $f : X \longrightarrow Y$, and the rule of the function is written as $y = f(x)$
* The set $\{(x , y) : x \in X , y \in Y , y = f(x)\}$ is called the set of ordered pairs of the function.
* The function can be represented by an arrow diagram or cartesian diagram.

For example :

* If $X = \{1 , 2 , 3 , 4\}$, $Y = \{1 , 2 , 3 , 4 , 5 , 6\}$ and the function $f : X \longrightarrow Y$ where $f(x) = x + 1$, then the set of ordered pairs of the function $= \{(1 , 2) , (2 , 3) , (3 , 4) , (4 , 5)\}$



Arrow diagram



Cartesian diagram

Notice that :

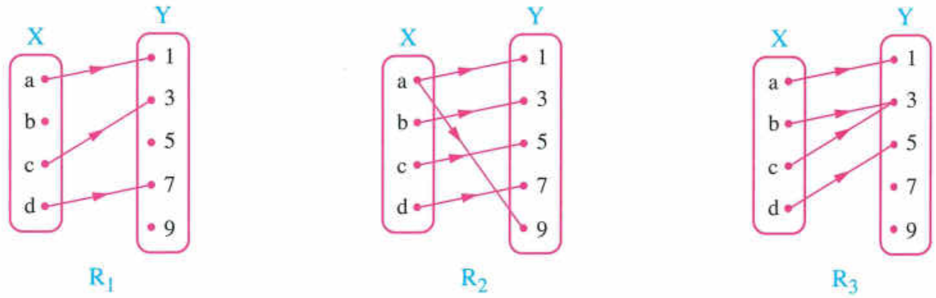
* Not each relation from X to Y is a function but all functions from X to Y are relations satisfy that :
  - Each element in X appears once as a first projection in one of the ordered pairs of the relation.

- Each element in X has only one arrow going out to an element of Y in the arrow diagram which represents the relation.
- Each vertical line has only one point from the points of the relation.

* The function $f : f(X) = a_0 + a_1 X + a_2 X^2 + a_3 X^3 + \cdots + a_n X^n$ where : $a_0 , a_1 , a_2 , a_3 , \ldots , a_n$ are constants , $a_n \in \mathbb{R} - \{0\}$ is called polynomial function of nth degree and its domain and range are $\mathbb{R}$ if its not mention other than that.
* If $f(X) = a$ where $a \neq 0$, then the function f is called constant polynomial function and if $f(X) =$ zero , then the function f is called zero function and is represented graphically by the X-axis.
* The polynomial function is named according to the degree of the algebraic expression of the function.
* The polynomial function of first degree is called linear function , of second degree is called quadratic function and of third degree is called cubic function and so on.
* Set of zeroes of polynomial function f is the set of values of X that make $f(X) = 0$ and equals the set of X-coordinates of the points of intersection of the curve of the function with X-axis.
* The function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = a X^n$ where $a \in \mathbb{R}^*$, $n \in \mathbb{Z}^+$ is called power function , so at adding or subtracting power functions with constants , we get a polynomial function.

Example

Show with reasons , which of the following relations (represented by the shown arrow diagrams) represents a function , if so , mention each of the domain and the range for every function :



Solution

- R_1 is not a function because there is no arrow from $b \in X$ to an element in Y
- R_2 is not a function because there are two arrows going from $a \in X$ to two elements in Y
- R_3 is a function because there is one and only one arrow drawn from each element in X to a corresponding element in Y

, the domain $= \{a , b , c , d\}$ and the range $= \{1 , 3 , 5\}$



Lesson One

Real functions

(Determination the domain and range - Discuss the monotony)

Real function

The function $f : X \longrightarrow Y$ is called a real function if each of the domain (X) and the co-domain (Y) is the set of the real numbers or a proper subset of it.

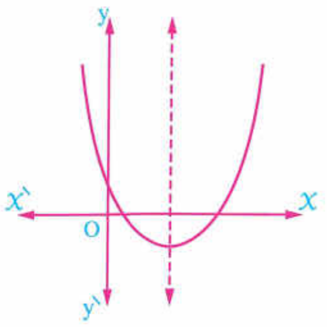
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Determining whether the relation from $X \longrightarrow Y$ is a function or not :

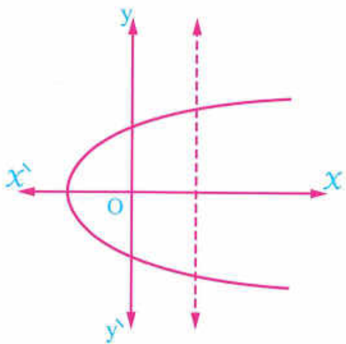
(1) Algebraically : The relation is a function if every value of the variable $x \in X$ is related with only one value of the variable $y \in Y$

(2) Graphically (The vertical line test) :

The relation is not a function if there exists at least one straight line parallel to y-axis and intersects the graph of the relation at more than one point.



Function from $X \longrightarrow Y$



Not Function from $X \longrightarrow Y$

Example 1

Show giving reasons , which of the following two relations is a function on $\mathbb{R}$:

(1) $y = x^2 + 3$ (2) $y^2 = x^2 + 9$

Solution

(1) The relation $y = x^2 + 3$ is a function because every real value of the variable x is related with a unique value of the variable y

For example : When $x = 3$, then $y = 12$ and when $x = -2$, then $y = 7$ and so on.

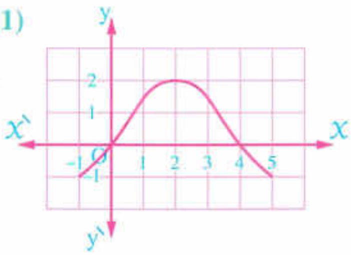
(2) The relation $y^2 = x^2 + 9$ is not a function because there is at least one real value of the variable x is related with two different values of the variable y

For example : When $x = 4$, then $y^2 = 25$ $\therefore y = \pm 5$

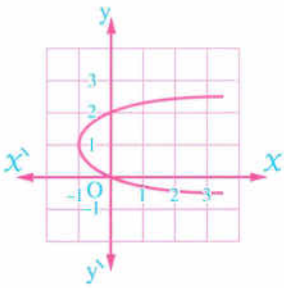
Example 2

Show which of the following graphs represents a function on $\mathbb{R}$, which doesn't represent a function giving reasons :

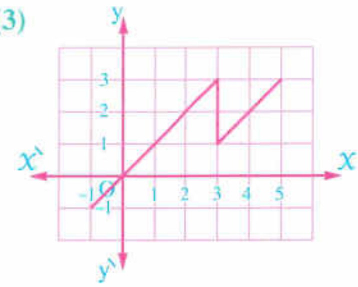
(1)



(2)



(3)



Solution

(1) Represents a function for each vertical line intersects the curve at one point at most.

(2) Does not represent a function for there are many vertical lines intersect the curve at two points.

(3) Does not represent a function for there is a vertical line passing through the point (3 , 0) and intersect the curve at a set of points.

Remarks

1. The relation $y = 4$ (represented by a horizontal straight line parallel to x-axis) is a function from X to Y because each element in X is related with only one element in Y

2. The relation $x = 4$ (represented by a vertical straight line parallel to y-axis is not a function from X to Y because the element $x = 4$ is related with infinite number of elements in Y

Identifying the domain of the real functions

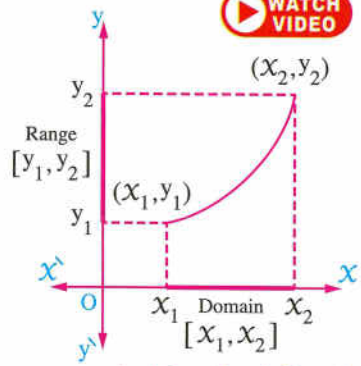
The domain of the function is identified by its rule or its graph.

First Identifying the domain and range of the function from its graph

From the graph of the function we can deduce the domain and the range of the function to be :

(1) Domain of the function is the set of the x-coordinates of all the points that lie on the curve of the function.

(2) Range of the function is the set of the y-coordinates of all the points that lie on the curve of the function.



Example 3

Determine the domain and range for each function represented by the following figures :

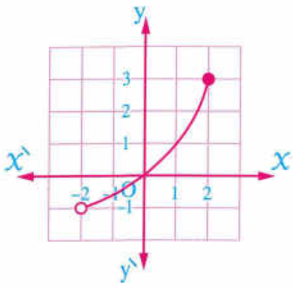


Fig. (1)

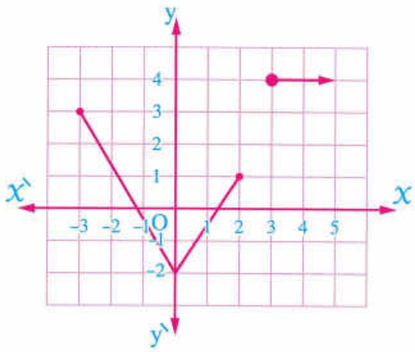
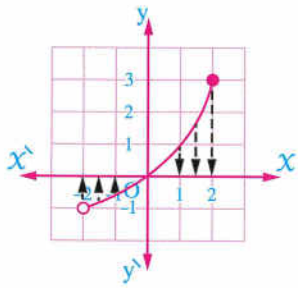


Fig. (2)

Solution

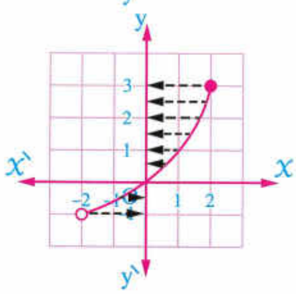
In fig. (1) : * The x-coordinates of all points on the curve of the function are on the interval $]-2, 2]$

$\therefore$ The domain $=]-2, 2]$



* The y-coordinates of all points on the curve of the function are on the interval $]-1, 3]$

$\therefore$ The range $=]-1, 3]$

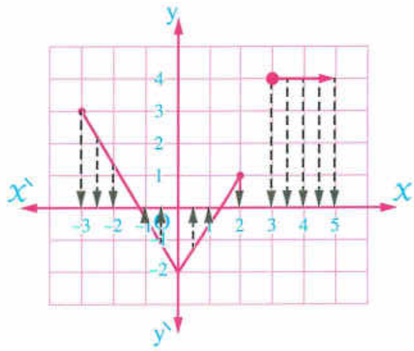


Notice that :

* The unshaded circle at point (– 2 , – 1) shows that the point ∉ the function and so – 2 ∉ the domain of the function and – 1 ∉ the range of the function.
* The shaded circle at point (2 , 3) shows that the point ∈ the function and so 2 ∈ the domain of the function and 3 ∈ the range of the function.

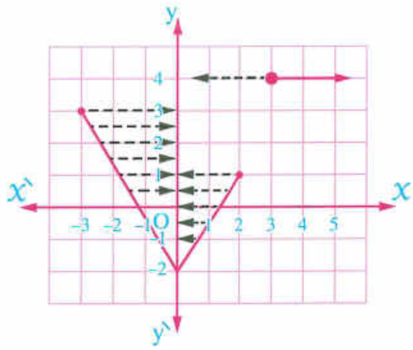
In fig. (2) : * The X-coordinates of all points on the curve of the function are on the two intervals [– 3 , 2] and [3 , ∞[

∴ The domain = [– 3 , 2] ∪ [3 , ∞[



* The y-coordinates of the points at the horizontal ray is y = 4
, the y-coordinates of the other points of the curve are on the interval [– 2 , 3]

∴ The range = [– 2 , 3] ∪ {4}



Second Identifying the domain of the function from its rule

1 Polynomial function

The domain of the polynomial function is ℝ unless it is defined on a subset of it.

For example : $f : f(X) = 3$ (Constant polynomial) , its domain = ℝ

, $f : f(X) = 2X + 1$ (First degree polynomial) , its domain = ℝ

, $f : f(X) = X^2 - 4X + 3$ (Second degree polynomial) , its domain = ℝ

2 Rational function

If f is a rational function where $f(X) = \frac{h(X)}{g(X)}$, h and g are two polynomials , then the domain of the function f = ℝ – the set of zeroes of the denominator.

Example 4

State the domain of each of the rational functions defined by the following rules :

(1) $f(x) = \frac{1}{x}$

(2) $f(x) = \frac{3}{x-2}$

(3) $f(x) = \frac{x-1}{2x^2+5x}$

(4) $f(x) = \frac{x-3}{x^2-5x+6}$

(5) $f(x) = \frac{x+1}{x^2-4x+4}$

(6) $f(x) = \frac{x}{x^2+25}$

Solution

(1) The domain $= \mathbb{R} - \{0\}$

(2) The domain $= \mathbb{R} - \{2\}$

(3) Let $2x^2 + 5x = 0$ $\quad \therefore x(2x+5) = 0$

$\therefore x = 0$ or $x = \frac{-5}{2}$ $\quad \therefore$ The domain $= \mathbb{R} - \{0, \frac{-5}{2}\}$

(4) Let $x^2 - 5x + 6 = 0$ $\quad \therefore (x-2)(x-3) = 0$

$\therefore x = 2$ or $x = 3$ $\quad \therefore$ The domain $= \mathbb{R} - \{2, 3\}$

(5) Let $x^2 - 4x + 4 = 0$ $\quad \therefore (x-2)^2 = 0$

$\therefore x = 2$ $\quad \therefore$ The domain $= \mathbb{R} - \{2\}$

(6) Let $x^2 + 25 = 0$ and this equation has no solution in $\mathbb{R}$

i.e. There are no real zeroes of the denominator

$\therefore$ The domain $= \mathbb{R}$

3 The nth root function

If $f(x) = \sqrt[n]{h(x)}$ where $n \in \mathbb{Z}^+$, $n > 1$, $h(x)$ is a polynomial

First : When (n) is an odd number, then the domain of $f = \mathbb{R}$

Second : When (n) is an even number, then :

The domain of f is the set of all values of x which satisfy $h(x) \geq 0$, n is called the index of the root.

Example 5

State the domain of each of the real functions which are defined by the following rules :

(1) $f(x) = \sqrt{x+2}$

(2) $f(x) = \sqrt{-2x+3}$

(3) $f(x) = \sqrt[3]{9-x^2}$

(4) $f(x) = \frac{3}{\sqrt{x-4}}$

Solution

(1) $\because$ The index of the root is an even number.

$\therefore$ The function is defined where $x + 2 \geq 0$

$\therefore x \geq -2$ $\qquad$ $\therefore$ The domain $= [-2, \infty[$

(2) $\because$ The index of the root is an even number.

$\therefore -2x + 3 \geq 0$ $\qquad$ $\therefore x \leq \frac{3}{2}$

$\therefore$ The domain $=]-\infty, \frac{3}{2}]$

(3) $\because$ The index of the root is an odd number. $\qquad$ $\therefore$ The domain $= \mathbb{R}$

(4) $\because$ The index of the root is an even number

$\therefore$ The function is defined where : $x - 4 > 0$

$\therefore x > 4$ $\qquad$ $\therefore$ The domain $=]4, \infty[$

4 Piecewise function

It is the function that is defined by different rules for different parts of its domain.

Example 6

Determine the domain of each of the two functions defined by the following rules :

(1) $f(x) = \begin{cases} 2 - x & , \quad x < 0 \\ x - 2 & , \quad x > 0 \end{cases}$

(2) $f(x) = \begin{cases} x^2 & , \quad -2 \leq x < 0 \\ x & , \quad 0 \leq x \leq 1 \\ \frac{1}{x} & , \quad x > 1 \end{cases}$

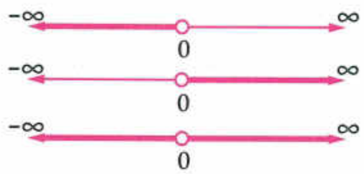
Solution

(1) The function f is defined on two intervals as the following :

Defined when $x \in]-\infty, 0[$

, defined when $x \in]0, \infty[$

$\therefore$ Domain of $f =]-\infty, 0[\cup]0, \infty[= \mathbb{R} - \{0\}$



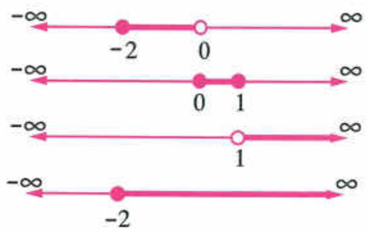
(2) The function f is defined on three intervals as the following :

Defined when $x \in [-2, 0[$

, defined when $x \in [0, 1]$

, defined when $x \in]1, \infty[$

$\therefore$ Domain of $f = [-2, 0[\cup [0, 1] \cup]1, \infty[= [-2, \infty[$



UNIT 1

Discussing the monotony of a function from its graph

Discussion of the monotony (monotonicity) of a function means identifying the intervals on which the function is increasing , the intervals on which the function is decreasing , and the intervals on which the function is constant.

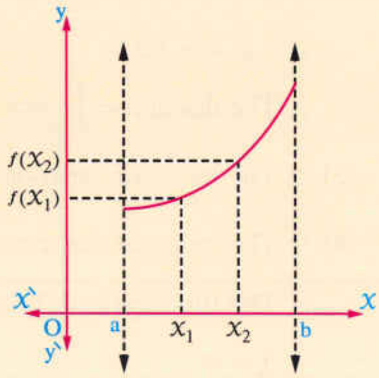
Definition (1) ***(Increasing function) :***

The function

f is said to be increasing on an interval]a , b[if :

$x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$ for every

$x_1 , x_2 \in]a , b[$



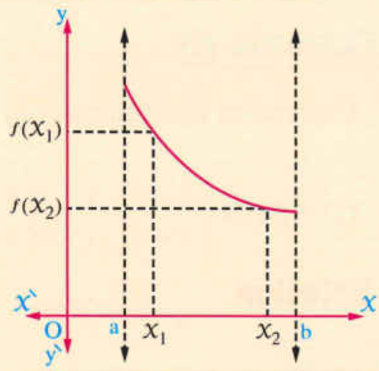
Definition (2) ***(Decreasing function) :***

The function

f is said to be decreasing on an interval]a , b[if :

$x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$ for every

$x_1 , x_2 \in]a , b[$



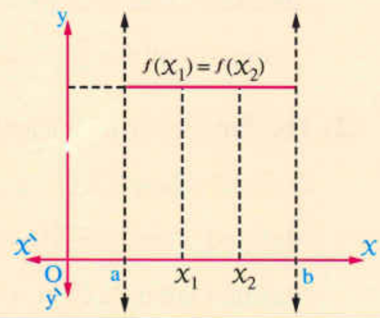
Definition (3) ***(Constant function) :***

The function

f is said to be constant on an interval]a , b[if :

$x_2 > x_1 \Rightarrow f(x_2) = f(x_1)$ for every

$x_1 , x_2 \in]a , b[$



Example 7

Discuss the monotonicity of each of the functions represented by the following graphs :

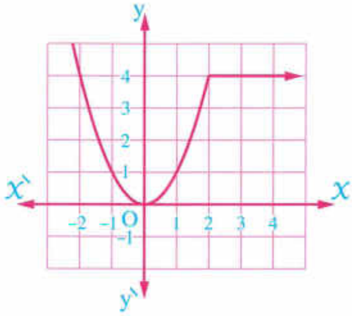


Fig. (1)

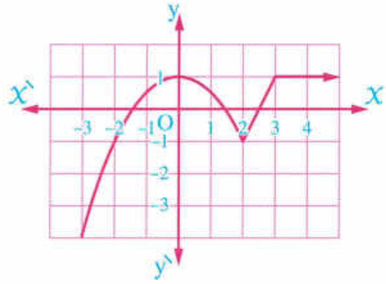


Fig. (2)

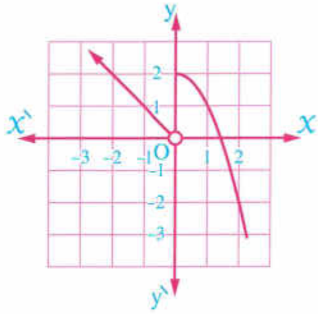


Fig. (3)

Solution

Fig. (1) : The function is decreasing on the interval $]-\infty , 0[$

, increasing on the interval $]0 , 2[$ and constant on the interval $]2 , \infty[$

Fig. (2) : The function is increasing on the interval $]-\infty , 0[$

, decreasing on the interval $]0 , 2[$, increasing on the interval $]2 , 3[$

and constant on the interval $]3 , \infty[$

Fig. (3) : The function is decreasing on each of the two intervals $]-\infty , 0[$ and $]0 , \infty[$

Activity *(Operations on functions)*

If f_1 , f_2 are two functions whose domains are D_1 and D_2 respectively , then :

(1) $(f_1 \pm f_2)(x) = f_1(x) \pm f_2(x)$ and the domain of $(f_1 \pm f_2)$ is $D_1 \cap D_2$

(2) $(f_1 \times f_2)(x) = f_1(x) \times f_2(x)$ and the domain of $(f_1 \times f_2)$ is $D_1 \cap D_2$

(3) $\left(\frac{f_1}{f_2}\right)(x) = \frac{f_1(x)}{f_2(x)}$ such that $f_2(x) \neq$ zero

, the domain of $\left(\frac{f_1}{f_2}\right)$ is $(D_1 \cap D_2) - Z(f_2)$ where $Z(f_2)$ is the set of zeroes of f_2

Noticing that in all the operations on the functions , the domain of the resulting function equals the intersection of the domains of the two functions except the zeroes of the divisor in the division operation.

Example

If $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$ where $f(x) = 2x^2 - 7x + 5$

and $g :]-\infty, 4] \longrightarrow \mathbb{R}$ where $g(x) = 2x - 5$

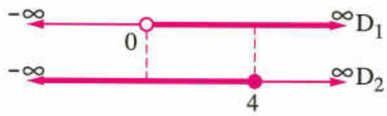
Find : (1) $(f + g)(x)$ | **(2)** $(f - g)(x)$

(3) $(f \times g)(x)$ | **(4)** $\left(\frac{f}{g}\right)(x)$

, then state the domain of each of them and calculate :

$(f + g)(3)$, $(f - g)(0)$, $(f \times g)(-3)$ and $\left(\frac{f}{g}\right)(1)$

Solution



- The domain of $f = D_1 = \mathbb{R}^+$
- The domain of $g = D_2 =]-\infty, 4]$

$\therefore$ The common domain of the two functions $= D_1 \cap D_2 = \mathbb{R}^+ \cap]-\infty, 4] =]0, 4]$

(1) $(f + g)(x) = (2x^2 - 7x + 5) + (2x - 5) = 2x^2 - 5x$ and the domain $=]0, 4]$

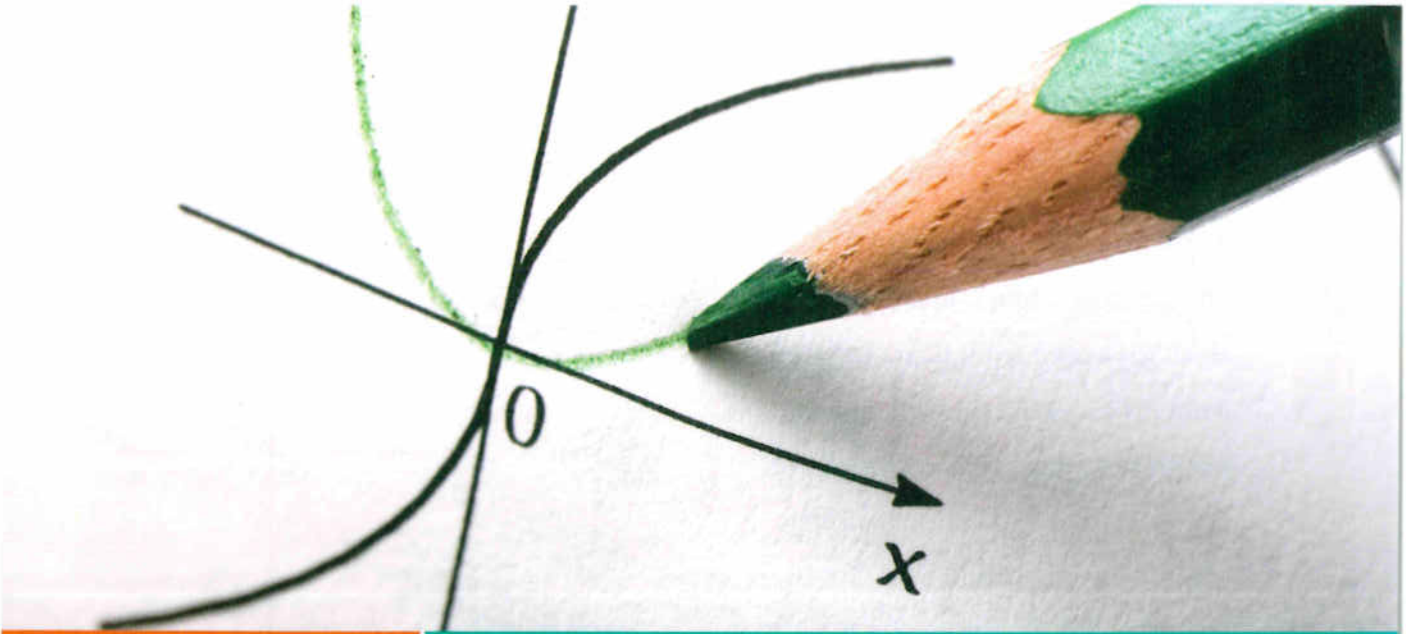
(2) $(f - g)(x) = (2x^2 - 7x + 5) - (2x - 5) = 2x^2 - 9x + 10$ and the domain $=]0, 4]$

(3) $(f \times g)(x) = (2x^2 - 7x + 5)(2x - 5) = 4x^3 - 24x^2 + 45x - 25$ and the domain $=]0, 4]$

(4) $\left(\frac{f}{g}\right)(x) = \frac{2x^2 - 7x + 5}{2x - 5} = \frac{(2x - 5)(x - 1)}{(2x - 5)} = x - 1$ and the domain $=]0, 4] - \left\{\frac{5}{2}\right\}$

The numerical values :

- $(f + g)(3) = 2(9) - 5(3) = 3$
- $(f - g)(0)$ is undefined because $0 \notin]0, 4]$
- $(f \times g)(-3)$ is undefined because $-3 \notin]0, 4]$
- $\left(\frac{f}{g}\right)(1) = 0$

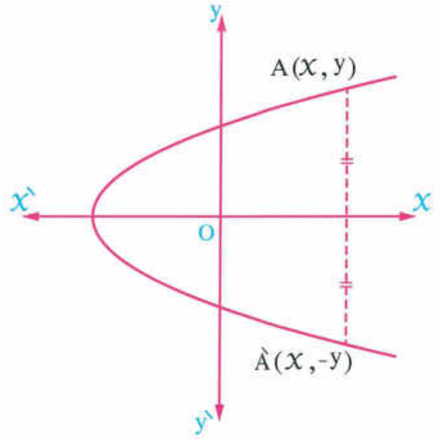


Lesson Two — Even and odd functions

Prelude

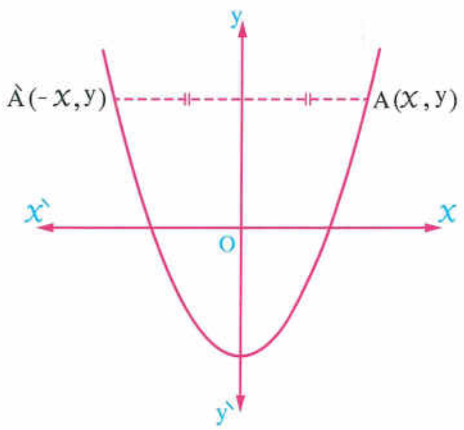
1 Symmetry about x-axis

The graph of a function is symmetric about x-axis if for each point A (x , y) lies on the graph there is a corresponding point Á (x , –y) lies on the same graph where Á is the image of A by reflection in x-axis.



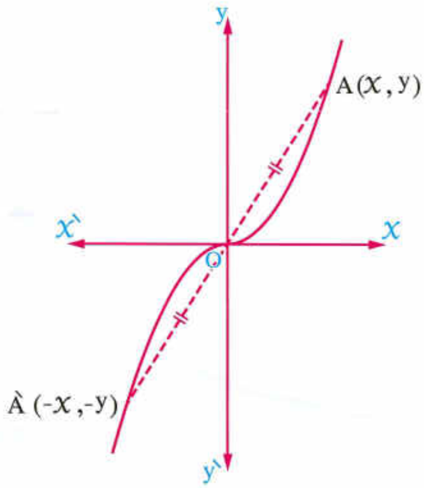
2 Symmetry about y-axis

The graph of a function is symmetric about y-axis if for each point A (x , y) lies on the graph there is a corresponding point Á (– x , y) lies on the same graph where Á is the image of A by reflection in y-axis.



3 Symmetry about the origin point "O"

The graph of a function is symmetric about the origin point (O) if for each point A (x , y) lies on the graph of the function there is a corresponding point À ($-x$, $-$ y) lies on the same graph where À is the image of A by reflection on the origin point (O)



Even function and odd function

- **Even function :** The function f is said to be even if $f(-x) = f(x)$ for each x , $-x \in$ the domain of the function f
  The curve of the even function is symmetric about y-axis.
- **Odd function :** The function f is said to be odd if $f(-x) = -f(x)$ for each x , $-x \in$ the domain of the function f
  The curve of the odd function is symmetric about the origin point.

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Remarks

1. If $f(-x) \neq f(x)$, $f(-x) \neq -f(x)$, then the function f is neither even nor odd.
2. When we investigate whether the function f is even or odd , the two elements x , $-x$ must belong to the domain of the function. If this condition is not satisfied , then the function is neither even nor odd without getting $f(-x)$
3. If the domain of the function is $\mathbb{R} - \{a\}$, $a \neq 0$, then the function is neither odd nor even.
4. If the function is even and its curve passes through (a , b) , then the curve must pass through ($-$ a , b)
5. If the function is odd and its curve passes through (a , b) , then the curve must pass through ($-$ a , $-$ b)
6. The zero function $f : f(x) = 0$ is an even and odd function at the same time.

Example 1

Determine which of the functions defined by the following rules is even , odd or otherwise :

(1) $f(x) = x^2$

(2) $f(x) = 2x^3$

(3) $f(x) = \sqrt{x-1}$

(4) $f(x) = \cos x$

Solution

(1) $\because$ f is polynomial. $\therefore$ The domain of $f = \mathbb{R}$

$\therefore$ For each x , $-x \in \mathbb{R}$, then $f(-x) = (-x)^2 = x^2 = f(x)$

$\therefore$ f is even.

(2) $\because$ f is polynomial. $\therefore$ The domain of $f = \mathbb{R}$

$\therefore$ For each x , $-x \in \mathbb{R}$, then $f(-x) = 2(-x)^3 = 2(-x^3) = -2x^3 = -f(x)$

$\therefore$ f is odd.

(3) $\because$ The domain of f is the set of values of x satisfying

$x - 1 \geq 0$ *i.e.* $x \geq 1$

$\therefore$ The domain of $f = [1, \infty[$

$\therefore$ For each $x \in [1, \infty[$

there is not $-x \in [1, \infty[$

$\therefore$ f is neither even nor odd.

Notice that :

$3 \in [1, \infty[$

while $-3 \notin [1, \infty[$

(4) $\because$ The domain of $f : f(x) = \cos x$ is $\mathbb{R}$

$\therefore$ For each x , $-x \in \mathbb{R}$, then

$f(-x) = \cos(-x) = \cos x = f(x)$

$\therefore$ f is even.

Remember that

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

Remarks

1. The function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = a x^n$ where $a \neq 0$, $n \in \mathbb{Z}^+$ is called the power function , and it is :

* Even when n is an even number.

* Odd when n is an odd number.

2. $f(x) = \cos x$, $f(x) = \sec x$ are even functions

but $f(x) = \sin x$, $f(x) = \csc x$, $f(x) = \tan x$ and $f(x) = \cot x$ are odd functions.

Example 2

If the function f is an even function where $f(X) = a X^2 + b X + 5$ and the curve of the function passes through the point (1 , 6) find the value of each of a and b

Solution

$\because$ The function is even and passes through (1 , 6)

$\therefore$ The curve passes through (– 1 , 6)

At the point (1 , 6) : $\therefore 6 = a + b + 5$ (1)

At the point (– 1 , 6) : $\therefore 6 = a - b + 5$ (2)

By adding (1) , (2) : $\therefore 12 = 2a + 10$ $\therefore 2a = 2$ $\therefore a = 1$

By substituting in (1) : $\therefore 6 = 1 + b + 5$ $\therefore b = \text{zero}$

Important properties

If each of f_1 , f_2 is an even function , and each of g_1 , g_2 is an odd function , then :

(1) $f_1 \pm f_2$ is even.

(2) $g_1 \pm g_2$ is odd.

(3) $f_1 \pm g_1$ is neither even nor odd.

(4) Each of $f_1 \times f_2$ and $\frac{f_1}{f_2}$ is even.

(5) Each of $g_1 \times g_2$ and $\frac{g_1}{g_2}$ is even.

(6) Each of $f_1 \times g_1$ and $\frac{f_1}{g_1}$ is odd.

Example 3

Determine which of the functions defined by the following rules is even , odd or otherwise :

(1) $f(X) = X^2 + \cos X$

(2) $f(X) = X^3 + \sin X$

(3) $f(X) = 3X^4 \tan X$

Solution

(1) $\because f(-X) = (-X)^2 + \cos(-X) = X^2 + \cos X = f(X)$

$\therefore f$ is even.

Another solution :

Let $f(X) = f_1(X) + f_2(X)$ where $f_1(X) = X^2$, $f_2(X) = \cos X$

$\therefore f_1(-X) = (-X)^2 = X^2 = f_1(X)$ $\therefore f_1$ is even.

, $\because f_2(-X) = \cos(-X) = \cos X = f_2(X)$ $\therefore f_2$ is even.

$\therefore f_1 + f_2$ is even. $\therefore f$ is even.

(2) $\because f(-x) = (-x)^3 + \sin(-x) = -x^3 - \sin x = -(x^3 + \sin x) = -f(x)$

$\therefore f$ is odd.

Note that : The function resulted from adding two odd functions is odd.

(3) $\because f(-x) = 3(-x)^4 \tan(-x) = 3x^4(-\tan x) = -3x^4 \tan x = -f(x)$

$\therefore f$ is odd.

Note that : The function resulted from multiplying an even function by an odd function is odd.

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Example 4

Each of the following graphs represents the curve of the function f , determine from the graph whether the function f is even , odd or otherwise verifying your answer algebraically :

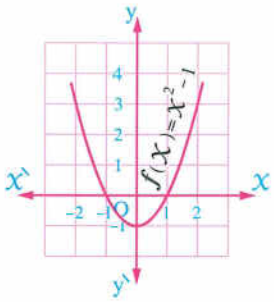


Fig. (1)

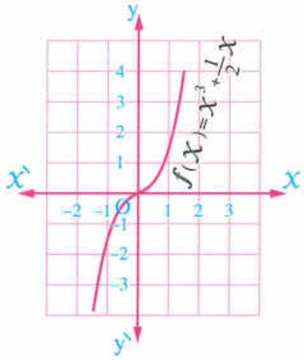


Fig. (2)

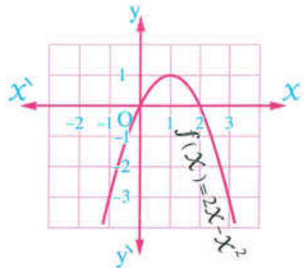


Fig. (3)

Solution

Fig. (1) : $f(x) = x^2 - 1$

$\because$ The domain of the function $f = \mathbb{R}$ and the curve is symmetric about y-axis.

$\therefore f$ is even.

Algebraically satisfaction :

$\because$ For each $x , -x \in \mathbb{R}$, then $f(-x) = (-x)^2 - 1 = x^2 - 1 = f(x)$ $\quad \therefore f$ is even.

Fig. (2) : $f(x) = x^3 + \frac{1}{2}x$

$\because$ The domain of the function $f = \mathbb{R}$ and the curve is symmetric about origin point O

$\therefore f$ is odd.

Algebraically satisfaction :

$\because$ For each $x , -x \in \mathbb{R}$

, then $f(-x) = (-x)^3 + \frac{1}{2}(-x) = -x^3 - \frac{1}{2}x = -\left(x^3 + \frac{1}{2}x\right) = -f(x)$ $\quad \therefore f$ is odd.

Fig. (3) : $f(x) = 2x - x^2$

$\because$ The domain of the function $f = \mathbb{R}$ and the curve is neither symmetric about y-axis nor about the origin point .

$\therefore f$ is neither even nor odd.

Algebraically satisfaction :

$\because$ For each x , $-x \in \mathbb{R}$, then $f(-x) = 2(-x) - (-x)^2 = -2x - x^2 = -(2x + x^2)$

$\therefore f(-x) \neq f(x)$, $f(-x) \neq -f(x)$ $\quad \therefore f$ is neither even nor odd.

Example 5

Determine which of the functions defined by the following rules is even , odd or otherwise :

(1) $f(x) = 3x^4 - 5x^2 + 1$

(2) $f(x) = x^3 + 2x - 5$

(3) $f(x) = \frac{x - \sin 3x}{1 + x^2}$

(4) $f(x) = \frac{x - \tan x}{x^3 + x}$

Solution

(1) $\because f(-x) = 3(-x)^4 - 5(-x)^2 + 1 = 3x^4 - 5x^2 + 1 = f(x)$

$\therefore f$ is even.

(2) $\because f(-x) = (-x)^3 + 2(-x) - 5 = -x^3 - 2x - 5 = -(x^3 + 2x + 5)$

$\therefore f(-x) \neq f(x)$, $f(-x) \neq -f(x)$

$\therefore f$ is neither even nor odd.

(3) $\because f(-x) = \frac{(-x) - \sin 3(-x)}{1 + (-x)^2} = \frac{(-x) - (-\sin 3x)}{1 + x^2} = \frac{-(x - \sin 3x)}{1 + x^2} = -f(x)$

$\therefore f$ is odd.

(4) $\because f(-x) = \frac{(-x) - \tan(-x)}{(-x)^3 + (-x)}$

$$= \frac{-x - (-\tan x)}{-x^3 - x} = \frac{-x + \tan x}{-x^3 - x} = \frac{-(x - \tan x)}{-(x^3 + x)} = \frac{x - \tan x}{x^3 + x} = f(x)$$

$\therefore f$ is even.



Lesson Three — Graphical representation of basic functions and graphing piecewise functions

Representing the linear function

* The linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ is represented graphically by a straight line passes through the point (0 , b) and its slope = a

Example 1

Represent graphically the function f in each of the following and deduce from the graph the range of the function :

(1) $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = -\frac{1}{3}X$

(2) $f : [-1 , 2[\longrightarrow \mathbb{R}$, $f(X) = 2X - 1$

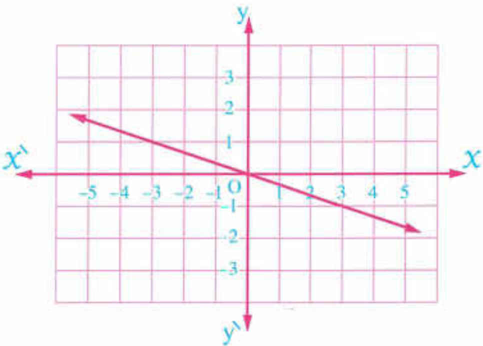
(3) $f :]-\infty , 1[\longrightarrow \mathbb{R}$, $f(X) = 2X - 1$

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Solution

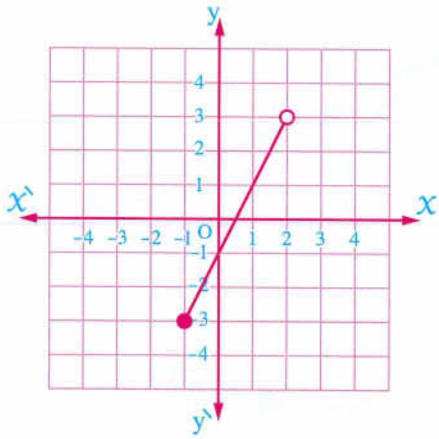
(1) $\because$ The domain = $\mathbb{R}$

$\therefore$ The function is represented by a straight line passes through the point (0 , 0) and its slope = $-\frac{1}{3}$



(2) $\because$ The domain $= [-1 , 2[$

x	-1	0	2
$f(x)$	-3	-1	3

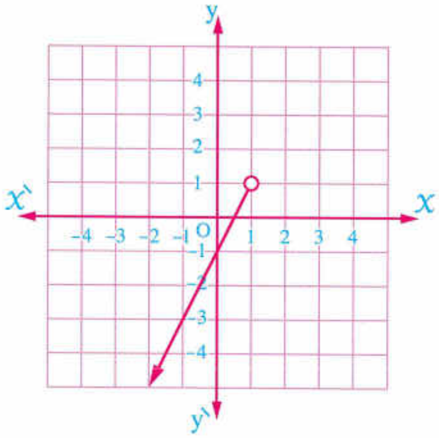


Notice that the point $(2 , 3)$ $\notin$ the function so it is excluded from the graph by drawing unshaded circle at this point.

From the graph : The range $= [-3 , 3[$

(3) $\because$ The domain $=]-\infty , 1[$

x	1	0	-1
$f(x)$	1	-1	-3



Notice that the point $(1 , 1)$ $\notin$ the function so it is excluded from the graph by drawing unshaded circle at this point.

From the graph : The range $=]-\infty , 1[$

Example 2

Represent graphically the function $f : \mathbb{R} - \{0\} \longrightarrow \mathbb{R}$, $f(x) = \frac{x^2 - x}{x}$, from the graph deduce the range of the function.

Solution

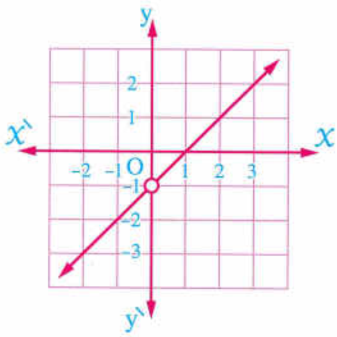
$\because$ Domain of the function $f = \mathbb{R} - \{0\}$

, $f(x) = \frac{x^2 - x}{x} = \frac{x(x-1)}{x} = x - 1$

, represented by a straight line

x	-1	0	1
$f(x)$	-2	-1	0

$\therefore$ The range $= \mathbb{R} - \{-1\}$



Notice that :

The unshaded circle at the point whose x-coordinate $= 0$ because it does not belong to the domain.

Graphing the piecewise-defined function

Example 3

Graph the function $f : f(X) = \begin{cases} 2 - X & , -1 \le X < 2 \\ X - 2 & , 2 \le X < 5 \end{cases}$, then from the graph :

(1) Determine the domain and the range of f (2) Discuss the monotonicity of f

(3) Determine whether f is even , odd or otherwise , giving reason.

Solution

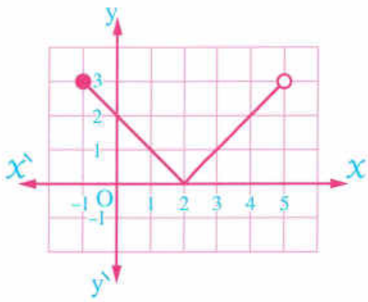
The function f is defined by two rules

- $f_1(X) = 2 - X$, $X \in [-1 , 2[$

X	-1	0	(2)
$f_1(X)$	3	2	(0)

- $f_2(X) = X - 2$, $X \in [2 , 5[$

X	2	3	(5)
$f_2(X)$	0	1	(3)



(1) The domain of $f = [-1 , 2[\cup [2 , 5[= [-1 , 5[$

, the range of $f = [0 , 3]$

(2) The function f is decreasing on $]-1 , 2[$

and increasing on $]2 , 5[$

(3) The function f is neither even nor odd

because it is not symmetric about y-axis

nor the origin point O

Notice that :

$2 \notin [-1 , 2[$, while $2 \in [2 , 5[$

so $(2 , 5) \in f$

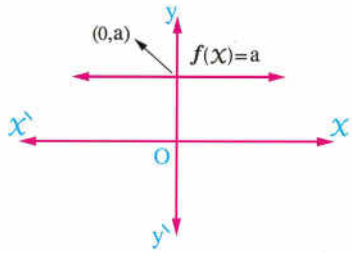
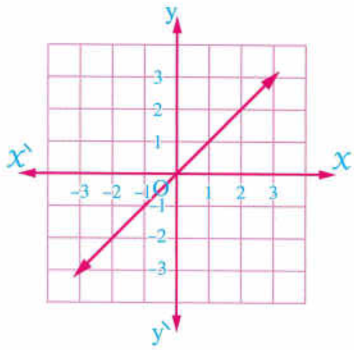
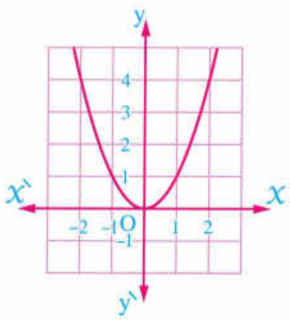
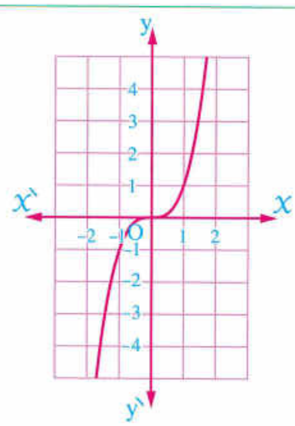
i.e. We don't put unshaded circle on the point (2 , 5) in the graph.

The basic forms of some functions

Now we will recognize the graph of simple forms (basic forms), (standard forms) for the real functions and this is preface to use it in representing the real functions in their different forms next lesson.

UNIT 1

1 The simplest forms of some polynomial functions

	The constant function	The first degree (linear) function
The simplest form	$f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(X) = a$ where $a \in \mathbb{R}$	$f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(X) = X$
The graph	 • A straight line parallel to X-axis and intersects y-axis at the point (0 , a)	 • A straight line passes through the origin point , its slope = 1
The range, monotony and some properties	• Range of the function = $\{a\}$ • The function is constant on its domain. • The function is even (symmetric about y-axis)	• Range of the function = $\mathbb{R}$ • The function is increasing on its domain $\mathbb{R}$ • The function is odd (symmetric about the origin point)
	The second degree (quadratic) function	**The third degree (cube) function**
The simplest form	$f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(X) = X^2$	$f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(X) = X^3$
The graph		

The range, monotony and some properties	• Range of the function = $[0, \infty[$ • The function is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$ • The function is even (symmetric about y-axis)	• Range of the function = $\mathbb{R}$ • The function is increasing on its domain $\mathbb{R}$ • The function is odd (symmetric about the origin point)

2 The basic form of the absolute function

• **Simplest form**

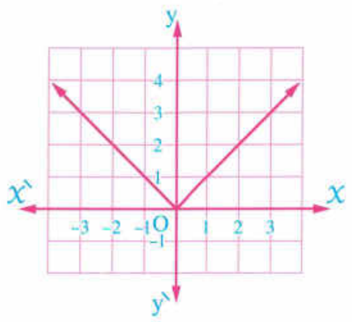
$f: \mathbb{R} \longrightarrow \mathbb{R}, f(x) = |x|$ and

it is redefined as follows :

$$f(x) = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

It is represented graphically by two rays their start points is the origin point (0 , 0) and the slope of the straight line which carries one of the two rays = 1 and the slope of the other carrier straight line = – 1

The graph



• **Range , monotony and some properties :**

* The range of the function = $[0, \infty[$
* The function is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$
* The function is even (symmetric about y-axis)

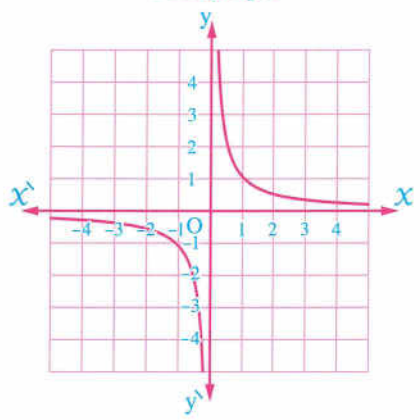
3 The basic form of the rational function

• **Simplest form**

$f: \mathbb{R} - \{0\} \longrightarrow \mathbb{R}, f(x) = \frac{1}{x}$

"approaching each of the two parts of the curve to the two axes without intersection with them , then the two axes $\overleftrightarrow{xx'}$ and $\overleftrightarrow{yy'}$ are called asymptotical lines of the curve"

The graph



• **Range , monotony and some properties :**

* Range of the function = $\mathbb{R} - \{0\}$
* The function is decreasing on $]-\infty, 0[$ and decreasing on $]0, \infty[$
* The function is odd (symmetric about the origin point)

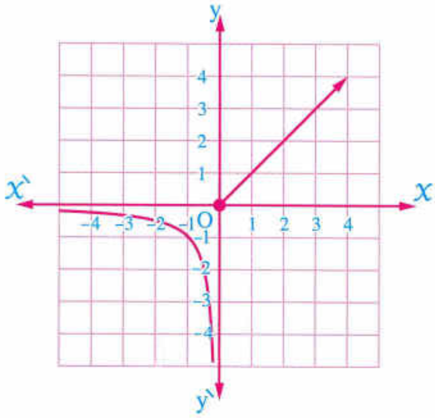
Example 4

Graph each of the functions which are defined by the following rules and from the graph find the domain , the range of the function and deduce its monotony and state whether the function is even , odd or otherwise :

(1) $f(x) = \begin{cases} \frac{1}{x} & , x < 0 \\ |x| & , x \geq 0 \end{cases}$

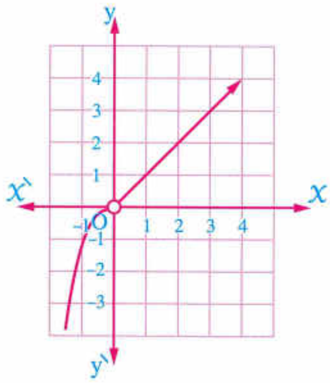
(2) $f(x) = \begin{cases} x^3 & , x < 0 \\ x & , x > 0 \end{cases}$

Solution



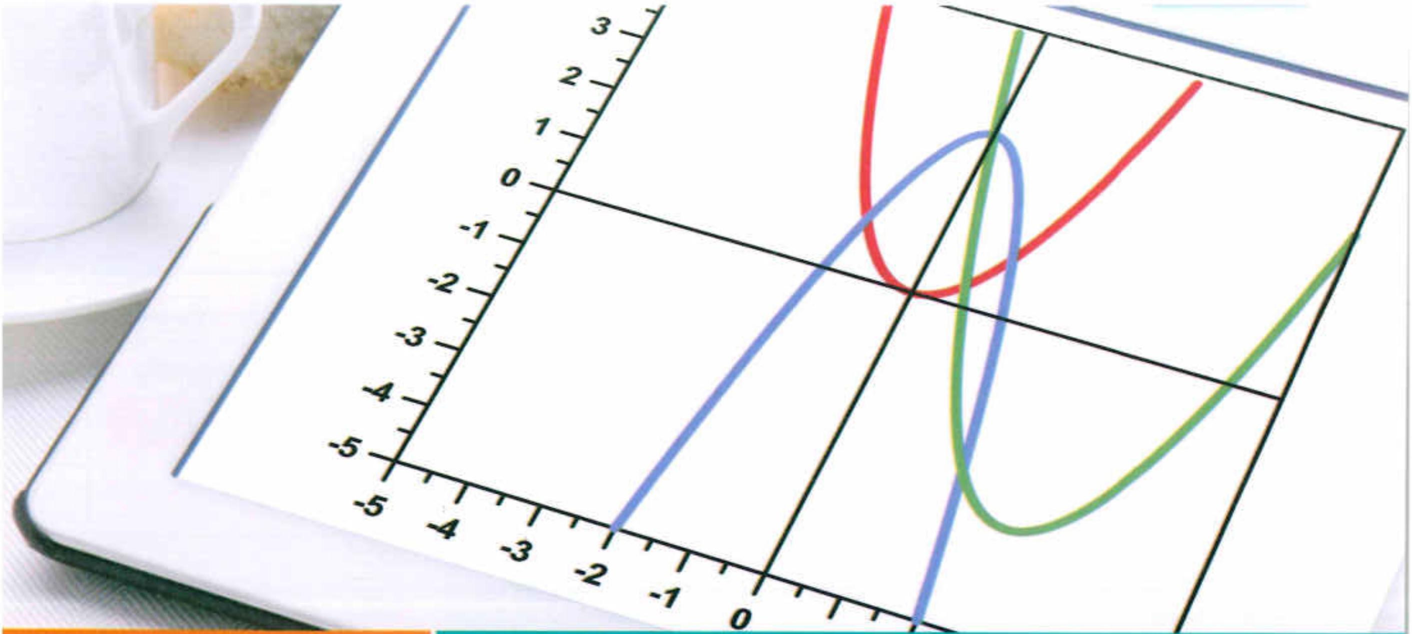
(1) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is decreasing on $]-\infty , 0[$ and is increasing on $]0 , \infty[$
* The function is neither odd nor even.



(2) * The domain = $\mathbb{R} - \{0\}$

* The range = $\mathbb{R} - \{0\}$
* The function is increasing on its domain.
* The function is neither odd nor even.



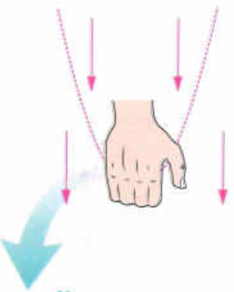
Lesson Four — Geometrical transformations of basic function curves

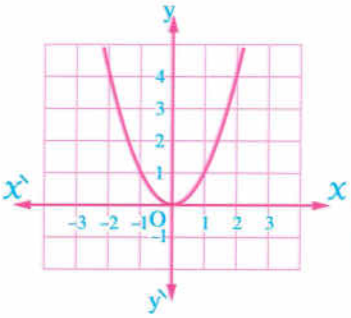
First — Vertical translation of the function curve

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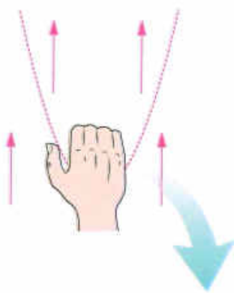
The simplest form of the function

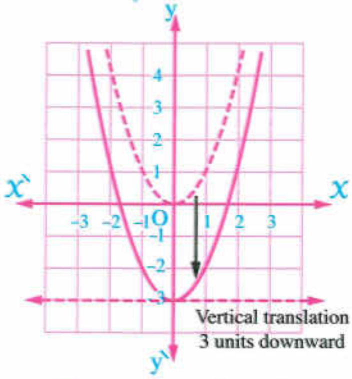
$y = x^2 - 3$ ← vertical translation 3 units downward ← $y = x^2$ → vertical translation 2 units upward → $y = x^2 + 2$



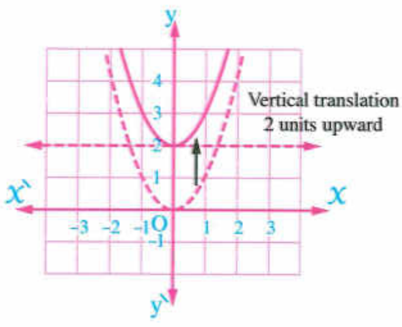


The vertex $(0, 0)$





The vertex $(0, -3)$



The vertex $(0, 2)$

UNIT
1

In general

For any function f , the curve of $y = f(x) + a$, $a \in \mathbb{R} - \{0\}$

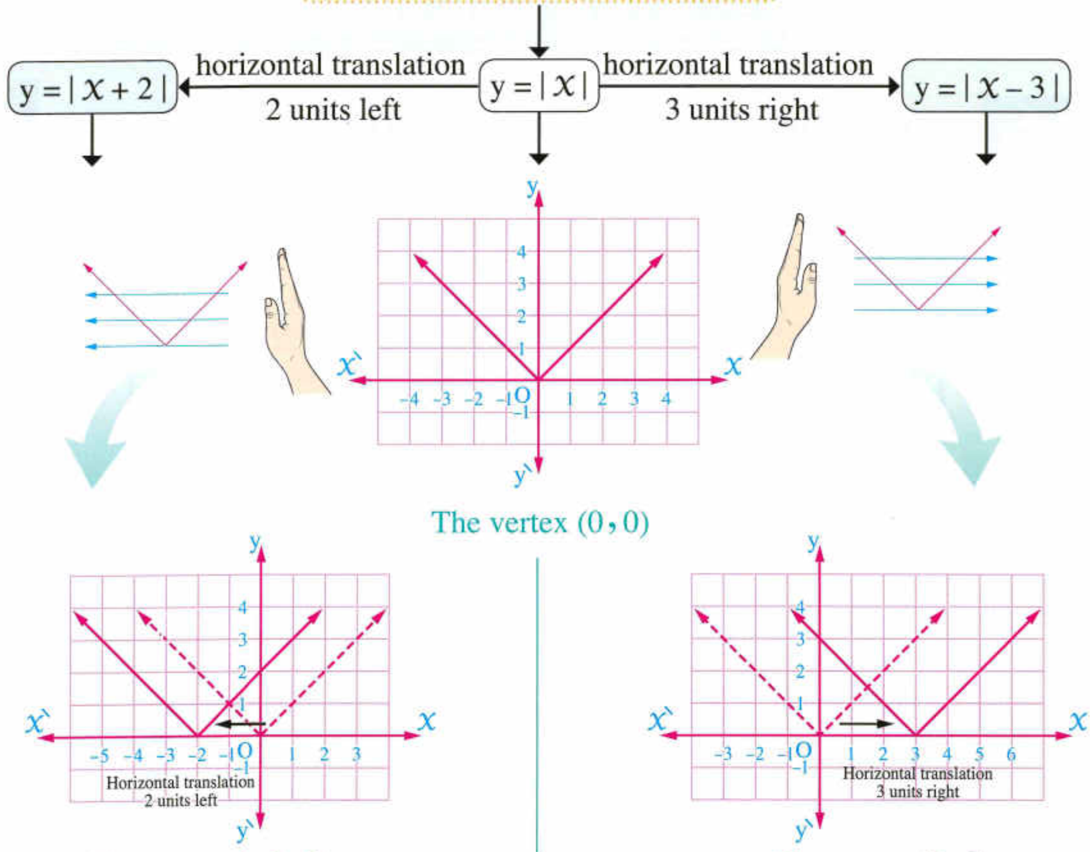
is the same curve of $y = f(x)$ by a vertical translation

, its value is $|a|$ length unit in the direction : $\begin{cases} \overrightarrow{Oy} & (\textit{i.e.}\text{ Upward}) \quad \text{at } a > 0 \\ \overrightarrow{Oy'} & (\textit{i.e.}\text{ Downward}) \quad \text{at } a < 0 \end{cases}$

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Second Horizontal translation of the function curve

The simplest form of the function



The vertex (− 2 , 0)

The vertex (3 , 0)

In general

For any function f , the curve of $y = f(x + a)$, $a \in \mathbb{R} - \{0\}$

is the same curve of $y = f(x)$ by a horizontal translation

, its value is $|a|$ length unit in the direction : $\begin{cases} \overrightarrow{Ox} & (\textit{i.e.}\text{ To the right}) \quad \text{at } a < 0 \\ \overrightarrow{Ox'} & (\textit{i.e.}\text{ To the left}) \quad \text{at } a > 0 \end{cases}$

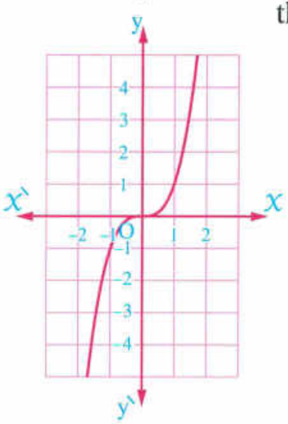
Third Horizontal translation followed by vertical translation of the function curve

The simplest form of the function

$y = x^3$

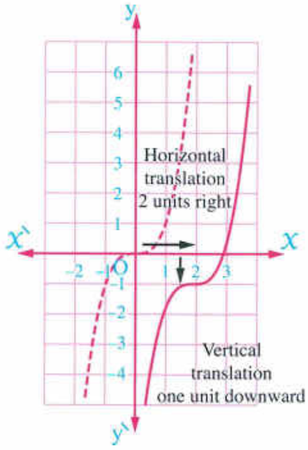
horizontal translation two units to the right followed by vertical translation one unit downward

horizontal translation two units to the left followed by vertical translation three units upward



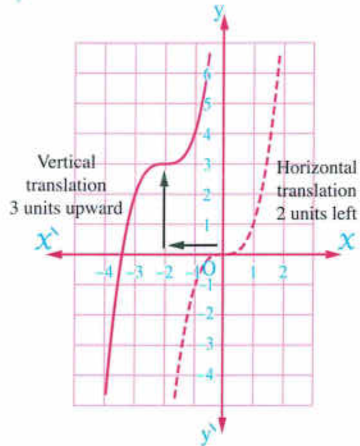
Symmetric point (0 , 0)

$y = (x - 2)^3 - 1$



Symmetric point (2 , – 1)

$y = (x + 2)^3 + 3$



Symmetric point (– 2 , 3)

In general

For any function f , the curve of $y = f(x + a) + b$ where a , $b \in \mathbb{R} - \{0\}$ is the same curve of $y = f(x)$ by a horizontal translation , its value $|a|$ length unit in the direction $\overrightarrow{Ox}$ if $a < 0$ or in the direction $\overrightarrow{Ox'}$ if $a > 0$, then a vertical translation , its value is $|b|$ length unit in the direction $\overrightarrow{Oy}$ if $b > 0$ or in the direction $\overrightarrow{Oy'}$ if $b < 0$

Example 1

Use the curves of the basic functions to graph the curves of the functions which are defined by the following rules , then from the graph determine the domain and the range of each function and discuss its monotony and state whether the function is even , odd or otherwise :

(1) $g(x) = |x - 2| - 1$ (2) $g(x) = (2 - x)^2 + 1$

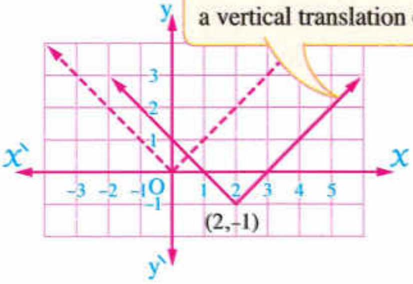
Solution

(1) • The domain of g = $\mathbb{R}$, the range of g = $[-1 , \infty[$

• The function g is decreasing on $]-\infty , 2[$ and is increasing on $]2 , \infty[$

• The function g is neither even nor odd.

The curve of the function g is the same curve of the function $f : f(x) = |x|$ by a horizontal translation two units in the direction $\overrightarrow{Ox}$, then a vertical translation one unit in the direction $\overrightarrow{Oy}$



(2) $\because (2-x)^2 = (x-2)^2$

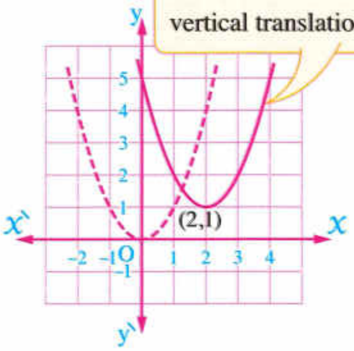
$\therefore g(x) = (x-2)^2 + 1$

• The domain of g = $\mathbb{R}$, the range of g = $[1 , \infty[$

• The function g is decreasing on $]-\infty , 2[$ and is increasing on $]2 , \infty[$

• The function g is neither even nor odd.

The curve of the function g is the same curve of the function $f : f(x) = x^2$ by a horizontal translation 2 units in the direction $\overrightarrow{Ox}$, then a vertical translation one unit in the direction $\overrightarrow{Oy}$



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Example 2

Use the curve of the function $f : f(x) = \frac{1}{x}$ to represent the functions g , h and k where :

(1) $g(x) = \frac{1}{x-2} + 1$ **(2)** $h(x) = \frac{1}{x} + 3$ **(3)** $k(x) = \frac{2x-1}{x-1}$

From the graph , determine the domain and the range of each function , then discuss its monotony.

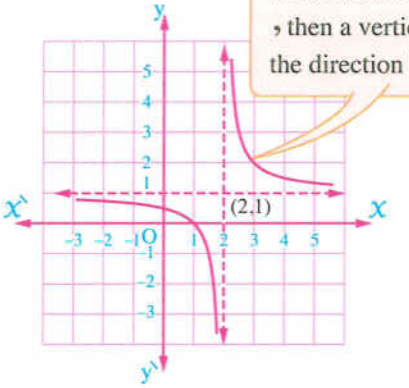
Solution

(1) • The domain of $g = \mathbb{R} - \{2\}$

• The range of $g = \mathbb{R} - \{1\}$

• The function is decreasing on $]-\infty , 2[$ and also decreasing on $]2 , \infty[$

The curve of the function g is the same curve of the function f by a horizontal translation 2 units in the direction $\overrightarrow{OX}$, then a vertical translation 1 unit in the direction $\overrightarrow{Oy}$

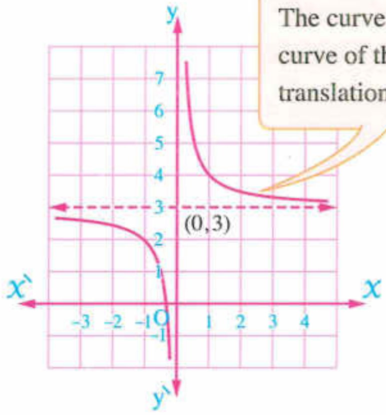


(2) • The domain of $h = \mathbb{R} - \{0\}$

• The range of $h = \mathbb{R} - \{3\}$

• The function is decreasing on $]-\infty , 0[$ and also decreasing on $]0 , \infty[$

The curve of the function h is the same curve of the function f by a vertical translation 3 units in the direction $\overrightarrow{Oy}$



(3) $k(X) = \dfrac{2X - 1}{X - 1} = \dfrac{2X - 2 + 1}{X - 1}$

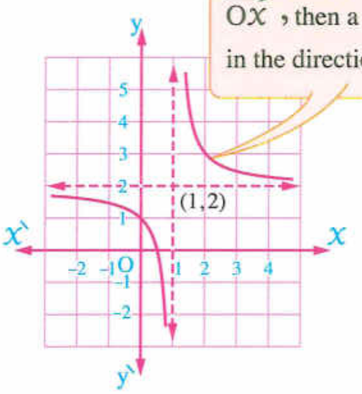
$= \dfrac{2\ (X - 1) + 1}{X - 1} = 2 + \dfrac{1}{X - 1}$

• The domain of $k = \mathbb{R} - \{1\}$

• The range of $k = \mathbb{R} - \{2\}$

• The function is decreasing on $]-\infty , 1[$ and also decreasing on $]1 , \infty[$

The curve of the function k is the same curve of the function f by a horizontal translation one unit in the direction $\overrightarrow{OX}$, then a vertical translation 2 units in the direction $\overrightarrow{Oy}$

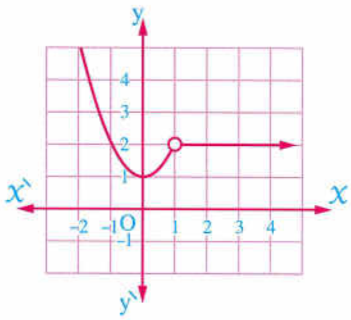


Example 3

Graph the function $f : f(x) = \begin{cases} x^2 + 1 & , \quad x < 1 \\ 2 & , \quad x > 1 \end{cases}$ and from the graph, find the domain, the range of the function and deduce its monotony and state whether the function is even, odd or otherwise :

Solution

- The domain $= \mathbb{R} - \{1\}$
- The range $= [1 , \infty[$
- The function is decreasing on $]-\infty , 0[$ and increasing on $]0 , 1[$, constant on $]1 , \infty[$
- The function is neither odd nor even.



Example 4

If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^3$, graph the function g where $g(x) = f(x + 1) - 2$, from the graph, determine the domain of g and its range, then discuss its monotony and state whether the function g is even, odd or otherwise.

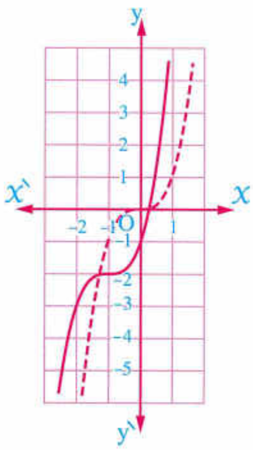
Solution

$\because g(x) = f(x + 1) - 2$

$\therefore g(x) = (x + 1)^3 - 2$

$\therefore$ The curve of the function g is the same curve of the function f by a horizontal translation one unit in the direction $\overrightarrow{Ox'}$, then a vertical translation 2 units in the direction $\overrightarrow{Oy'}$

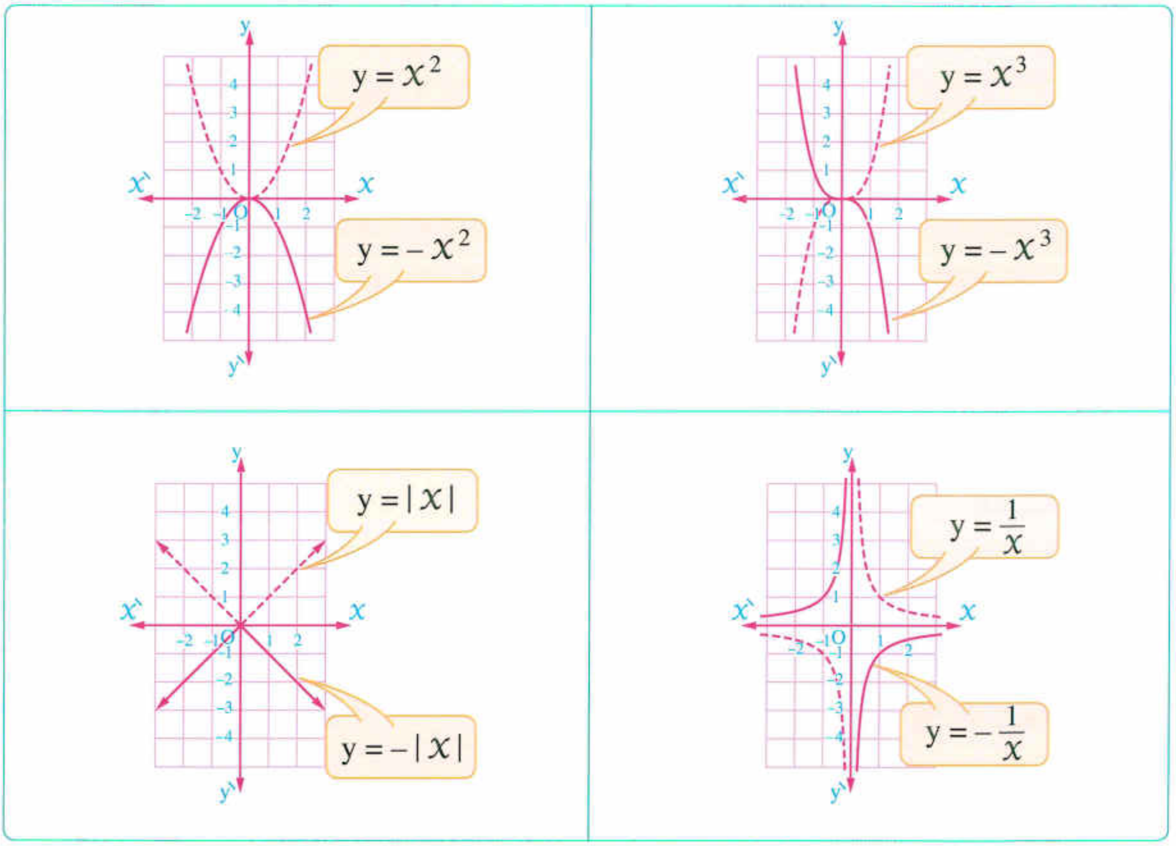
- The domain of g $= \mathbb{R}$, the range of g $= \mathbb{R}$
- The function g is increasing on its domain $\mathbb{R}$
- The function g is neither even nor odd.



Fourth Reflection of the function curve in x-axis

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For any function f , the curve of $y = -f(x)$ is the same curve $y = f(x)$ by reflection in x-axis



Important remark

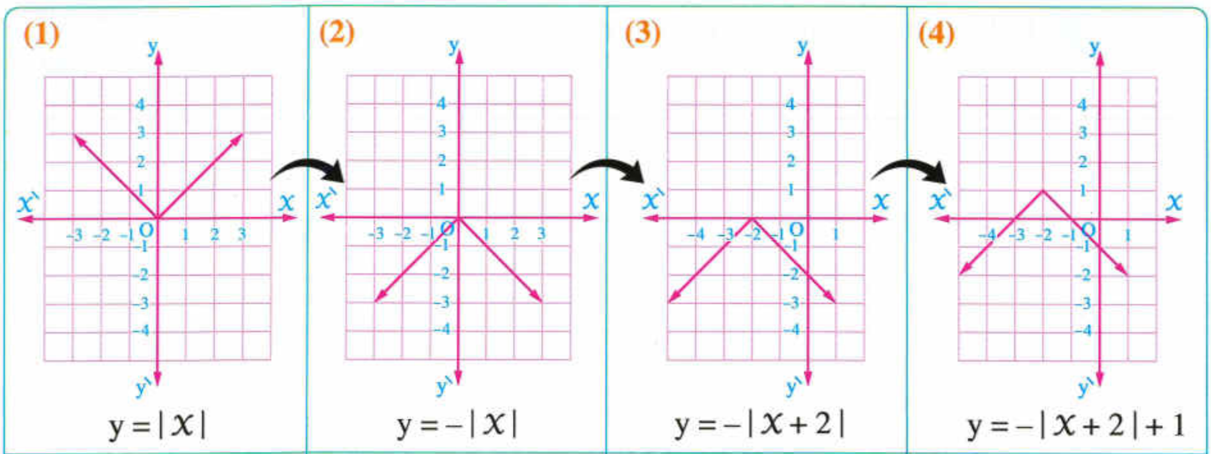
It is necessary that ordering the performing of transformations on the curve $y = f(x)$ to get from it the curve $y = -f(x + a) + b$ as follows :

1. Reflection in x-axis.
2. Horizontal translation.
3. Vertical translation.

If we reverse the order of performing the vertical translation before performing the reflection in x-axis , then we get another curve not the required curve.

For example :

From the curve of the simplest form of the function $y = |x|$ we can get the curve of the function $y = -|x + 2| + 1$ as follows :



The simplest form of the function	then	Reflect in x-axis	then	Horizontal translation 2 units left	then	Vertical translation 1 unit upward

Example 5

Using the curves of the basic functions , graph the curves of the functions g , k and z where :

(1) $g(x) = -(x-2)^3$

(2) $k(x) = \frac{1}{2-x} + 3$

(3) $z(x) = 4x - x^2 - 3$

From the graph , determine the range of each function , discuss its monotony and its symmetry , and state whether the function is even , odd or otherwise.

Solution

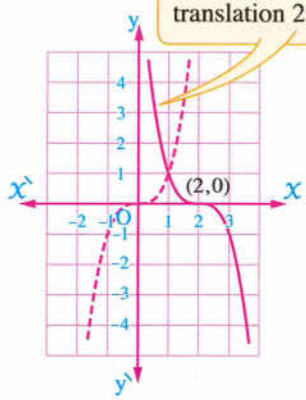
(1) • The range of g = $\mathbb{R}$

• The function g is decreasing on its domain $\mathbb{R}$

• The function g is symmetric about the point (2 , 0)

• The function g is neither even nor odd.

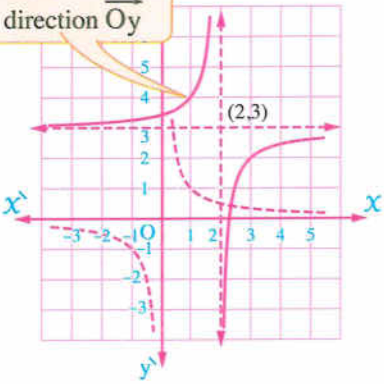
The curve of the function g is the same curve of the function $f : f(x) = x^3$ by reflection in x-axis , then a horizontal translation 2 units in the direction $\overrightarrow{Ox}$



(2) $k(X) = \frac{1}{-X+2} + 3 = \frac{1}{-(X-2)} + 3$

$= \frac{-1}{X-2} + 3$

The curve of the function k is the same curve of the function $f : f(X) = \frac{1}{X}$ by reflection in X-axis followed by a horizontal translation 2 units in the direction $\overrightarrow{OX}$, then a vertical translation 3 units in the direction $\overrightarrow{Oy}$



- The range of $k = \mathbb{R} - \{3\}$
- The function k is increasing on the interval $]-\infty , 2[$ and also is increasing on the interval $]2 , \infty[$
- The function k is symmetric about the point (2 , 3)
- The function k is neither even nor odd.

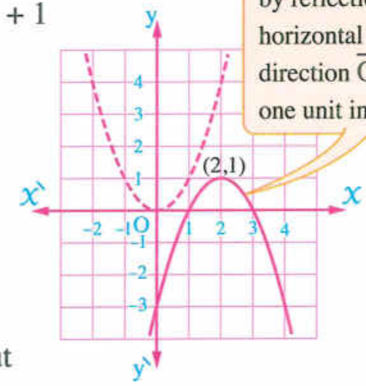
(3) $z(X) = -X^2 + 4X - 3 = -(X^2 - 4X + 3)$

$= -(X^2 - 4X + 4 - 1)$

$= -[(X-2)^2 - 1]$

$= -(X-2)^2 + 1$

The curve of the function z is the same curve of the function $f : f(X) = X^2$ by reflection in X-axis followed by a horizontal translation two units in the direction $\overrightarrow{OX}$, then a vertical translation one unit in the direction $\overrightarrow{Oy}$



- The range of $z =]-\infty , 1]$
- The function z is increasing on the interval $]-\infty , 2[$ and is decreasing on the interval $]2 , \infty[$
- The function z is symmetric about the line $X = 2$
- The function z is neither even nor odd.

Notice that :

The vertex of the curve of the function z is (2 , 1) we can get it from the law :

The vertex of the curve $= \left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right)\right)$

for the functions whose rules are in the form :

$f(X) = aX^2 + bX + c$

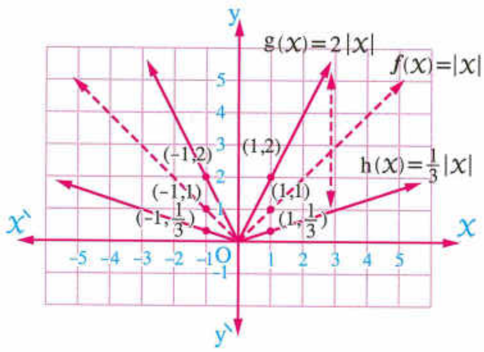
Fifth Stretching of the function curve

For any function f , the curve of $y = a f(x)$ where $a \in \mathbb{R}^*$

- Vertical stretch for the curve $y = f(x)$ if $a > 1$
- Vertical shrinking for the curve $y = f(x)$ if $0 < a < 1$

For example :

In the opposite figure :



- The curve of the function g :
  $g(x) = 2|x|$ is vertical stretch for the curve of the function $f : f(x) = |x|$
  because : $a > 1$
  i.e. For each $(x , y) \in f$, then $(x , 2y) \in g$
- The curve of the function h :
  $h(x) = \frac{1}{3}|x|$ is vertical shrinking for the curve of the function $f : f(x) = |x|$
  because : $0 < a < 1$
  i.e. For each $(x , y) \in f$, then $\left(x , \frac{1}{3} y\right) \in h$

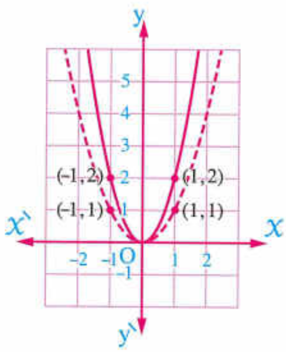
Example 6

Use the curve of the function $f : f(x) = x^2$ to represent each of the following curves :

(1) $g(x) = 2x^2$ (2) $h(x) = -\frac{1}{2} x^2$ (3) $k(x) = 2(x-1)^2 - 3$

From the graph , determine the range of each one , discuss its monotony and state whether the function is even , odd or otherwise.

Solution



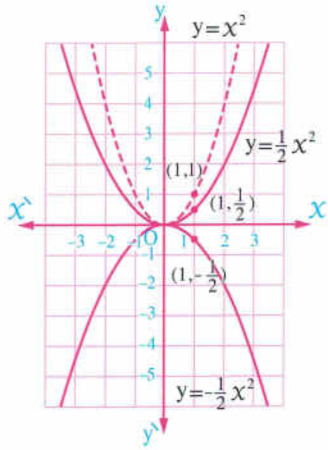
(1) The curve of the function g is vertical stretch for the curve of the function f where $a = 2 > 1$

i.e. For each $(x , y) \in f$, then $(x , 2y) \in g$

- Range of g = $[0 , \infty[$
- The function g is decreasing on $]-\infty , 0[$ and is increasing on $]0 , \infty[$
- The function g is even.

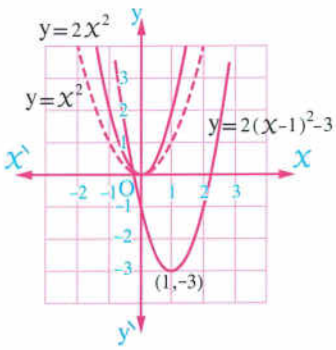
(2) The curve of the function h is vertical shrinking for the curve of the function f where $a = \frac{1}{2} < 1$, then reflection in X-axis

i.e. For each $(X , y) \in f$, then $\left(X , -\frac{1}{2} y\right) \in h$



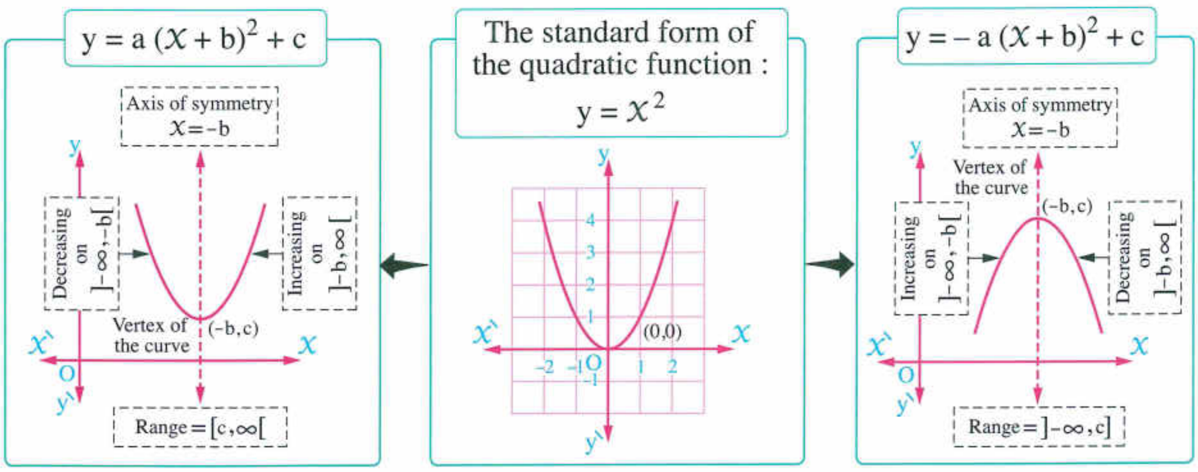
- Range of h = $]-\infty , 0]$
- The function h is increasing on $]-\infty , 0[$ and is decreasing on $]0 , \infty[$
- The function h is even.

(3) The curve of the function k is vertical stretch for the curve of the function f where $a = 2 > 1$, then a horizontal translation one unit in the direction $\overrightarrow{OX}$ followed by a vertical translation three units in the direction $\overrightarrow{Oy}$



- Range of k = $[-3 , \infty[$
- The function k is decreasing on $]-\infty , 1[$ and is increasing on $]1 , \infty[$
- The function k is neither even nor odd.

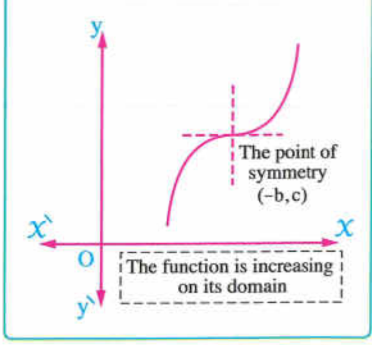
Generally



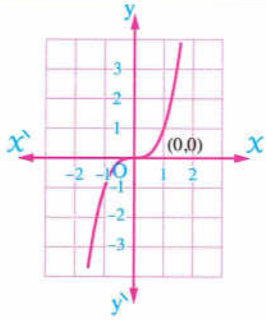
Domain = $\mathbb{R}$, the function is neither even nor odd except if b = 0 , then it is even.

UNIT 1

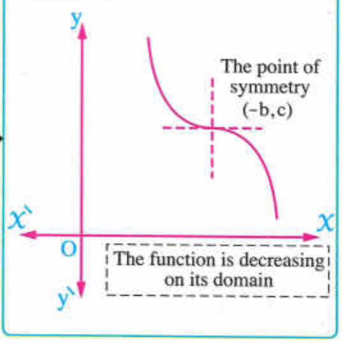
$y = a(x+b)^3 + c$



The standard form of the cubic function : $y = x^3$

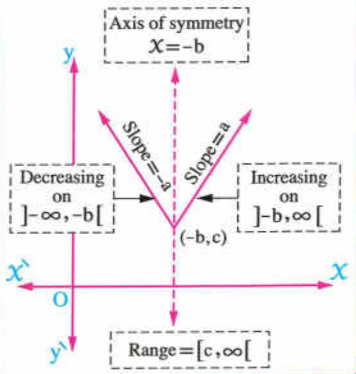


$y = -a(x+b)^3 + c$

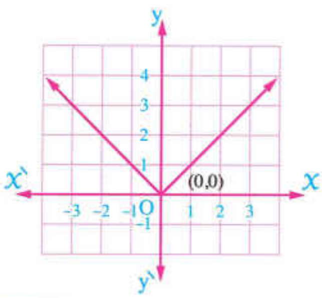


Domain = $\mathbb{R}$, range = $\mathbb{R}$, the function is neither even nor odd except if b = 0 , c = 0 , then it is odd.

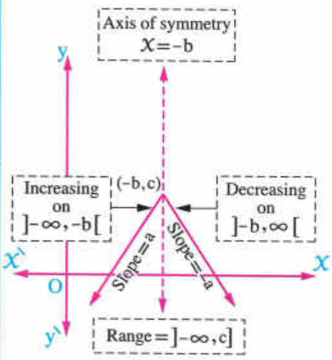
$y = a|x+b| + c$



The standard form of the absolute value function : $y = |x|$

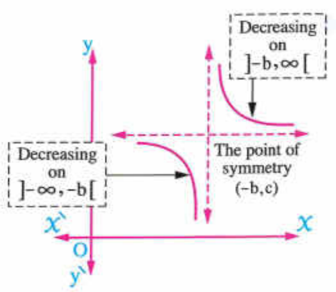


$y = -a|x+b| + c$

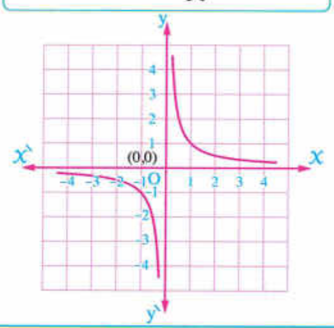


Domain = $\mathbb{R}$, the function is neither even nor odd except if b = 0 , then it is even.

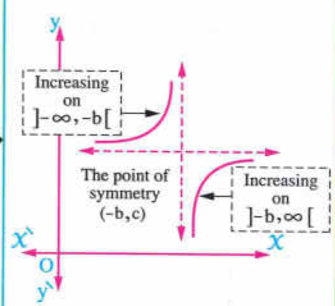
$y = \frac{a}{x+b} + c$



The standard form of the rational function : $y = \frac{1}{x}$



$y = \frac{-a}{x+b} + c$



Domain = $\mathbb{R} - \{-b\}$, range = $\mathbb{R} - \{c\}$, the function is neither even nor odd except if b = 0 , c = 0 , then it is odd.



Lesson Five — Solving absolute value equations

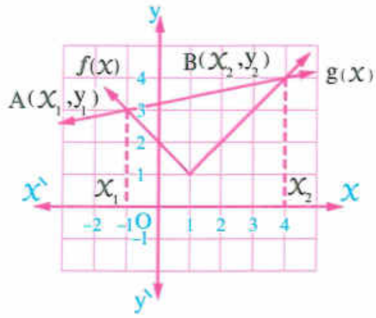
There are two methods for solving absolute value equations :

(1) Graphical method

In this method , we use graphing the real functions in solving equations , noticing that for any two functions f and g the solutions set of the equation $f(X) = g(X)$ is the set of X-coordinates of the intersecting points of the curves of the two functions f and g

In the opposite figure :

If the two curves of the two functions f and g intersecting at the two points A (X_1 , y_1) and B (X_2 , y_2) , then the solution set of the equation $f(X) = g(X)$ in $\mathbb{R}$ is $\{X_1 , X_2\}$



(2) Algebraic method

In this method , we use the definition of the absolute value function and some properties of the absolute value of the real number in solving the equations.

Definition of the absolute value

If X is a real variable , a , b are real numbers , then $|X| = \begin{cases} X & , X \geq 0 \\ -X & , X < 0 \end{cases}$

and so $|X + a| = \begin{cases} X + a & , X \geq -a \\ -X - a & , X < -a \end{cases}$, $|aX + b| = \begin{cases} aX + b & , X \geq \frac{-b}{a} \\ -aX - b & , X < \frac{-b}{a} \end{cases}$

UNIT 1

Properties of the absolute value of the real number

(1) $|a| \geq 0$

(2) $|a\,b| = |a| \times |b|$

(3) $|a + b| \leq |a| + |b|$

i.e. The absolute value of the sum of two numbers is smaller than or equal to the sum of their absolute values and the equality is happened if a , b are negative together , positive together or each of them equals zero.

For example :

i.e. $|4 + (-7)| < |4| + |-7|$

, $|-4 + (-7)| = |-4| + |-7|$

Remarks

1. For any real number a , then : $|a| = |-a|$

For example : $|3| = |-3|$

2. $|a - x| = |x - a|$

For example : $|2 - x| = |x - 2|$

3. $|x| = c \text{ , } c > 0 \Leftrightarrow x = \pm c$

For example : If $|x| = 3$, then : $x = \pm 3$ and if $a = \pm 5$, then : $|a| = 5$

4. If a and b are two real numbers , then : $|a| = |b| \Leftrightarrow a = \pm b$

5. For any real number a , then : $\left(|a|\right)^2 = a^2$

For example : $\left(|-2|\right)^2 = 4$, $\left(\left|-\frac{1}{2}\right|\right)^2 = \frac{1}{4}$

6. For any real number a , then : $\sqrt{a^2} = |a|$

For example : $\sqrt{(5)^2} = |5| = 5$, $\sqrt{(-3)^2} = |-3| = 3$

7. If $|x| = x$, then : $x \in [0 , \infty[$

8. If $|x| = -x$, then : $x \in]-\infty , 0]$

1 Solving the equation in the form : $|a\,x + b| = c$, $c \in [0, \infty[$

"i.e. Absolute of first degree expression = non negative real number"

The algebraic solution

(1) Using the definition

(2) Using the property value inside the absolute sign = ± the real number

The graphical solution

The x-coordinates of the intersection points of the two curves $f(x) = |a\,x + b|$, $g(x) = c$

Remark

If $|a\,x + b| = c$, $c \in]-\infty, 0[$, then the solution set in $\mathbb{R} = \varnothing$

For example the solution set of the equation $|3\,x - 4| = -5$ in $\mathbb{R}$ is $\varnothing$

Example 1

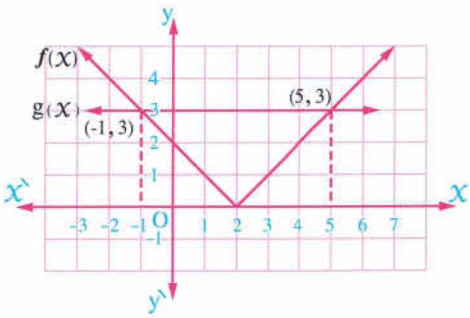
Find graphically , then perform algebraically the solution set in $\mathbb{R}$ for each of the following equations :

(1) $|x - 2| = 3$ (2) $|2\,x + 3| = 2$ (3) $|5 - x| = -1$

Solution

(1) Graphical solution :

Putting $f(x) = |x - 2|$, $g(x) = 3$



- We draw the curve of the function $f : f(x) = |x - 2|$ and it is the same curve of $y = |x|$ with a horizontal translation 2 units in the direction $\overrightarrow{Ox}$
- We draw the curve of the function $g : g(x) = 3$ and it is a constant function represented by a straight line parallel to the x-axis and intersects the y-axis at the point $(0, 3)$
- We find the intersection points of the two curves are $(-1, 3)$ and $(5, 3)$

$\therefore$ The solution set $= \{-1, 5\}$

Algebraic solution :

First : Using the definition of the absolute value function

$$f(x) = |x-2| = \begin{cases} x-2 & , \quad x-2 \geq 0 \\ -x+2 & , \quad x-2 < 0 \end{cases}$$

$$\therefore f(x) = \begin{cases} x-2 & , \quad x \geq 2 \\ -x+2 & , \quad x < 2 \end{cases}$$

At $x \geq 2 : x - 2 = 3$ $\qquad \therefore x = 5 \in [2, \infty[$

At $x < 2 : -x + 2 = 3$ $\qquad \therefore x = -1 \in]-\infty, 2[$

$\therefore$ The solution set $= \{-1, 5\}$

Second : Using the property "what inside the absolute sign = ± the real number"

We can summarize the steps of algebraic solution as the following :

$\because |x-2| = 3 \qquad \therefore x - 2 = \pm 3$

$\therefore x - 2 = 3 \qquad$ *i.e.* $x = 5$

or $\quad x - 2 = -3 \qquad$ *i.e.* $x = -1$

$\therefore$ The solution set $= \{-1, 5\}$

(2) $\because |2x + 3| = 2 \qquad \therefore \left|2\left(x + \frac{3}{2}\right)\right| = 2$

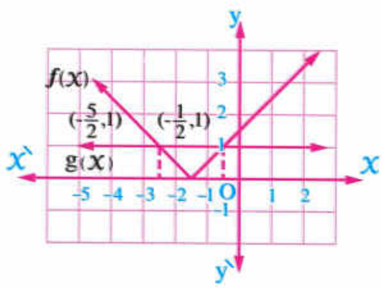
$\therefore 2\left|x + \frac{3}{2}\right| = 2 \qquad \therefore \left|x + \frac{3}{2}\right| = 1$

Graphical solution :

Putting $f(x) = \left|x + \frac{3}{2}\right|$, $g(x) = 1$

$\because f$ is represented graphically by the curve of $y = |x|$ with a horizontal translation $\frac{3}{2}$ unit in the direction of $\overrightarrow{Ox'}$

, g is represented graphically by a straight line parallel to the x-axis and intersects the y-axis at (0 , 1)



$\therefore$ The two curves intersect at $\left(-\frac{1}{2}, 1\right)$ and $\left(-\frac{5}{2}, 1\right)$

$\therefore$ The solution set $= \left\{-\frac{1}{2}, -\frac{5}{2}\right\}$

Algebraic solution :

By using the definition of the absolute value function :

$$f(x) = \left|x + \frac{3}{2}\right| = \begin{cases} x + \frac{3}{2} & , \quad x + \frac{3}{2} \geq 0 \\ -x - \frac{3}{2} & , \quad x + \frac{3}{2} < 0 \end{cases} \qquad \therefore f(x) = \begin{cases} x + \frac{3}{2} & , \quad x \geq \frac{-3}{2} \\ -x - \frac{3}{2} & , \quad x < \frac{-3}{2} \end{cases}$$

At $x \geq \frac{-3}{2}$: $x + \frac{3}{2} = 1$, then $x = \frac{-1}{2} \in [\frac{-3}{2}, \infty[$

At $x < \frac{-3}{2}$: $-x - \frac{3}{2} = 1$, then $x = \frac{-5}{2} \in]-\infty, \frac{-3}{2}[$

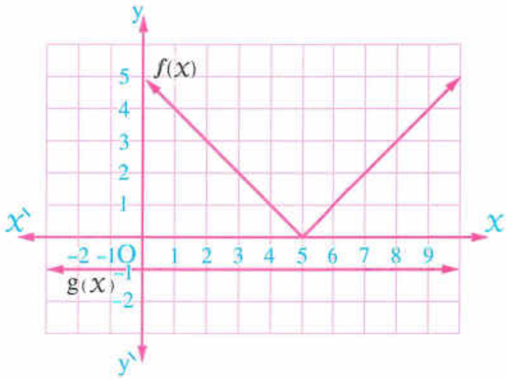
$\therefore$ The solution set $= \{\frac{-1}{2}, \frac{-5}{2}\}$

(3) $\because |5 - x| = -1$ $\therefore |x - 5| = -1$ (Notice that : $|5 - x| = |x - 5|$)

Graphical solution :

Putting $f(x) = |x - 5|$, $g(x) = -1$

$\because$ The function f is represented graphically by the curve of $y = |x|$ with a horizontal translation 5 units in the direction $\overrightarrow{Ox}$, g is represented by a straight line parallel to the x-axis and intersect y-axis at (0 , – 1) , from the graph , the two curves do not intersect at any point.



$\therefore$ The solution set $= \varnothing$

Algebraic solution :

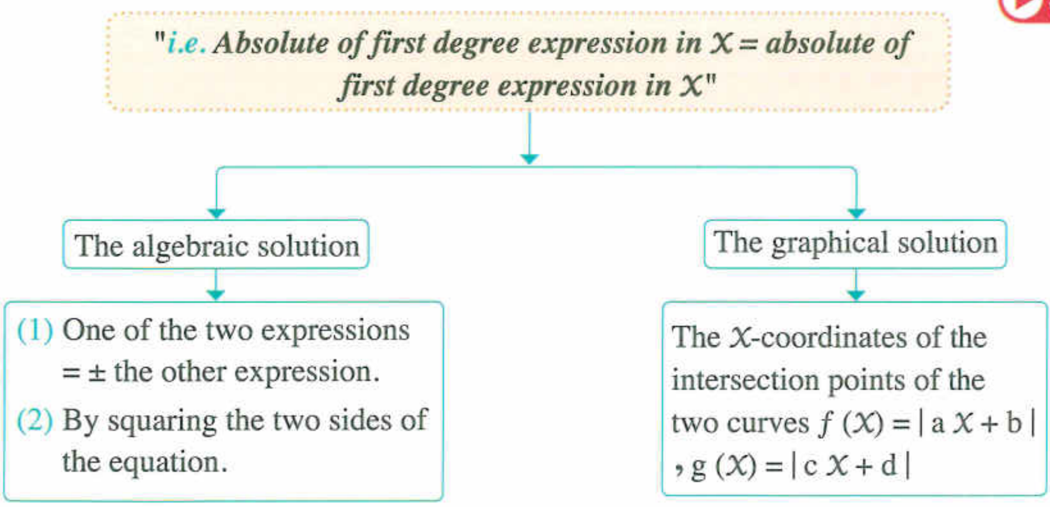
$\because$ The absolute value of any real number is a non-negative real number.

$\therefore$ There is no solution to the equation : $|x - 5| = -1$ in $\mathbb{R}$

$\therefore$ The solution set $= \varnothing$

2 Solving the equation in the form : $|a x + b| = |c x + d|$

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Example 2

Find graphically , then perform algebraically the solution set of the equation :

$|x-4| = |2x-5|$ in $\mathbb{R}$

Solution

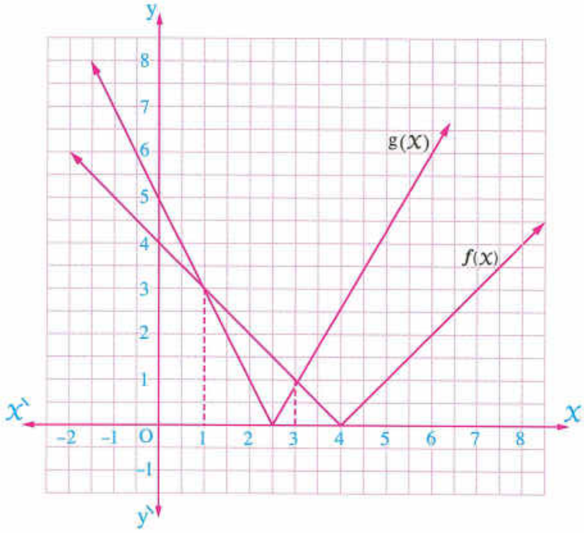
Put $f(x) = |x-4|$, $g(x) = |2x-5| = 2\left|x - 2\frac{1}{2}\right|$

Graphical solution :

The function f is represented graphically by the curve $y = |x|$ with horizontal translation 4 units in $\overrightarrow{Ox}$ directions , the function g is represented graphically by the graph $y = 2|x|$ with horizontal translation $2\frac{1}{2}$ units in $\overrightarrow{Ox}$ direction.

, ∵ the two curves are intersecting at the two points (1 , 3) , (3 , 1)

∴ The solution set = {1 , 3}



Algebraic solution :

First : By using the property "one of the two expressions = ± the other expression"

∵ $|x-4| = |2x-5|$

∴ $x-4 = \pm(2x-5)$ (from absolute property)

∴ $x-4 = 2x-5$ and so $x = 1$

or $x-4 = -2x+5$

∴ $3x = 9$ and so $x = 3$

∴ The solution set = {1 , 3}

Second : By squaring both sides

∴ $(x-4)^2 = (2x-5)^2$

∴ $x^2 - 8x + 16 = 4x^2 - 20x + 25$

∴ $3x^2 - 12x + 9 = 0$

∴ $x^2 - 4x + 3 = 0$

∴ $(x-3)(x-1) = 0$

∴ $x = 3$ or $x = 1$

∴ The solution set = {1 , 3}

Example 3

Find algebraically the solution set in $\mathbb{R}$ for each of the following equations :

(1) $|3x-9|-|3-x|=10$ (2) $\sqrt{x^2-4x+4}=10$

Solution

(1) $\because |3x-9|-|3-x|=10$

$\therefore 3|x-3|-|x-3|=10$

(Notice that : $|x-3|=|3-x|$)

$\therefore 2|x-3|=10$ $\therefore |x-3|=5$

$\therefore x-3=\pm 5$ $\therefore x-3=5$, then $x=8$

or $x-3=-5$, then $x=-2$

$\therefore$ The solution set $=\{8, -2\}$

(2) $\because \sqrt{x^2-4x+4}=10$ $\therefore \sqrt{(x-2)^2}=10$

$\therefore |x-2|=10$ $\therefore x-2=\pm 10$

$\therefore x-2=10$, then $x=12$ or $x-2=-10$, then $x=-8$

$\therefore$ The solution set $=\{12, -8\}$

Remember that

$\sqrt{a^2}=|a|$

Example 4

Find in $\mathbb{R}$ the solution set of each of the following equations :

(1) $|x-3|-|x+1|=0$ (2) $|x-3|+|x-1|=0$

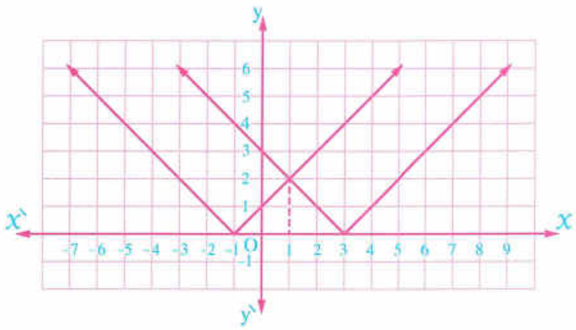
Solution

(1) $|x-3|=|x+1|$

Putting $f(x)=|x-3|$, $g(x)=|x+1|$

$\because f$ is represented by the curve of $y=|x|$ with a horizontal translation 3 units in the direction $\overrightarrow{Ox}$

, g is represented by the curve of $y=|x|$ with a horizontal translation one unit in the direction $\overrightarrow{Ox}$



, $\because$ the two curves intersect at the point (1 , 2)

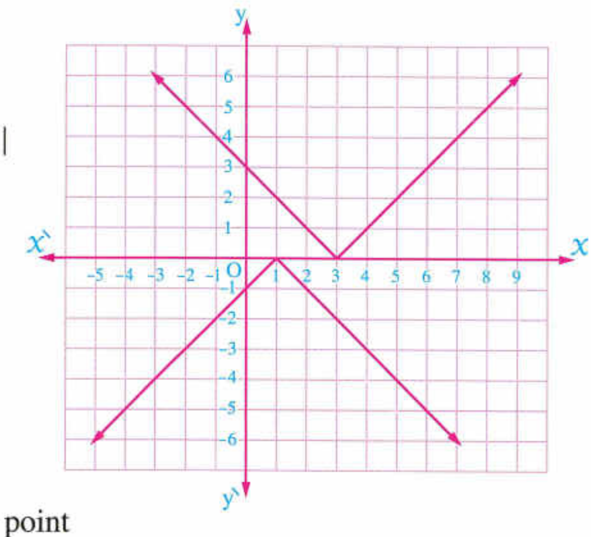
$\therefore$ The solution set $=\{1\}$

(2) $|X - 3| = -|X - 1|$

Putting $f(X) = |X - 3|$, $g(X) = -|X - 1|$

$\because$ f is represented by the curve of $y = |X|$ with a horizontal translation 3 units in the direction $\overrightarrow{OX}$

, $g(X)$ is represented by the curve of $y = |X|$ with reflection in the X-axis , then a horizontal translation one unit in the direction $\overrightarrow{OX}$



, $\because$ the two curves do not intersect at any point

$\therefore$ The solution set $= \varnothing$

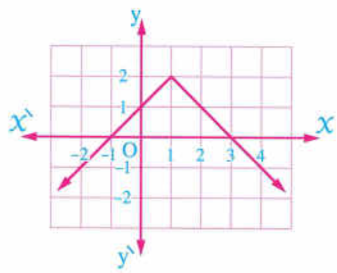
Example 5

Graph the function $f : f(X) = 2 - |X - 1|$
and from the graph deduce in $\mathbb{R}$ the solution set of the equation : $f(X) = 0$

Solution

$\because$ $f(X) = -|X - 1| + 2$

$\therefore$ f is represented graphically by the image of the curve of $y = |X|$ with the reflection in the X-axis , then a horizontal translation one unit in the direction $\overrightarrow{OX}$ and a vertical translation 2 units in the direction $\overrightarrow{Oy}$



$\therefore$ The solution set of the equation $f(X) = 0$ is the set of the X-coordinates for the intersecting points of the curve of the function f with the X-axis

i.e. With the line $y = 0$ and they are 3 and -1

$\therefore$ The solution set of the equation $= \{3 , -1\}$

Example 6

Two ways , the first way is represented by the curve of the function f where $f(X) = 4 - |X - 2|$ and the second one is represented by the curve of the function g where $g(X) = 1$, if the two ways intersect at A and B , then find the distance between A and B , knowing that the length unit is one kilometre.

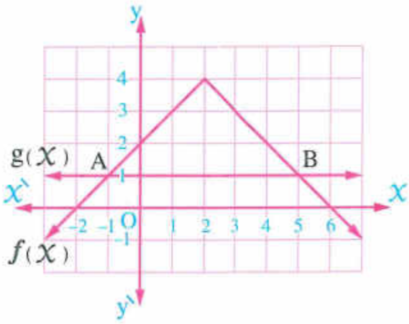
Solution

f is represented by the image of the curve of $y = |x|$ with reflection in the x-axis, then a horizontal translation 2 units in the direction $\overrightarrow{Ox}$ and a vertical translation 4 units in the direction $\overrightarrow{Oy}$

From the graph : A (– 1 , 1) and B (5 , 1)

$\therefore$ The length of $\overline{AB} = 5 - (-1) = 6$ length units.

$\therefore$ The distance between A and B = 6 kilometres.



Example 7

If $f(x) = x^2|x|$, then state whether the function f is even, odd or otherwise, then find in $\mathbb{R}$ the solution set of the equation : $f(x) = 8$

Solution

$\because f(-x) = (-x)^2|-x| = x^2|x| = f(x)$

$\therefore f$ is even.

, $\because x^2|x| = 8$ $\qquad\qquad$ $\therefore x^2|x| - 8 = 0$

, $\because x^2|x| - 8 = \begin{cases} x^2(x) - 8 & , \quad x \geq 0 \\ x^2(-x) - 8 & , \quad x < 0 \end{cases} = \begin{cases} x^3 - 8 & , \quad x \geq 0 \\ -x^3 - 8 & , \quad x < 0 \end{cases}$

At $x \geq 0$:

$x^3 - 8 = 0$ $\qquad\qquad$ $\therefore x^3 = 8$

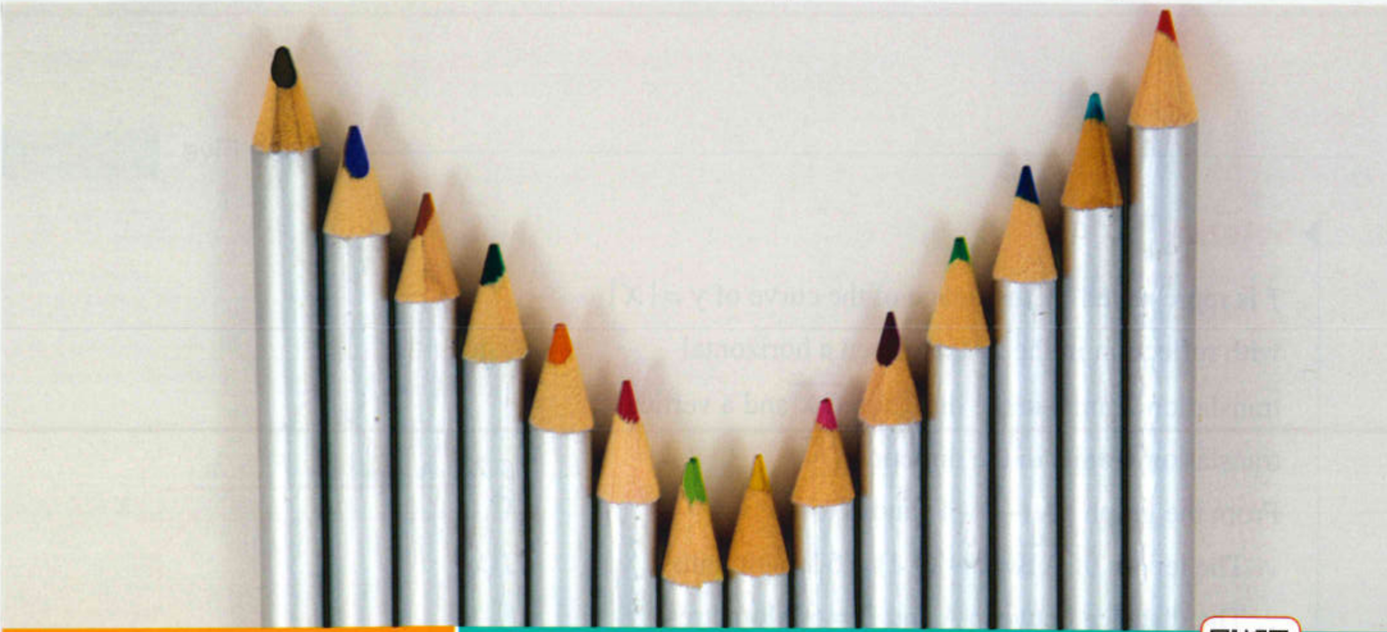
$\therefore x = 2$

At $x < 0$:

$-x^3 - 8 = 0$ $\qquad\qquad$ $\therefore x^3 = -8$

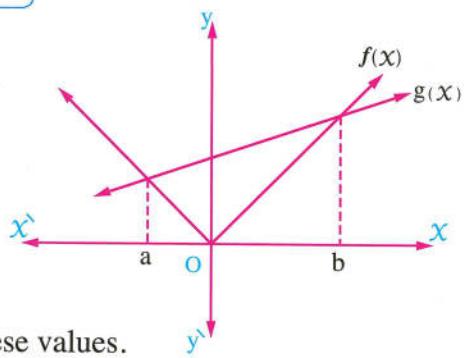
$\therefore x = -2$

$\therefore$ The solution set = $\{2, -2\}$



Lesson Six Solving absolute value inequalities

1 Graphical solution of the absolute value inequalities



In the opposite figure :

For any two functions f and g :

- The solution set of the inequality : $f(X) < g(X)$ is $]a , b[$

and this is the set of values of X where the curve of the function f is under the curve of the function g at these values.

- The solution set of the inequality : $f(X) > g(X)$ is $]-\infty , a[\cup]b , \infty[= \mathbb{R} - [a , b]$

and this is the set of values of X where the curve of the function f is up the curve of the function g at these values.

From the graph , notice that : a

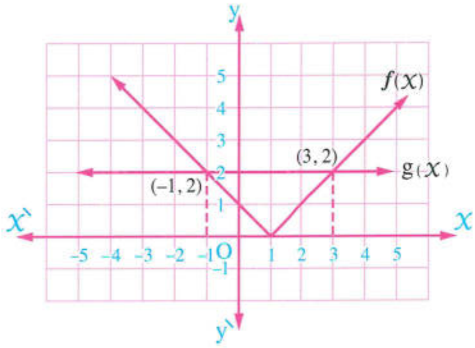
The solution set of the equation $f(X) = g(X)$ is $\{a , b\}$

, then :

- The solution set of the inequality $f(X) \leq g(X)$ is $[a , b]$
- The solution set of the inequality $f(X) \geq g(X)$ is $]-\infty , a] \cup [b , \infty[= \mathbb{R} -]a , b[$

For example :

In the opposite figure :



- The solution set of the inequality :

  $f(X) < g(X)$ is $]-1, 3[$
- The solution set of the inequality :

  $f(X) > g(X)$ is $\mathbb{R} - [-1, 3]$
- The solution set of the inequality :

  $f(X) \leq g(X)$ is $[-1, 3]$
- The solution set of the inequality :

  $f(X) \geq g(X)$ is $\mathbb{R} -]-1, 3[$

Example 1

Find graphically in $\mathbb{R}$ the solution set of each of the following inequalities :

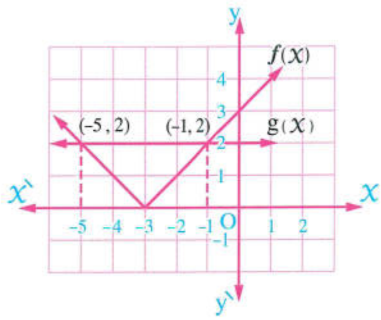
(1) $|X + 3| < 2$

(2) $|2X - 8| \leq 6$

(3) $|X - 2| > 1$

(4) $|2X - 3| \geq 4$

Solution



(1) Putting $f(X) = |X + 3|$, $g(X) = 2$

From the graph of the two functions

f and g in the opposite figure

, we get that : $f(X) < g(X)$

i.e. : $|X + 3| < 2$ on the interval $]-5, -1[$

$\therefore$ The solution set of the inequality $=]-5, -1[$

(2) $\because |2(X - 4)| \leq 6$

$\therefore 2|X - 4| \leq 6 \quad \therefore |X - 4| \leq 3$

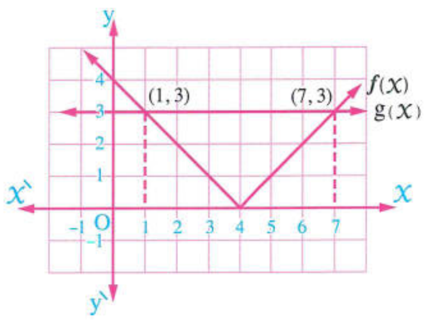
Putting $f(X) = |X - 4|$, $g(X) = 3$

From the graph of the two functions f and g

in the opposite figure , we get that : $f(X) \leq g(X)$

i.e. $|X - 4| \leq 3$ on the interval $[1, 7]$

$\therefore$ The solution set of the inequality $= [1, 7]$



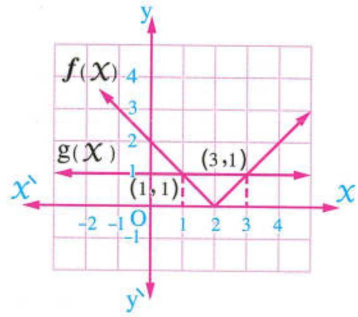
(3) Putting $f(x) = |x - 2|$, $g(x) = 1$

From the graph of the two functions f and g in the opposite figure , we get that :

$f(x) > g(x)$ on the interval

$]-\infty, 1[\cup]3, \infty[= \mathbb{R} - [1, 3]$

$\therefore$ The solution set of the inequality $= \mathbb{R} - [1, 3]$



(4) $\because |2(x - \frac{3}{2})| \geq 4$

$\therefore 2|x - \frac{3}{2}| \geq 4$ $\therefore |x - \frac{3}{2}| \geq 2$

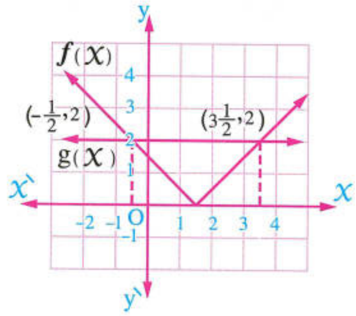
Putting $f(x) = |x - \frac{3}{2}|$, $g(x) = 2$

From the graph of the two functions f and g in the opposite figure , we get that :

$f(x) \geq g(x)$ on the interval

$]-\infty, -\frac{1}{2}] \cup [3\frac{1}{2}, \infty[= \mathbb{R} -]-\frac{1}{2}, 3\frac{1}{2}[$

$\therefore$ The solution set of the inequality $= \mathbb{R} -]-\frac{1}{2}, 3\frac{1}{2}[$



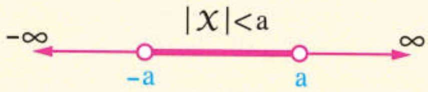
2 Algebraic solution of the absolute value inequalities

Corollaries

* For each $a \in \mathbb{R}^+$

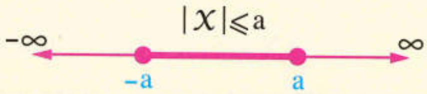
(1) If $|x| < a$, then $-a < x < a$

i.e. $x \in]-a, a[$



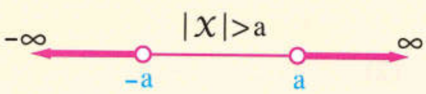
(2) If $|x| \leq a$, then $-a \leq x \leq a$

i.e. $x \in [-a, a]$



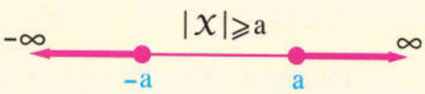
(3) If $|x| > a$, then $x > a$ or $x < -a$

i.e. $x \in \mathbb{R} - [-a, a]$



(4) If $|x| \geq a$, then $x \geq a$ or $x \leq -a$

i.e. $x \in \mathbb{R} -]-a, a[$



* For every $a \in \mathbb{R}^-$

(1) The solution set of the inequality $|x| < a$ or $|x| \leq a$ in $\mathbb{R}$ equals $\varnothing$

(2) The solution set of the inequality $|x| > a$ or $|x| \geq a$ in $\mathbb{R}$ equals $\mathbb{R}$

Example 2

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Find in $\mathbb{R}$ the solution set for each of the following inequalities :

(1) $|x-3| \le 4$

(2) $|x+2| > 3$

(3) $|2x-5| < 1$

(4) $\sqrt{4x^2+12x+9} \le 1$

(5) $|2x-5| + |5-2x| < 14$

Solution

(1) $\because |x-3| \le 4$ $\quad \therefore -4 \le x-3 \le 4$

$\therefore -4+3 \le x \le 4+3$ $\quad \therefore -1 \le x \le 7$

$\therefore$ The solution set $= [-1 , 7]$

(2) $\because |x+2| > 3$ $\quad \therefore x+2 > 3$ or $x+2 < -3$

$\therefore x > 1$ or $x < -5$ $\quad \therefore$ The solution set $= \mathbb{R} - [-5 , 1]$

(3) $\because |2x-5| < 1$ $\quad \therefore -1 < 2x-5 < 1$

$\therefore -1+5 < 2x < 1+5$ $\quad \therefore 4 < 2x < 6$ (dividing by 2)

$\therefore 2 < x < 3$ $\quad \therefore$ The solution set $=]2 , 3[$

(4) $\because \sqrt{4x^2+12x+9} \le 1$ $\quad \therefore \sqrt{(2x+3)^2} \le 1$

$\therefore |2x+3| \le 1$ $\quad \therefore -1 \le 2x+3 \le 1$

$\therefore -4 \le 2x \le -2$ $\quad \therefore -2 \le x \le -1$

$\therefore$ The solution set $= [-2 , -1]$

(5) $\because |2x-5| + |5-2x| < 14$ $\quad \therefore |2x-5| + |2x-5| < 14$

$\therefore 2|2x-5| < 14$ (dividing by 2)

$\therefore |2x-5| < 7$ $\quad \therefore -7 < 2x-5 < 7$

$\therefore -2 < 2x < 12$ $\quad \therefore -1 < x < 6$

$\therefore$ The solution set $=]-1 , 6[$

Example 3

Write the absolute value inequality which expresses :

(1) Student's mark in an exam ranges from 70 to 90 marks.

(2) The depth that some fish live in under the water level in an aquarium with interior height 40 cm.

Solution

(1) Let the mark of the student be X

$\therefore 70 \le X \le 90$ (By adding – 80 to the terms of the inequality)

$\therefore 70 - 80 \le X - 80 \le 90 - 80$

$\therefore -10 \le X - 80 \le 10$

$\therefore$ The absolute value inequality is $|X - 80| \le 10$

Notice that :

80 is the arithmetic mean of the two numbers 70 and 90

(2) Let the depth that these fish live in be X cm.

$\therefore 0 < X < 40$ (By adding – 20 to the terms of the inequality)

$\therefore 0 - 20 < X - 20 < 40 - 20$

$\therefore -20 < X - 20 < 20$

$\therefore$ The absolute value inequality is $|X - 20| < 20$

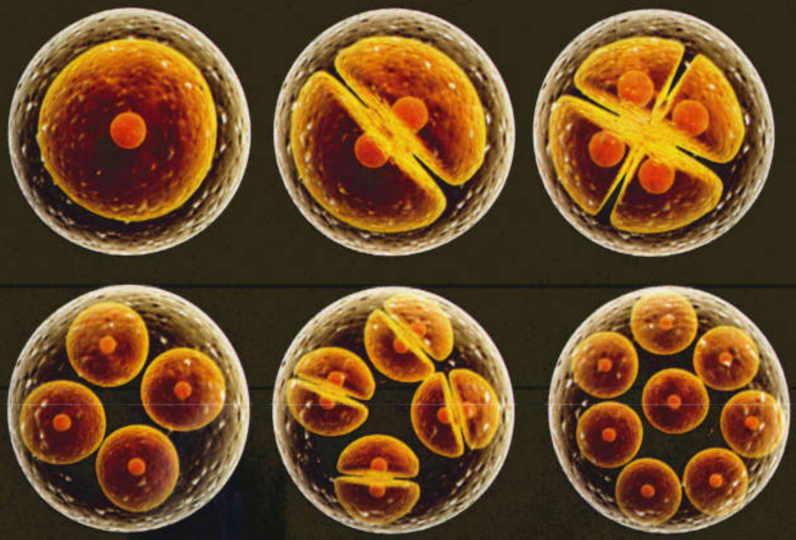
Notice that :

20 is the arithmetic mean of the two numbers 0 and 40

UNIT 2

Exponents, logarithms and their applications

Unit Lessons



Lesson One — Rational exponents and exponential equations

The nth root

The nth root of the number a is the inverse operation of raising this number to the power (n) , and the nth root of a is denoted by $\sqrt[n]{a}$ where n is called the index of the root.

For example : $\sqrt[5]{32}$ (the fifth root of 32) = 2 because $2^5 = 32$

i.e. $\sqrt[n]{a}$ (the nth root of the number a) $= x$ if $x^n = a$

∗ ***Note that :*** The equation $x^n = a$, $a \in \mathbb{R}$, $n \in \mathbb{Z}^+$ has n roots.

Let's study the following cases :

(1) If n is an even number , $a > 0$, then the equation $x^n = a$ has 2 real roots , one of them is positive and the other is negative and the other roots are complex not real numbers (when $n > 2$) and the two real roots denoted by $\sqrt[n]{a}$, $-\sqrt[n]{a}$

For example : The equation $x^6 = 64$ has two real roots : $\sqrt[6]{64} = 2$, $-\sqrt[6]{64} = -2$ and there are four another complex not real roots.

(2) If n is an even number , $a < 0$, then the equation $x^n = a$ has no real roots. (Its roots are complex not real numbers).

For example : To solve the equation : $x^2 = -16$, then $x = \pm\sqrt{-16} = \pm 4i$ (Complex not real numbers)

(3) If n is an odd number , $a \in \mathbb{R} - \{0\}$, then the equation $x^n = a$ has only one real root which is $\sqrt[n]{a}$ and the other roots are complex not real numbers.

For example : The equation $x^3 = -27$ has only one real root which is $\sqrt[3]{-27} = -3$ and there are two complex not real roots.

(4) If $n \in \mathbb{Z}^+$, $a = 0$, then the equation $x^n = 0$ has only one real root which is $x = 0$

(The number of roots for the equation equals n and each of them = 0 when $n > 1$)

For example : The equation $x^3 = 0$ has three equal real roots and each of them = 0

Remark

$\sqrt[n]{a^n} = |a|$ if n is an even number , $\sqrt[n]{a^n} = a$ if n is an odd number

For example : $\sqrt[4]{(-4)^4} = |-4| = 4$, $\sqrt[3]{(-3)^3} = -3$

The properties of the nth root

If a and b are two real numbers , $\sqrt[n]{a}$, $\sqrt[n]{b} \in \mathbb{R}$, then :

(1) $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$

(2) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, $b \neq 0$

Notice that : $\sqrt[n]{a \pm b} \neq \sqrt[n]{a} \pm \sqrt[n]{b}$

For example : $\sqrt[3]{27 b^{15}} = \sqrt[3]{27} \times \sqrt[3]{b^{15}} = 3 b^5$, $\sqrt[4]{81 x^4 y^8} = \sqrt[4]{81} \times \sqrt[4]{x^4} \times \sqrt[4]{y^8} = 3 |x| y^2$

Rational exponents

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Definition

(1) If $n \in \mathbb{Z}^+ - \{1\}$, $a \in \mathbb{R}$, then $a^{\frac{1}{n}} = \sqrt[n]{a}$

taking in considerations that if : n is an even number , $a < 0$, then $a^{\frac{1}{n}} = \sqrt[n]{a} \notin \mathbb{R}$

For example :

$9^{\frac{1}{2}} = \sqrt{9} = 3 \in \mathbb{R}$, $\left(\frac{-1}{8}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{-1}{8}} = -\frac{1}{2} \in \mathbb{R}$ but $(-16)^{\frac{1}{4}} = \sqrt[4]{-16} \notin \mathbb{R}$

(2) If m , n are two integers with no common factor , $n > 1$, $\sqrt[n]{a} \in \mathbb{R}$

, then $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

For example :

$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$, $(81)^{-\frac{3}{4}} = \left(\sqrt[4]{81}\right)^{-3} = 3^{-3} = \frac{1}{27}$

UNIT 2

Exponential Rules

If a , b are two real numbers , m , n are two rational numbers and by excluding the cases in which the denominator = zero , and cases in which both the base = zero and the index = zero and all expressions should be defined , then :

(1) $a^{zero} = 1$

(2) $a^{-n} = \frac{1}{a^n}$

(3) $a^m \times a^n = a^{m+n}$

(4) $\frac{a^m}{a^n} = a^{m-n}$

(5) $(a^m)^n = a^{mn}$

(6) $(ab)^n = a^n b^n$

(7) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

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Remarks

1. If $a \in \mathbb{R}^-$, then : $a^n > 0$ when n is an even integer

, $a^n < 0$ when n is an odd integer

For example : $(-4)^2 = 16 > 0$ but $(-4)^3 = -64 < 0$

2. * If $x^{\frac{m}{n}} = a$, then $x = a^{\frac{n}{m}}$ where m is an odd number

* If $x^{\frac{m}{n}} = a$, then $x = \pm a^{\frac{n}{m}}$ where m is an even number

where m , n have no common factors $\left(\textit{i.e. } \frac{m}{n} \text{ is a rational in simplest form}\right)$

and if one of them is even , then a must be greater than or equal to zero.

3. *Common mistake* * $(-32)^{\frac{2}{10}} = \sqrt[10]{(-32)^2} = 2$ (wrong answer)

* $(-32)^{\frac{2}{10}} = \left(\sqrt[10]{-32}\right)^2 =$ undefined in $\mathbb{R}$ (wrong answer)

because the power $\frac{2}{10}$ is not in the simplest form and should be simplified first $\left(\frac{2}{10} = \frac{1}{5}\right)$

$\therefore (-32)^{\frac{2}{10}} = (-32)^{\frac{1}{5}} = \sqrt[5]{-32} = -2$ (the correct answer)

Example 1

Find the result of each of the following in the simplest form :

(1) $\sqrt[5]{a^3} \times \sqrt{a^3}$

(2) $\left(\sqrt[5]{x}\right)^2 \times \sqrt[3]{x^2}$

(3) $\left(\sqrt[3]{a^{-5}}\right)^2 \times \left(\sqrt[4]{a^3}\right)^3$

Solution

(1) $\sqrt[5]{a^3} \times \sqrt{a^3} = a^{\frac{3}{5}} \times a^{\frac{3}{2}} = a^{\frac{3}{5}+\frac{3}{2}} = a^{\frac{21}{10}} = a^2 \times a^{\frac{1}{10}} = a^2\sqrt[10]{a}$

(2) $\left(\sqrt[5]{x}\right)^2 \times \sqrt[3]{x^2} = x^{\frac{2}{5}} \times x^{\frac{2}{3}} = x^{\frac{2}{5}+\frac{2}{3}} = x^{\frac{16}{15}} = x \times x^{\frac{1}{15}} = x\sqrt[15]{x}$

(3) $\left(\sqrt[3]{a^{-5}}\right)^2 \times \left(\sqrt[4]{a^3}\right)^3 = (a^{-\frac{5}{3}})^2 \times (a^{\frac{3}{4}})^3 = a^{-\frac{10}{3}} \times a^{\frac{9}{4}} = a^{-\frac{13}{12}} = \frac{1}{a^{\frac{13}{12}}} = \frac{1}{|a| \times a^{\frac{1}{12}}} = \frac{1}{|a|\sqrt[12]{a}}$

Example 2

Put in the simplest form : $\dfrac{\sqrt[4]{8} \times \sqrt[8]{0.01} \times 125}{\sqrt[4]{(15)^3} \times \sqrt[8]{4^5} \times (36)^{\frac{-3}{8}}}$

Solution

The expression $= \dfrac{\sqrt[4]{2^3} \times \sqrt[8]{(10)^{-2}} \times 5^3}{(15)^{\frac{3}{4}} \times 4^{\frac{5}{8}} \times (36)^{\frac{-3}{8}}} = \dfrac{2^{\frac{3}{4}} \times (2 \times 5)^{-\frac{1}{4}} \times 5^3}{(3 \times 5)^{\frac{3}{4}} \times (2^2)^{\frac{5}{8}} \times (2^2 \times 3^2)^{-\frac{3}{8}}}$

$= \dfrac{2^{\frac{3}{4}} \times 2^{-\frac{1}{4}} \times 5^{-\frac{1}{4}} \times 5^3}{3^{\frac{3}{4}} \times 5^{\frac{3}{4}} \times 2^{\frac{5}{4}} \times 2^{-\frac{3}{4}} \times 3^{-\frac{3}{4}}} = 2^{\frac{3}{4}-\frac{1}{4}-\frac{5}{4}+\frac{3}{4}} \times 3^{-\frac{3}{4}+\frac{3}{4}} \times 5^{-\frac{1}{4}+3-\frac{3}{4}}$

$= 2^0 \times 3^0 \times 5^2 = 1 \times 1 \times 25 = 25$

Example 3

Find the solution set in $\mathbb{R}$ for each of the following :

(1) $3x^5 = -96$

(2) $x^6 = -64$

(3) $(x-2)^4 = 81$

(4) $x^{\frac{3}{4}} = 27$

(5) $\sqrt[5]{x^2} = 1$

(6) $\sqrt[4]{(3x+2)^3} = 8$

(7) $x^{\frac{4}{3}} - 5x^{\frac{2}{3}} + 4 = 0$

Solution

Notice that : The required is the solution set in $\mathbb{R}$ *i.e.* The required is the real roots only.

(1) $\because 3x^5 = -96$ $\quad\therefore x^5 = -32$

$\therefore x = \sqrt[5]{-32} = -2$ $\quad\therefore$ S.S. $= \{-2\}$

(2) $x^6 = -64$ $\quad\because -64 < 0$, 6 is an even number.

$\therefore$ S.S. $= \varnothing$

(3) $\because (x-2)^4 = 81$

$\therefore x - 2 = \sqrt[4]{81} = 3$ or $x - 2 = -\sqrt[4]{81} = -3$

$\therefore x = 3 + 2 = 5$ or $x = -3 + 2 = -1$

$\therefore$ S.S. $= \{5, -1\}$

(4) $\because x^{\frac{3}{4}} = 27$ $\therefore x = 27^{\frac{4}{3}} = (3^3)^{\frac{4}{3}} = 3^4$

$\therefore x = 81$ $\therefore$ S.S. $= \{81\}$

(5) $\because \sqrt[5]{x^2} = 1$ $\therefore x^{\frac{2}{5}} = 1$

$\therefore x = \pm 1^{\frac{5}{2}}$ $\therefore x = \pm 1$

$\therefore$ S.S. $= \{1 , -1\}$

(6) $\because \sqrt[4]{(3x+2)^3} = 8$ $\therefore (3x+2)^{\frac{3}{4}} = 8$

$\therefore 3x + 2 = 8^{\frac{4}{3}}$ $\therefore 3x + 2 = (2^3)^{\frac{4}{3}}$

$\therefore 3x + 2 = 16$ $\therefore x = \frac{14}{3}$

$\therefore$ S.S. $= \left\{\frac{14}{3}\right\}$

(7) $\because x^{\frac{4}{3}} - 5 \times x^{\frac{2}{3}} + 4 = 0$

$\therefore (x^{\frac{2}{3}} - 1)(x^{\frac{2}{3}} - 4) = 0$

$\therefore x^{\frac{2}{3}} = 1$ and hence $x = \pm 1^{\frac{3}{2}} = \pm 1$

or $x^{\frac{2}{3}} = 4$ and hence $x = \pm 4^{\frac{3}{2}} = \pm (2^2)^{\frac{3}{2}}$

$= \pm 2^3 = \pm 8$ $\therefore$ S.S. $= \{1 , -1 , 8 , -8\}$

Another solution :

Let $x^{\frac{2}{3}} = k$

$\therefore k^2 - 5k + 4 = 0$ $\therefore (k - 4)(k - 1) = 0$

$\therefore k = 4$ $\therefore x^{\frac{2}{3}} = 4$ $\therefore x = \pm\sqrt{4^3} = \pm 8$

or $k = 1$ $\therefore x^{\frac{2}{3}} = 1$ $\therefore x = \pm\sqrt{1^3} = \pm 1$

$\therefore$ S.S. $= \{1 , -1 , 8 , -8\}$

Exponential equations

The exponential equation is an equation which contains a variable (unknown) in the power as ($2^{x+1} = 8$)

Laws of exponents

- **For every $m, n \in \mathbb{Z}$ and $a, b \in \mathbb{R} - \{-1, 0, 1\}$ we have :**

(1) If $a^n = 1$, then $n = \text{zero}$

(2) If $a^m = a^n$, then $m = n$

(3) If $a^n = b^n$,
- if n is an odd number, then $a = b$
- if n is an even number, then $a = \pm b$
- if $a \neq b$, then $n = \text{zero}$

Example 4

Find the value of x that satisfies each of the following equations :

(1) $2^{x+5} = 8$

(2) $3^{x^2-4} = 1$

(3) $4^{x+2} = x^{x+2}$

(4) $4^{x-3} = 3^{2x-6}$

(5) $\left(\frac{2}{3}\right)^{|x-5|} = \left(3\frac{3}{8}\right)^{-2}$

Solution

(1) $\because 2^{x+5} = 8$ $\therefore 2^{x+5} = 2^3$

$\therefore x + 5 = 3$ $\therefore x = -2$

(2) $\because 3^{x^2-4} = 1$

$\therefore x^2 - 4 = 0$ $\therefore x = \pm 2$

(3) $\because 4^{x+2} = x^{x+2}$ $\therefore x = \pm 4$ or $x + 2 = 0$, then $x = -2$

$\therefore x \in \{-2, 4, -4\}$

(4) $\because 4^{x-3} = 3^{2x-6}$ $\therefore 4^{x-3} = 3^{2(x-3)}$

$\therefore 4^{x-3} = 9^{x-3}$, $\because 4 \neq 9$ $\therefore x - 3 = 0$

$\therefore x = 3$

(5) $\because \left(\frac{2}{3}\right)^{|x-5|} = \left(\frac{27}{8}\right)^{-2}$ $\therefore \left(\frac{2}{3}\right)^{|x-5|} = \left(\left(\frac{3}{2}\right)^3\right)^{-2}$

$\therefore \left(\frac{2}{3}\right)^{|x-5|} = \left(\frac{3}{2}\right)^{-6} = \left(\frac{2}{3}\right)^6$ $\therefore |x-5| = 6$ $\therefore x - 5 = \pm 6$

$\therefore x - 5 = 6$ or $x - 5 = -6$

$\therefore x = 11$ | $\therefore x = -1$

Example 5

Find in $\mathbb{R}$ the S.S. of each of the following equations :

(1) $2^{x} \times 5^{-x} = \frac{125}{8}$

(2) $\left(3\sqrt{3}\right)^{x+1} = 27$

(3) $2^{x} \times \sqrt[3]{4} = \left(\sqrt[3]{16}\right)^{-1}$

(4) $4^{x^2-1} = 8^{-x}$

Solution

(1) $\because 2^{x} \times 5^{-x} = \frac{125}{8}$ $\quad \therefore \frac{2^{x}}{5^{x}} = \left(\frac{5}{2}\right)^{3}$

$\therefore \left(\frac{2}{5}\right)^{x} = \left(\frac{2}{5}\right)^{-3}$ $\quad \therefore x = -3$ $\quad \therefore$ S.S. $= \{-3\}$

(2) $\because \left(3\sqrt{3}\right)^{x+1} = 27$ $\quad \therefore (3 \times 3^{\frac{1}{2}})^{x+1} = 3^{3}$

$\therefore (3^{\frac{3}{2}})^{x+1} = 3^{3}$ $\quad \therefore 3^{\frac{3}{2}(x+1)} = 3^{3}$

$\therefore \frac{3}{2}(x+1) = 3$ $\quad \therefore x + 1 = 3 \times \frac{2}{3} = 2$

$\therefore x = 1$ $\quad \therefore$ S.S. $= \{1\}$

(3) $\because 2^{x} \times 4^{\frac{1}{3}} = (16)^{-\frac{1}{3}}$ $\quad \therefore 2^{x} \times \left(2^{2}\right)^{\frac{1}{3}} = \left(2^{4}\right)^{-\frac{1}{3}}$

$\therefore 2^{x} \times 2^{\frac{2}{3}} = 2^{-\frac{4}{3}}$ $\quad \therefore 2^{x+\frac{2}{3}} = 2^{-\frac{4}{3}}$

$\therefore x + \frac{2}{3} = -\frac{4}{3}$ $\quad \therefore x = -2$

$\therefore$ S.S. $= \{-2\}$

(4) $\because 4^{x^2-1} = 8^{-x}$ $\quad \therefore (2^{2})^{x^2-1} = (2^{3})^{-x}$

$\therefore 2^{2x^2-2} = 2^{-3x}$ $\quad \therefore 2x^{2} - 2 = -3x$

$\therefore 2x^{2} + 3x - 2 = 0$ $\quad \therefore (2x-1)(x+2) = 0$

$\therefore 2x - 1 = 0$, then $2x = 1$ $\quad \therefore x = \frac{1}{2}$ or $x + 2 = 0$, then $x = -2$

$\therefore$ S.S. $= \left\{\frac{1}{2}, -2\right\}$

Example 6

Find in $\mathbb{R}$ the S.S. of each of the following equations :

(1) $2^{x+1} + 2^{x-1} = 5$

(2) $5^{x} + \frac{125}{5^{x}} = 30$

(3) $9^{x} + 3^{x+1} = 18$

Solution

(1) Taking 2^{x-1} as a common factor

$\therefore 2^{x-1}(2^2+1)=5$ $\qquad \therefore 2^{x-1}(4+1)=5$

$\therefore 2^{x-1}=1$ $\qquad \therefore x-1=0$

$\therefore x=1$ $\qquad \therefore \text{S.S.}=\{1\}$

Another solution :

$\because 2^{x+1}+2^{x-1}=5$ $\qquad \therefore 2^x\times 2+2^x\times 2^{-1}=5$

$\therefore 2^x\left[2+\frac{1}{2}\right]=5$ $\qquad \therefore 2^x\times\frac{5}{2}=5$

$\therefore 2^x=2$ $\qquad \therefore x=1$ $\qquad \therefore \text{S.S.}=\{1\}$

(2) Multiplying the two sides by 5^x

$\therefore 5^{2x}+125=30\times 5^x$ $\qquad \therefore 5^{2x}-30\times 5^x+125=0$ and by factorizing

$\therefore (5^x-5)(5^x-25)=0$

$\therefore 5^x-5=0$ or $5^x-25=0$

$\therefore 5^x=5$ $\qquad \therefore 5^x=5^2$

$\therefore x=1$ $\qquad \therefore x=2$ $\qquad \therefore \text{S.S.}=\{1, 2\}$

Another solution :

Putting $5^x=y$ $\qquad \therefore y+\frac{125}{y}=30$

Multiplying the two sides by y $\therefore y^2+125=30y$

$\therefore y^2-30y+125=0$ $\qquad \therefore (y-5)(y-25)=0$

$\therefore y=5$ or $y=25$

$\therefore 5^x=5$ $\qquad \therefore x=1$

or $5^x=5^2$ $\qquad \therefore x=2$

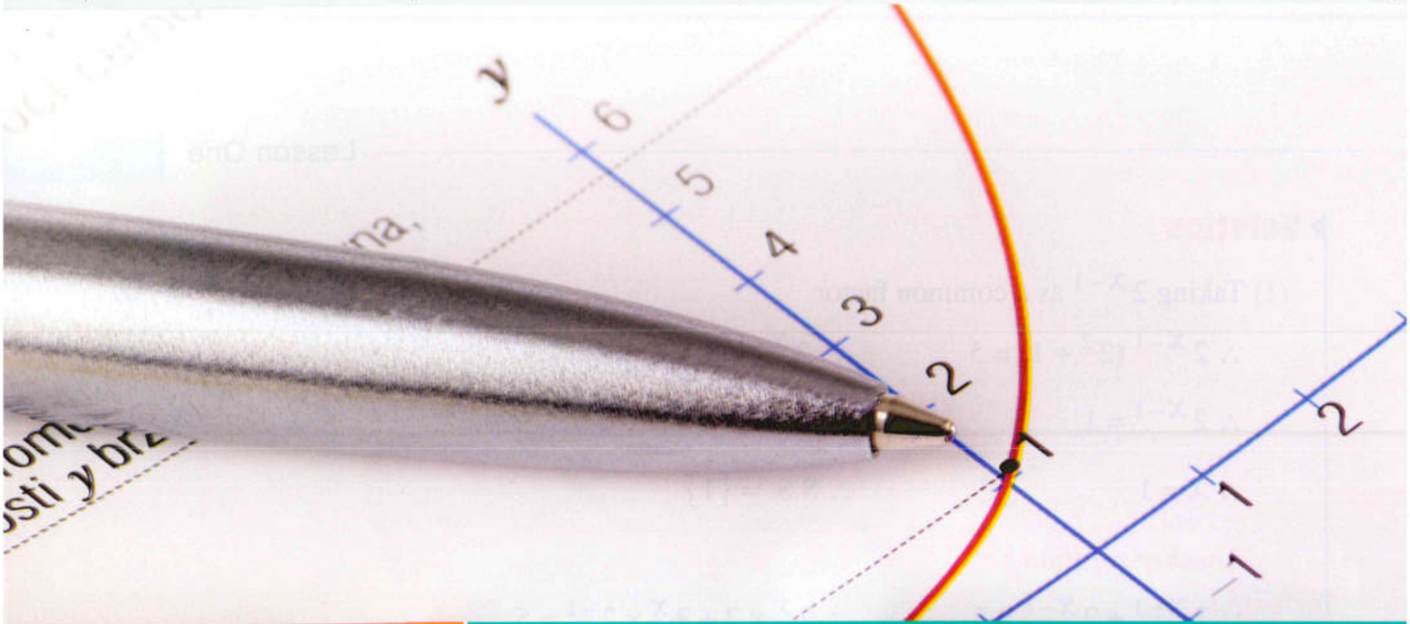
$\therefore \text{S.S.}=\{1, 2\}$

(3) $\because 3^{2x}+3\times 3^x-18=0$ $\qquad \therefore (3^x-3)(3^x+6)=0$

$\therefore 3^x-3=0$ $\qquad \therefore 3^x=3$ $\qquad \therefore x=1$

or $3^x+6=0$

$\therefore 3^x=-6$ (refused) $\qquad \therefore \text{S.S.}=\{1\}$



Lesson Two — Exponential function and its applications

Definition

If $a \in \mathbb{R}^+ - \{1\}$

, then the function $f : \mathbb{R} \longrightarrow \mathbb{R}^+$ where $f(X) = a^X$

is called an exponential function whose base is "a"

For example :

- $f : f(X) = 3^X$ is an exponential function whose base = 3 and its power = X
- $f : f(X) = \left(\frac{1}{2}\right)^{X+1}$ is an exponential function whose base = $\frac{1}{2}$ and its power = $X + 1$

Remark

Notice the difference between the algebraic function and the exponential function :

* In the algebraic function , the independent variable X is the base in the rule of the function while the power is a real number.

For example : $f : f(X) = X^2 - 3X + 1$ or $f : f(X) = (X - 3)^3$

* In the exponential function , the independent variable X is the power in the rule of the function while the base is a positive real number $\neq 1$

For example : $f : f(X) = 3^X$ or $f : f(X) = 3^{X-1} + 2$ are exponential functions

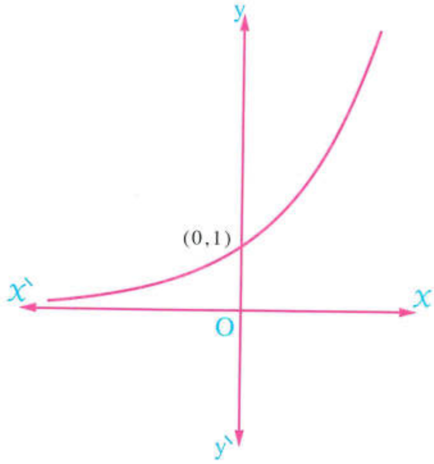
but : $f : f(X) = (-3)^X$ or $f(X) = (1)^X$ are not exponential functions.

The graphical representation of the exponential function

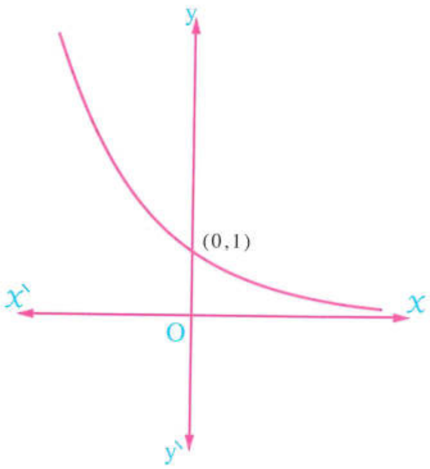
The general diagram of the graph of the function

$f : f(x) = a^x$ is as shown in the following two graphs :

a > 1

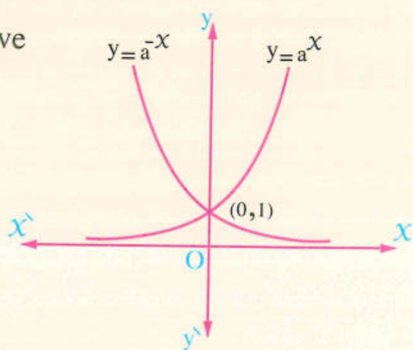


0 < a < 1



Some properties of the exponential function $f : f(x) = a^x$

- The domain = $\mathbb{R}$
- The range = $\mathbb{R}^+$ and its curve lies completely above the x-axis.
- The function is increasing on its domain $\mathbb{R}$ when $a > 1$ and is called an exponential growth function , its coefficient is a and the curve of the function approach to x-axis by the decreasing of the value of x
- The function is decreasing on its domain $\mathbb{R}$ when $0 < a < 1$ and is called an exponential decay function , its coefficient is a and the curve of the function approach to x-axis by the increasing of the value of x
- The curve of the exponential function passes through the point (0 , 1)
- If $f(x) = a^x$, then $f(-x) = a^{-x} = \left(\frac{1}{a}\right)^x$ and the curve $y = \left(\frac{1}{a}\right)^x$ is the image of the curve $y = a^x$ by reflection in y-axis.



UNIT 2

Example 1

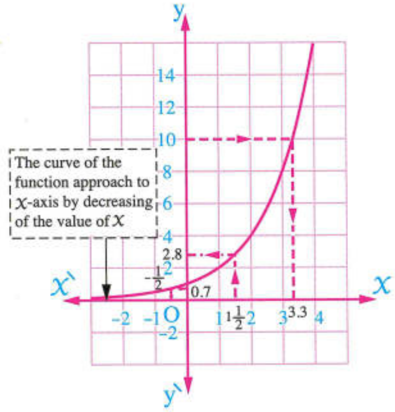
Graph the function $f : \mathbb{R} \longrightarrow \mathbb{R}^+$, $f(x) = 2^x$ taking $x \in [-3, 4]$ and from the graph find an approximated value for each of the following :

(1) $f(1.5)$, $f\left(-\frac{1}{2}\right)$ (2) The value of x when $f(x) = 10$

Solution

We form the following table :

x	-3	-2	-1	0	1	2	3	4
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16



(1) Finding $f(1.5)$ and $f\left(-\frac{1}{2}\right)$:

At $x = 1.5$ we draw a straight line parallel to y-axis to cut the curve at a point, then read the corresponding value of y on y-axis we get it 2.8 approximately.

$\therefore f(1.5) \simeq 2.8$ similarly $f\left(-\frac{1}{2}\right) \simeq 0.7$

Remark

$f(x) = 2^x$ is an exponential growth function where $a > 1$

(2) Finding x when $f(x) = 10$ i.e. when $2^x = 10$:

At $y = 10$ we draw a straight line parallel to x-axis to cut the curve at a point, then read the corresponding value of x on x-axis to get it $= 3.3$ approximately.

$\therefore$ When $2^x = 10$, then $x \simeq 3.3$

Example 2

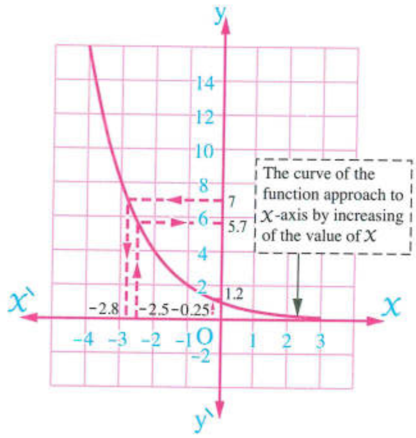
Graph the function $f : \mathbb{R} \longrightarrow \mathbb{R}^+$, $f(x) = \left(\frac{1}{2}\right)^x$ taking $x \in [-4, 3]$, from the graph find an approximated value for each of the following :

(1) $f(-2.5)$ (2) $\sqrt[4]{2}$ (3) The value of x when $\left(\frac{1}{2}\right)^x = 7$

Solution

We form the following table :

x	-4	-3	-2	-1	0	1	2	3
$y=\left(\frac{1}{2}\right)^{x}$	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



From the graph we find that :

(1) $f(-2.5) \simeq 5.7$

(2) $\because \sqrt[4]{2} = 2^{\frac{1}{4}} = \left(2^{-1}\right)^{-\frac{1}{4}} = \left(\frac{1}{2}\right)^{-\frac{1}{4}} = f\left(-\frac{1}{4}\right)$

$\therefore f\left(-\frac{1}{4}\right) \simeq 1.2$

(3) When $\left(\frac{1}{2}\right)^{x} = 7$ i.e. $f(x) = 7$

$\therefore x \simeq -2.8$

Remark

$f(x) = \left(\frac{1}{2}\right)^{x}$ is an exponential decay function where $0 < a < 1$

Notice that :

In example (1) , example (2) : the curve $f : f(x) = 2^{x}$ is the image of the curve of the function $f : f(x) = \left(\frac{1}{2}\right)^{x}$ by reflection in y-axis.

Remark

If $f(x) = a^{x}$, then $y = f(x + b)$

i.e. $y = a^{x+b}$ is represented graphically by the curve $y = a^{x}$ by horizontal displacement of magnitude $|b|$

* In direction of $\overrightarrow{Ox}$ if $b < 0$
* In direction of $\overrightarrow{Ox}$ if $b > 0$

Example 3

Graph each of the two functions defined by the given rules , from the graph find the domain , the range and determine which of them is increasing and which is decreasing :

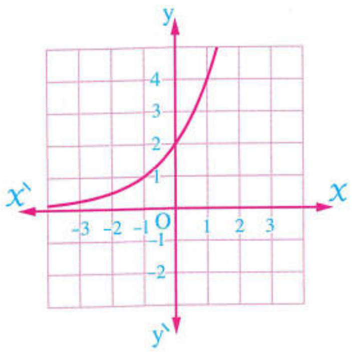
(1) $y = 2^{x+1}$ (2) $y = 3^{x-1}$

Solution

(1)

Notice that :

The curve $y = 2^{x+1}$ is the image of the curve $y = 2^{x}$ with horizontal displacement one unit in the direction $\overrightarrow{Ox}$

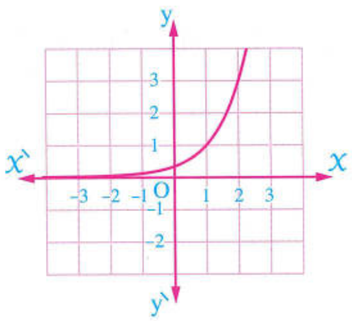


The domain = $\mathbb{R}$, the range = $]0 , \infty[$, the function is increasing on its domain.

(2)

Notice that :

The curve $y = 3^{x-1}$ is the image of the curve $y = 3^{x}$ with horizontal displacement one unit in the direction $\overrightarrow{Ox}$



The domain = $\mathbb{R}$, the range = $]0 , \infty[$, the function is increasing on its domain.

Solving the exponential equations graphically

The graphical solution for the exponential equation depends on supposing that the left hand side of the equation is an exponential function f , and by supposing the right hand side of the equation is another function g , then draw the two functions f , g in the same figure and then determine the x-coordinate of the point (points) of the intersection to get the solution set.

Example 4

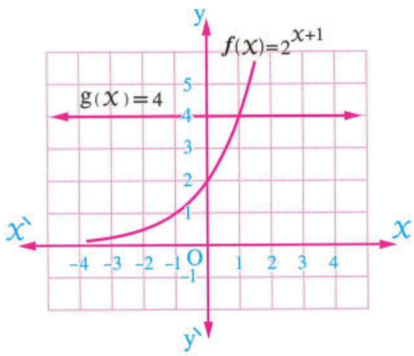
Find graphically in $\mathbb{R}$ the S.S. of the equation : $2^{x+1} = 4$

Solution

Let the left hand side of the equation be the rule of the function $f : f(x) = 2^{x+1}$ and the right hand side be the rule of the function g : g $(x) = 4$ and by drawing the two curves in the same figure , from the graph :

$\therefore$ The point of intersection is (1 , 4)

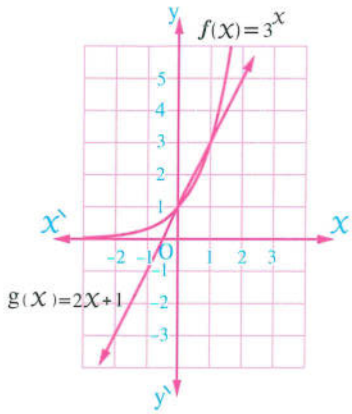
$\therefore x = 1$ $\therefore$ S.S. = {1}



Example 5

Find graphically in $\mathbb{R}$ the S.S. of the equation : $3^{x} = 2x + 1$

Solution



Let the left hand side of the equation be the rule of the function $f : f(x) = 3^{x}$ and the right hand side be the rule of the function $g : g(x) = 2x + 1$ and by drawing the two curves in the same figure , from the graph :

$\therefore$ The x-coordinates of the points of intersection are 0 , 1

$\therefore$ S.S. $= \{0 , 1\}$

Example 6

If $f : \mathbb{R} \longrightarrow \mathbb{R}^{+}$, $f(x) = 3^{x}$, prove that : $\dfrac{f(x+5) + f(x+3)}{f(x+3) + f(x+1)} = f(2)$

Solution

$$\text{L.H.S.} = \frac{3^{x+5} + 3^{x+3}}{3^{x+3} + 3^{x+1}} = \frac{3^{x+3}(3^2 + 1)}{3^{x+1}(3^2 + 1)} = \frac{3^{x+3}}{3^{x+1}} = 3^2$$

R.H.S. $= f(2) = 3^2$ $\therefore$ The two sides are equal.

Another solution :

$$\text{L.H.S.} = \frac{3^{x+5} + 3^{x+3}}{3^{x+3} + 3^{x+1}} = \frac{3^{x}(3^5 + 3^3)}{3^{x}(3^3 + 3)} = 9 = 3^2 = f(2)$$

Example 7

If $f(x) = 5^{x}$, find the value of x if : $f(2x - 1) + f(2x + 1) = \frac{26}{25}$

Solution

$\because f(2x - 1) + f(2x + 1) = \frac{26}{25}$ $\therefore 5^{2x-1} + 5^{2x+1} = \frac{26}{25}$

$\therefore 5^{2x-1}(1 + 5^2) = \frac{26}{25}$ $\therefore 5^{2x-1} \times 26 = \frac{26}{25}$ $\therefore 5^{2x-1} = \frac{1}{25}$

$\therefore 5^{2x-1} = 5^{-2}$ $\therefore 2x - 1 = -2$ $\therefore x = -\frac{1}{2}$

Another solution :

$\because f(2x - 1) + f(2x + 1) = \frac{26}{25}$ $\therefore 5^{2x-1} + 5^{2x+1} = \frac{26}{25}$

$\therefore 5^{2x}(5^{-1} + 5) = \frac{26}{25}$

$\therefore 5^{2x} = 5^{-1}$ $\therefore 2x = -1$ $\therefore x = -\frac{1}{2}$

Example 8

If $f(x) = 3^{x-2}$, find in $\mathbb{R}$ the S.S. of each of the following equations :

(1) $f(x) = \frac{1}{81}$

(2) $f(x-1) = 9$

(3) $f(2x) = \left(\frac{1}{3}\right)^{x}$

Solution

(1) $\because 3^{x-2} = \frac{1}{81}$ $\therefore 3^{x-2} = 3^{-4}$

$\therefore x - 2 = -4$ $\therefore x = -2$

$\therefore$ S.S. $= \{-2\}$

(2) $\because f(x) = 3^{x-2}$ $\therefore f(x-1) = 3^{(x-1)-2} = 3^{x-3}$

$\therefore 3^{x-3} = 9$ $\therefore 3^{x-3} = 3^2$

$\therefore x - 3 = 2$ $\therefore x = 5$

$\therefore$ S.S. $= \{5\}$

(3) $\because f(x) = 3^{x-2}$ $\therefore f(2x) = 3^{2x-2}$

$\therefore 3^{2x-2} = \left(\frac{1}{3}\right)^{x}$ $\therefore 3^{2x-2} = 3^{-x}$

$\therefore 2x - 2 = -x$ $\therefore 2x + x = 2$

$\therefore 3x = 2$ $\therefore x = \frac{2}{3}$

$\therefore$ S.S. $= \left\{\frac{2}{3}\right\}$

Life applications on the exponential growth and decay

1 Exponential growth

- The function $f : f(t) = a(1 + r)^t$ represents the exponential growth with a constant percentage during equal intervals of time, where a is the initial value, r is the percentage of the growth in a constant interval of time, t is the time interval.
- We can deduce this function by studying a phenomena such as the population :

If the number of population in a city in one of the years is "a" and this number increases annually by constant percentage rate "r", then the number of population after one year $= a + ra = a(1 + r)$, after 2 years $= a(1 + r) + ra(1 + r) = a(1 + r^2)$ and so on, then the number of population after n years $= a(1 + r)^n$

Example 9

Wael bought a house by 1350000 L.E. and its price increases at the rate of 2.5% per year :

(1) Write the exponential function which represents the price of the house after n year.

(2) Estimate to the nearest pound the price of the house after 6 years.

Solution

$a = 1350000$, $r = \frac{2.5}{100} = 0.025$, $t = 6$

(1) The exponential growth $f : f(t) = a(1 + r)^t$

$\therefore f(t) = 1350000 (1 + 0.025)^t$ $\therefore f(t) = 1350000 (1.025)^t$

(2) By substituting at $t = 6$ $\therefore f(6) = 1350000 (1.025)^6 \approx 1565586$ L.E.

The compound interest

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If principal (P) is deposited in one of the banks at interest rate (r) as a percentage and compounded (n) times per year for a period of (t) years , then the accumulated value A is given by : $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Example 10

A man deposited a capital of 15000 L.E. in one of the banks with annual compound interest 7% , find the sum of the capital after 10 years in each of the following :

(1) The interest compounded annually.

(2) The interest compounded quarter annually.

(3) The interest compounded monthly.

Solution

$\because A = P\left(1 + \frac{r}{n}\right)^{nt}$

(1) $\because$ The interest is annually *i.e.* The number of divided intervals = 1

$\therefore n = 1$ $\therefore A = 15000 (1 + 0.07)^{10} \approx 29507.27$ L.E.

(2) $\because$ The interest is quarter annually *i.e.* The number of divided intervals = 4

$\therefore n = 4$ $\therefore A = 15000 \left(1 + \frac{0.07}{4}\right)^{10 \times 4} \approx 30023.96$ L.E.

(3) $\because$ The interest is monthly *i.e.* The number of divided intervals = 12

$\therefore n = 12$ $\therefore A = 15000 \left(1 + \frac{0.07}{12}\right)^{10 \times 12} \approx 30144.92$ L.E.

2 Exponential decay

The function $f : f(t) = a(1-r)^t$ represents the exponential decay where a is the initial value , r is the percentage of the decay in a constant interval of time , t is the time interval.

Example 11

The number of infection persons by Hepatitis C is decreased at the rate 15% annually as a result of discovering the new treatment , if the number of infection persons in one of the countries 8000000 infections , write the exponential function which represents the number of infections persons after n years , then estimate the number of the infections persons after 8 years.

Solution

$a = 8000000$, $r = 0.15$, $t = 8$

The exponential function $f : f(t) = 8000000(1-0.15)^t = 8000000(0.85)^t$

when $t = 8$, then the number of infections persons $= 8000000(0.85)^8 \approx 2179924$ persons.



Lesson Three — Logarithmic function and its graph

You know that the number 8 can be written as : $8 = 2^3$, the number (3) which is written as a power to the number (2) to get (8) is called the logarithm (8) to the base (2) and denoted by : $\log_2 8$

i.e. $\log_2 8 = 3$

Thus we find that every exponential form , whose base is a positive real number $\neq 1$ has an equivalent form called the logarithmic form

$$y = \log_a x \Leftrightarrow x = a^y \text{ where } a \in \mathbb{R}^+ - \{1\} , x \in \mathbb{R}^+ \text{ and } y \in \mathbb{R}$$

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For example :

$\log_3 81 = 4 \Leftrightarrow 3^4 = 81$, $\log_3 \frac{1}{9} = -2 \Leftrightarrow 3^{-2} = \frac{1}{9}$,

$4^2 = 16 \Leftrightarrow \log_4 16 = 2$, $2^{-3} = \frac{1}{8} \Leftrightarrow \log_2 \frac{1}{8} = -3$, and so on ....

Remarks

(1) The logarithm of a non-positive number is meaningless

i.e. Each of $\log_2 -3$, $\log_5 -8$ and $\log_6 0$ is meaningless.

(2) The base "a" must be a positive number differs "1"

i.e. Each of $\log_{-2} 8$, $\log_0 5$, $\log_1 4$ is meaningless.

(3) If the base of the logarithm = 10 , then the logarithm is called the common logarithm and the base is called a common base. It is agreed to omit this base in writing.

For example :

$\log_{10} 3$ is written as $\log 3$

UNIT 2

Example 1

Express each of the following by the equivalent exponential form :

(1) $\log_2 64 = 6$

(2) $\log_2 8\sqrt{2} = \frac{7}{2}$

(3) $\log_3 \frac{1}{27} = -3$

(4) $\log 0.01 = -2$

Solution

(1) $\log_2 64 = 6 \Leftrightarrow 64 = 2^6$

(2) $\log_2 8\sqrt{2} = \frac{7}{2} \Leftrightarrow 8\sqrt{2} = 2^{\frac{7}{2}}$

(3) $\log_3 \frac{1}{27} = -3 \Leftrightarrow \frac{1}{27} = 3^{-3}$

(4) $\log 0.01 = -2 \Leftrightarrow 0.01 = 10^{-2}$

Example 2

Write the logarithmic form that is equivalent to each of the following exponential forms :

(1) $243 = \left(\sqrt{3}\right)^{10}$

(2) $10^{-2} = 0.01$

(3) $3^{\frac{5}{2}} = 9\sqrt{3}$

(4) $c = a^x$

Solution

(1) $243 = \left(\sqrt{3}\right)^{10} \Leftrightarrow \log_{\sqrt{3}} 243 = 10$

(2) $10^{-2} = 0.01 \Leftrightarrow \log_{10} 0.01 = -2$

(3) $3^{\frac{5}{2}} = 9\sqrt{3} \Leftrightarrow \log_3 9\sqrt{3} = \frac{5}{2}$

(4) $c = a^x \Leftrightarrow \log_a c = x$

Example 3

Find the value of each of :

(1) $\log_2 64$

(2) $\log_6 1$

(3) $\log_{\sqrt{3}} \frac{1}{27}$

(4) $\log 0.0001$

Solution

(1) Putting : $\log_2 64 = x$ $\quad \therefore 2^x = 64$

$\therefore 2^x = 2^6$ $\quad \therefore x = 6$ $\quad \therefore \log_2 64 = 6$

(2) Putting : $\log_6 1 = x$ $\therefore 6^x = 1$

$\therefore x = \text{zero}$ $\therefore \log_6 1 = \text{zero}$

(3) Putting : $\log_{\sqrt{3}} \frac{1}{27} = x$ $\therefore \left(\sqrt{3}\right)^x = \frac{1}{27}$ $\therefore 3^{\frac{1}{2}x} = 3^{-3}$

$\therefore \frac{1}{2} x = -3$ $\therefore x = -6$ $\therefore \log_{\sqrt{3}} \frac{1}{27} = -6$

(4) Putting : $\log 0.0001 = x$ $\therefore 10^x = 0.0001 = 10^{-4}$

$\therefore x = -4$ $\therefore \log 0.0001 = -4$

Example 4

Find the value of x if :

(1) $\log_2 x = -4$ (2) $\log_9 81\sqrt{3} = x$ (3) $\log_{\frac{1}{2}} x = -3$

Solution

(1) $\because \log_2 x = -4$ $\therefore x = 2^{-4}$ $\therefore x = \frac{1}{2^4} = \frac{1}{16}$

(2) $\because \log_9 81\sqrt{3} = x$ $\therefore 9^x = 81\sqrt{3}$

$\therefore 3^{2x} = 3^4 \times 3^{\frac{1}{2}}$ $\therefore 3^{2x} = 3^{\frac{9}{2}}$

$\therefore 2x = \frac{9}{2}$ $\therefore x = \frac{9}{4}$

(3) $\because \log_{\frac{1}{2}} x = -3$ $\therefore x = \left(\frac{1}{2}\right)^{-3} = 2^3$ $\therefore x = 8$

Example 5

Find in $\mathbb{R}$ the solution set of each of the following equations :

(1) $\log_x 7x = 2$ (2) $\log_2 \left(x^2 + \frac{3}{4} x\right) = -2$ (3) $(\log_2 x)^2 - 3 \log_2 x = 4$

Solution

(1) $\because \log_x 7x = 2$

$\therefore x^2 = 7x$

$\therefore x^2 - 7x = 0$

$\therefore x(x - 7) = 0$

$\therefore x = 0$ (refused) or $x = 7$ (verify)

$\therefore$ S.S. $= \{7\}$

Notice that :

When you solve the equations you must verify the values that you obtained in the original equation and the solution is the value (s) which verify this equation , as we know the logarithm of non-positive number is meaningless or finding the set of the available values of the variable x for substituting by them before starting of solving the equations and this is for avoidance the substituting operation by the values of x that we obtained.

(2) $\because \log_2 (x^2 + \frac{3}{4} x) = -2$ $\therefore x^2 + \frac{3}{4} x = 2^{-2}$

$\therefore x^2 + \frac{3}{4} x = \frac{1}{4}$ $\therefore 4x^2 + 3x - 1 = 0$

$\therefore (x+1)(4x-1) = 0$ $\therefore x = -1$ (verify) or $x = \frac{1}{4}$ (verify)

$\therefore$ S.S. $= \{-1 , \frac{1}{4}\}$

(3) $\because (\log_2 x)^2 - 3 \log_2 x - 4 = 0$ $\therefore (\log_2 x - 4)(\log_2 x + 1) = 0$

$\therefore \log_2 x = 4$ $\therefore x = 2^4 = 16$ (verify)

or $\log_2 x = -1$ $\therefore x = 2^{-1} = \frac{1}{2}$ (verify)

$\therefore$ S.S. $= \{16 , \frac{1}{2}\}$

The logarithmic function

If $a \in \mathbb{R}^+ - \{1\}$, then the function $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$ where $f(x) = \log_a x$ is called the logarithmic function.

Example 6

Find the domain of each of the functions that are defined by the following rules :

(1) $f(x) = \log_4 (4 - x)$

(2) $f(x) = \log_{1-x} 5$

(3) $f(x) = \log_{x-3} x$

(4) $f(x) = \log_{3-x} x$

Solution

(1) The function is defined for all values of x which verify : $4 - x > 0$

i.e. $x < 4$

$\therefore$ The domain of $f =]-\infty , 4[$

(2) The function is defined for all values of x which verify : $\begin{cases} 1 - x > 0 \\ 1 - x \neq 1 \end{cases}$ *i.e.* $\begin{cases} x < 1 \\ x \neq 0 \end{cases}$

$\therefore$ The domain of $f =]-\infty , 1[- \{0\}$

(3) The function is defined for all values of x which verify : $\begin{cases} x > 0 \\ x - 3 > 0 \\ x - 3 \neq 1 \end{cases}$ *i.e.* $\begin{cases} x > 0 \\ x > 3 \\ x \neq 4 \end{cases}$

$\therefore$ The domain of $f =]3 , \infty[- \{4\}$

Remember

The function f : $f(x) = \log_a x$ is defined for all the values of x , a which verify : $\begin{cases} x > 0 \\ a > 0 \\ a \neq 1 \end{cases}$

(4) The function is defined for all values of x which

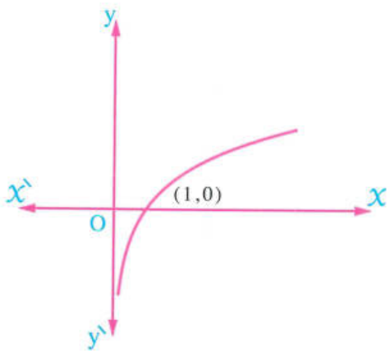
verify : $\begin{cases} x > 0 \\ 3 - x > 0 \\ 3 - x \neq 1 \end{cases}$ *i.e.* $\begin{cases} x > 0 \\ x < 3 \\ x \neq 2 \end{cases}$

$\therefore$ The domain of $f =]0 , 3[- \{2\}$

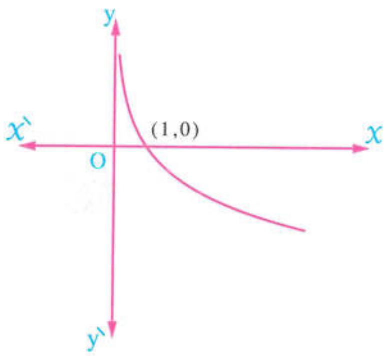
The graphical representation of the logarithmic function $f : f(x) = \log_a x$

- The graph of the logarithmic function will be in the shape of one of the following figures according to the value of the base a :

If a > 1



If 0 < a < 1



Some properties of the logarithmic function $f : f(x) = \log_a x$

(1) The domain of the logarithmic function $= \mathbb{R}^+$

(2) The range of the logarithmic function $= \mathbb{R}$

(3) The logarithmic function is increasing when $a > 1$ and it is decreasing when $0 < a < 1$

(4) All curves of the logarithmic functions for any positive base $\neq 1$ pass through the point $(1 , 0)$

Example 7

If the curve of the function $f : f(x) = \log_a x$ passes through the point $(27 , 3)$, find the value of a , then draw the graph of the function taking $x \in \left[\frac{1}{9} , 9\right]$, then from the graph :

(1) Deduce the domain , the range , monotonicity and the point of intersection with x-axis.

(2) Find an approximated value to the number $\log_3 6$

Solution

$\because f(x) = \log_a x$ for each $x > 0$, $a \in \mathbb{R}^+ - \{1\}$

, $\because$ the point $(27 , 3) \in$ the curve of the function

$\therefore 3 = \log_a 27$ $\therefore a^3 = 27 = 3^3$

$\therefore a = 3$ $\therefore f(x) = \log_3 x$

Form the following table : [Note the base = 3 > 1]

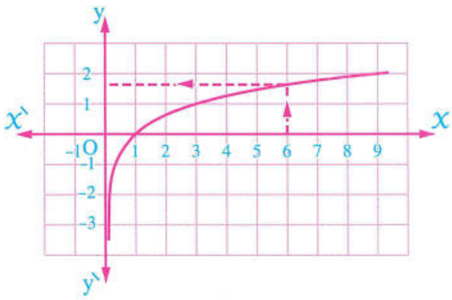
x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$y = \log_3 x$	-2	-1	0	1	2

Notice that :

Choosing the values of x from the powers of the base 3 $\{3^{-2} , 3^{-1} , 3^0 , 3^1 , 3^2\}$

From the graph we find that :

* The domain = $\mathbb{R}^+$, the range = $\mathbb{R}$
* The function is increasing on its domain
* The curve intersects the x-axis at the point (1 , 0)
* $\log_3 6 \approx 1.6$



Example 8

If the curve of the function $f : f(x) = \log_a x$ passes through the point $\left(\frac{1}{16} , 4\right)$, find the value of a , then draw the graph of the function taking $x \in \left[\frac{1}{4} , 4\right]$, then from the graph deduce the range , monotonicity then find an approximated value to the number $\log_{\frac{1}{2}} 3.5$

Solution

$\because f(x) = \log_a x$ for each $x > 0$, $a \in \mathbb{R}^+ - \{1\}$

, $\because$ the point $\left(\frac{1}{16} , 4\right) \in$ the curve of the function

$\therefore 4 = \log_a \frac{1}{16}$

$\therefore a^4 = \frac{1}{16} = \left(\frac{1}{2}\right)^4$

$\therefore a = \frac{1}{2}$ (negative solution is refused)

$\therefore f(x) = \log_{\frac{1}{2}} x$

Form the following table :

[Note the base = $\frac{1}{2} < 1$]

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
$y = \log_{\frac{1}{2}} x$	2	1	0	-1	-2

Notice that :

Choosing the values of x from the powers of the base $\frac{1}{2}$

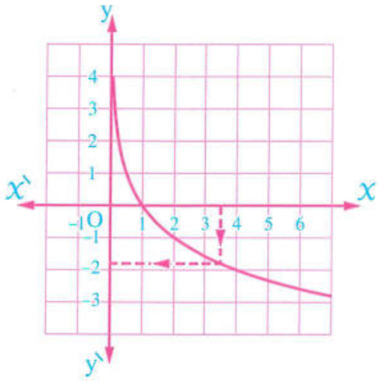
$\{(\frac{1}{2})^{-2}, (\frac{1}{2})^{-1}, (\frac{1}{2})^{0}, (\frac{1}{2})^{1}, (\frac{1}{2})^{2}\}$

From the graph we find that :

* The range = $\mathbb{R}$

* The function is decreasing on its domain

* $\log_{\frac{1}{2}} 3.5 \simeq -1.8$



Example 9

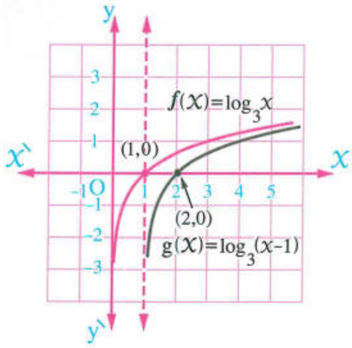
Use the curve of the function $f : f(x) = \log_3 x$ to graph the function $g : g(x) = \log_3 (x - 1)$ and from the graph find the domain , the range and the monotonicity.

Solution

The curve of the function g is the same curve of the function f by horizontal displacement 1 unit in the direction of $\overrightarrow{OX}$,

the domain = $]1 , \infty[$, the range = $\mathbb{R}$

, the function is increasing on its domain.



The relation between the exponential function and the logarithmic function

We had studied before the graph of the exponential function $f : f(x) = 3^x$ *i.e.* $y = 3^x$, then we form the following table :

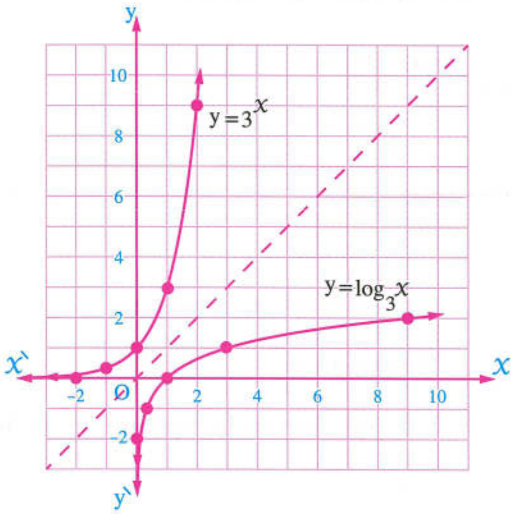
x	-2	-1	0	1	2
$y = 3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

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By replacing the two variables we get a function called the inverse function $x = 3^y$ which is equivalent to the logarithmic form $y = \log_3 x$

, to draw this function , we replace the values of x by the values of y in the previous table :

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$y = \log_3 x$	-2	-1	0	1	2



* From the opposite figure we notice that :
The two curves of the two functions are symmetric about the straight line $y = x$, the domain of the exponential function is $\mathbb{R}$ and the range = $]0 , \infty[$, the domain of the logarithmic function is $]0 , \infty[$ and the range is $\mathbb{R}$

Using the calculator

* The key of the logarithm for any base is [log▪□] , the key of the common logarithm is [log]

For example :

(1) To find $\log_3 24$ we use the keys of the calculator successively as shown below

Start → [log▪□] [3] [REPLAY ►] [2] [4] [=] 2.892789261

i.e. $\log_3 24 \approx 2.8928$ to the nearest 4 decimals digits

(2) To find log 8.4 we use the keys of the calculator successively as shown below

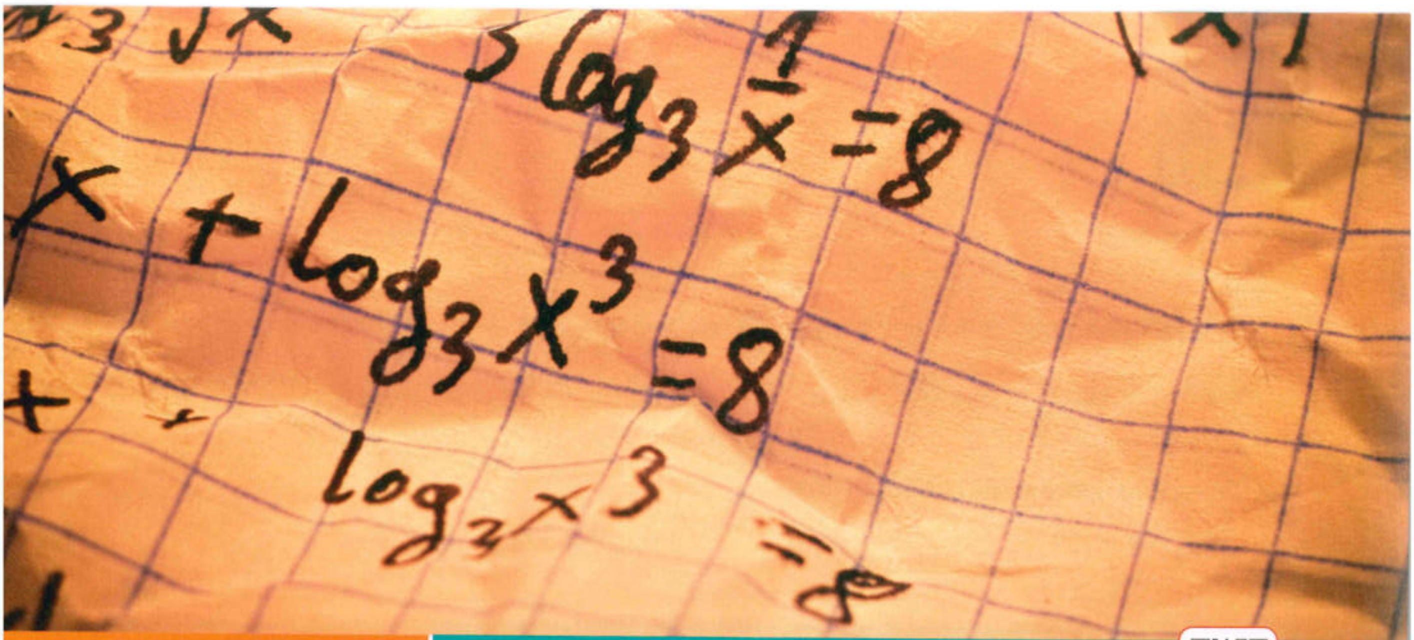
Start → [log] [8] [.] [4] [=] 0.9242792861

i.e. $\log 8.4 \approx 0.9243$ to the nearest 4 decimals digits.

(3) To evaluate the value of x which satisfies $\log x = 0.4572$, use the keys of the calculator as follow.

Start → [SHIFT] [$10^{\square}$ log] [0] [.] [4] [5] [7] [2] [=] 2.865497276

$\therefore x \approx 2.8655$ to the nearest 4 decimals digits.



Lesson Four — Some properties of logarithms

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1st Property

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- If $a \in \mathbb{R}^+ - \{1\}$, then $\log_a a = 1$

For example :

$\log_7 7 = 1$, $\log_5 5 = 1$, $\log_{\sqrt{3}} \sqrt{3} = 1$

Proof :

$\because a^1 = a$ Converting into the logarithmic form

$\therefore \log_a a = 1$

2nd Property

- If $a \in \mathbb{R}^+ - \{1\}$, then $\log_a 1 = 0$

For example :

$\log_3 1 = 0$, $\log_5 1 = 0$, $\log_{\sqrt{7}} 1 = 0$

Proof :

$\because a^0 = 1$ Converting into the logarithmic form

$\therefore \log_a 1 = 0$

3rd Property Multiplication property

- If $x , y \in \mathbb{R}^+$, $a \in \mathbb{R}^+ - \{1\}$

, then $\log_a xy = \log_a x + \log_a y$

For example :

$\log_3 (2 \times 5) = \log_3 2 + \log_3 5$

and vice versa : $\log_5 2 + \log_5 11 = \log_5 (2 \times 11) = \log_5 22$

Proof : Put $\log_a x = b$, $\log_a y = c$

$\therefore x = a^b$, $y = a^c$ $\quad\quad \therefore x y = a^b \times a^c$ $\quad\quad \therefore x y = a^{b+c}$

Converting into the logarithmic form

$\therefore \log_a x y = b + c$ $\quad\quad \therefore \log_a x y = \log_a x + \log_a y$

Corollary

If $x_1 , x_2 , x_3 , \cdots , x_n \in \mathbb{R}^+ , a \in \mathbb{R}^+ - \{1\}$, then

$\log_a (x_1 \times x_2 \times x_3 \times \cdots \times x_n) = \log_a x_1 + \log_a x_2 + \log_a x_3 + \cdots + \log_a x_n$

For example :

$\log_2 (3 \times 5 \times 7) = \log_2 3 + \log_2 5 + \log_2 7$

and vice versa :

$\log_a 75 + \log_a \frac{4}{9} + \log_a 0.06 = \log_a \left(75 \times \frac{4}{9} \times \frac{6}{100}\right) = \log_a 2$

An important remark

Remember very well that :

$\log_a (x + y) \neq \log_a x + \log_a y$, and $\log_a (x \times y) \neq \log_a x \times \log_a y$

4th Property Division property

- If $x , y \in \mathbb{R}^+$, $a \in \mathbb{R}^+ - \{1\}$, then $\log_a \frac{x}{y} = \log_a x - \log_a y$

For example :

$\log_5 \frac{2}{3} = \log_5 2 - \log_5 3$

and vice versa : $\log_5 11 - \log_5 2 = \log_5 \frac{11}{2}$

Proof : Put $\log_a x = b$, $\log_a y = c$

$\therefore x = a^b$, $y = a^c$ $\quad\quad \therefore \frac{x}{y} = \frac{a^b}{a^c} = a^{b-c}$

Converting into the logarithmic form :

$\therefore \log_a \frac{x}{y} = b - c$ $\quad\quad \therefore \log_a \frac{x}{y} = \log_a x - \log_a y$

Corollary

$\log_a \frac{x y}{z \ell} = \log_a x + \log_a y - \log_a z - \log_a \ell$

An important remark

Remember very well that : $\log_a (x - y) \neq \log_a x - \log_a y$, and $\log_a \left(\frac{x}{y}\right) \neq \log_a x \div \log_a y$

5th Property The power property

- If $x \in \mathbb{R}^+$, $a \in \mathbb{R}^+ - \{1\}$, $n \in \mathbb{R}$, then $\log_a x^n = n \log_a x$

For example :

$\log_2 125 = \log_2 5^3 = 3 \log_2 5$

and vice versa : $7 \log_5 2 = \log_5 2^7 = \log_5 128$

Proof :

$\log_a x^n = \log_a (x \times x \times x \times \dots \text{ to n terms})$

$= \log_a x + \log_a x + \dots \text{ to n terms}$

$= n \log_a x$

6th Property Base changing property

- If $x \in \mathbb{R}^+$, $y \in \mathbb{R}^+ - \{1\}$, $a \in \mathbb{R}^+ - \{1\}$, then $\log_y x = \frac{\log_a x}{\log_a y}$

For example :

$\log_5 7 = \frac{\log 7}{\log 5}$, $\log_3 2 = \frac{\log_{11} 2}{\log_{11} 3}$

Proof :

Put $\log_y x = z$

$\therefore y^z = x$ by taking logarithms to both sides for the base "a"

$\therefore z \log_a y = \log_a x$ $\therefore z = \frac{\log_a x}{\log_a y}$

$\therefore \log_y x = \frac{\log_a x}{\log_a y}$

7th Property The multiplicative inverse property

- If x , $y \in \mathbb{R}^+ - \{1\}$, then $\log_y x = \frac{1}{\log_x y}$

For example :

$\log_7 5 = \frac{1}{\log_5 7}$, then $\log_7 5 \times \log_5 7 = 1$

Proof :

$\because \log_y x = \frac{\log x}{\log y}$, $\log_x y = \frac{\log y}{\log x}$ $\therefore \log_y x \times \log_x y = 1$

$\therefore \log_y x = \frac{1}{\log_x y}$

Example 1

Without using the calculator , find the value of each of the following :

(1) $\log_3 15 + \log_3 6 - \log_3 10$

(2) $\log_5 100 - 3 \log_5 2 - \log_5 18 + \log_5 36$

(3) $\log_5 \frac{3}{5} + 2 \log_5 \frac{15}{2} - \log_5 \frac{5}{36} + \log_5 \frac{5}{243}$

(4) $\log_2 7 \times \log_7 11 \times \log_{11} 9 \times \log_3 2$

(5) $\frac{\log_2 243 - \log_3 32}{\log_2 27 - \log_3 8}$

Solution

(1) The value $= \log_3 \frac{15 \times 6}{10} = \log_3 9 = \log_3 3^2 = 2 \log_3 3 = 2 \times 1 = 2$

(2) The value $= \log_5 100 - \log_5 2^3 - \log_5 18 + \log_5 36$

$$= \log_5 \frac{100 \times 36}{8 \times 18} = \log_5 25 = \log_5 5^2 = 2 \log_5 5 = 2 \times 1 = 2$$

(3) The value $= \log_5 \frac{3}{5} + \log_5 \left(\frac{15}{2}\right)^2 - \log_5 \frac{5}{36} + \log_5 \frac{5}{243}$

$$= \log_5 \frac{\frac{3}{5} \times \frac{15}{2} \times \frac{15}{2} \times \frac{5}{243}}{\frac{5}{36}} = \log_5 \frac{3 \times 15 \times 15 \times 5 \times 36}{5 \times 2 \times 2 \times 243 \times 5} = \log_5 5 = 1$$

(4) The value $= \frac{\log 7}{\log 2} \times \frac{\log 11}{\log 7} \times \frac{\log 9}{\log 11} \times \frac{\log 2}{\log 3}$

$$= \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2 \log 3}{\log 3} = 2$$

(5) The value $= \frac{\log_2 3^5 - \log_3 2^5}{\log_2 3^3 - \log_3 2^3} = \frac{5 \log_2 3 - 5 \log_3 2}{3 \log_2 3 - 3 \log_3 2} = \frac{5 (\log_2 3 - \log_3 2)}{3 (\log_2 3 - \log_3 2)} = \frac{5}{3}$

Example 2

Without using the calculator , prove that :

(1) $3 \log 5 + 2 \log 6 - \log 9 + \log 0.2 = \log_5 25$

(2) $\frac{\log 30 - \log 6 + \log 5}{\log 12 - \log 3 + \log 25} = 1 - \log 2$

Solution

(1) L.H.S. $= \log 5^3 + \log 6^2 - \log 9 + \log \frac{2}{10}$

$$= \log \frac{5^3 \times 6^2 \times \frac{2}{10}}{9} = \log \frac{125 \times 36 \times 2}{9 \times 10}$$

$$= \log 100 = \log 10^2 = 2 \log 10 = 2$$

, R.H.S. $= \log_5 5^2 = 2 \log_5 5 = 2$

$\therefore$ The two sides are equal.

(2) L.H.S. $= \dfrac{\log \frac{30 \times 5}{6}}{\log \frac{12 \times 25}{3}} = \dfrac{\log 25}{\log 100} = \dfrac{\log 5^2}{\log 10^2} = \dfrac{2 \log 5}{2 \log 10} = \log 5$

, R.H.S. $= 1 - \log 2 = \log 10 - \log 2 = \log \frac{10}{2} = \log 5$

$\therefore$ The two sides are equal.

Example 3

If $\log_3 7 \simeq 1.771$

, find the value of each of the following in the simplest form , then verify your answer by using the calculator :

(1) $\log_3 21$ (2) $\log_3 63$ (3) $\log_3 \frac{7}{9}$

Solution

(1) $\log_3 21 = \log_3 (3 \times 7) = \log_3 3 + \log_3 7 = 1 + 1.771 = 2.771$

(Verifying by using the calculator : Start → log▫□ 3 REPLAY 2 1 = ...)

(2) $\log_3 63 = \log_3 (9 \times 7) = \log_3 9 + \log_3 7$

$$= \log_3 3^2 + \log_3 7 = 2 \log_3 3 + \log_3 7$$

$$= 2 + 1.771 = 3.771$$

(Verifying by using the calculator : Start → log▫□ 3 REPLAY 6 3 = ...)

(3) $\log_3 \frac{7}{9} = \log_3 7 - \log_3 9$

$$= \log_3 7 - \log_3 3^2$$

$$= \log_3 7 - 2 \log_3 3$$

$$= (\log_3 7) - 2 = 1.771 - 2 = -0.229$$

(Verifying by using the calculator : Start → log▫□ 3 REPLAY ▭/▭ 7 REPLAY 9 = ...)

Example 4

Find the value of each of the following in the simplest form :

(1) $\log_2 \sqrt[7]{32}$

(2) $\frac{1}{\log_x xyz} + \frac{1}{\log_y xyz} + \frac{1}{\log_z xyz}$

Solution

(1) $\log_2 \sqrt[7]{32} = \log_2 (2^5)^{\frac{1}{7}} = \log_2 2^{\frac{5}{7}} = \frac{5}{7} \log_2 2 = \frac{5}{7}$

(2) $\frac{1}{\log_x xyz} + \frac{1}{\log_y xyz} + \frac{1}{\log_z xyz} = \log_{xyz} x + \log_{xyz} y + \log_{xyz} z$

$= \log_{xyz} xyz = 1$

Example 5

Using the calculator , find the value of x to the nearest 2 decimal digits in each of the following :

(1) $5^x = 17$

(2) $2^{x-1} = 7$

(3) $5^{x+1} = 2^{4x-3}$

(4) $5^{x-2} = 3 \times 4^{x+1}$

Solution

(1) $\because 5^x = 17$ "taking logarithms of the two sides"

$\therefore \log 5^x = \log 17$ $\therefore x \log 5 = \log 17$

$\therefore x = \frac{\log 17}{\log 5}$, then by using the calculator $x \simeq 1.76$

(2) $\because 2^{x-1} = 7$ "taking logarithms of the two sides"

$\therefore \log 2^{x-1} = \log 7$ $\therefore (x-1) \log 2 = \log 7$

$\therefore x \log 2 - \log 2 = \log 7$ $\therefore x = \frac{\log 7 + \log 2}{\log 2} \simeq 3.81$

(3) $\because 5^{x+1} = 2^{4x-3}$ "taking logarithms of the two sides"

$\therefore \log 5^{x+1} = \log 2^{4x-3}$ $\therefore (x+1) \log 5 = (4x-3) \log 2$

$\therefore x \log 5 + \log 5 = 4x \log 2 - 3 \log 2$ $\therefore 4x \log 2 - x \log 5 = 3 \log 2 + \log 5$

$\therefore x (4 \log 2 - \log 5) = 3 \log 2 + \log 5$ $\therefore x = \frac{3 \log 2 + \log 5}{4 \log 2 - \log 5} \simeq 3.17$

(4) $\because 5^{x-2} = 3 \times 4^{x+1}$ "taking logarithms of the two sides"

$\therefore (x-2)\log 5 = \log 3 + (x+1)\log 4$

$\therefore x\log 5 - 2\log 5 = \log 3 + x\log 4 + \log 4$

$\therefore x\log 5 - x\log 4 = \log 3 + \log 4 + 2\log 5$

$\therefore x(\log 5 - \log 4) = \log 3 + \log 4 + 2\log 5$

$\therefore x = \dfrac{\log 3 + \log 4 + 2\log 5}{\log 5 - \log 4} \simeq 25.56$

Another solution :

$\because 5^{x-2} = 3 \times 4^{x+1}$

$\therefore \dfrac{5^x}{5^2} = 3 \times 4^x \times 4$

$\therefore \dfrac{5^x}{4^x} = 3 \times 4 \times 5^2$

$\therefore \left(\frac{5}{4}\right)^x = 300$

$\therefore x = \dfrac{\log 300}{\log \frac{5}{4}} \simeq 25.56$

Important remarks at solving logarithmic equation

(1) If $\log_a x = \log_a y$, then $x = y$

(2) If $x \in \mathbb{R}^*$ and m is an even number $\neq 0$ and $a \in \mathbb{R}^+ - \{1\}$

, then $\boxed{\log_a x^m = m\log_a |x|}$

For example : $\log_5 x^4 = 4\log_5 |x|$

Example 6

Find in $\mathbb{R}$ the S.S. of each of the following equations :

(1) $2\log x - \log(x+2) = 0$

(2) $\log x^2 = \log 4 + \log 9$

(3) $\log_2 x + \log_2 (x-2) = 3$

(4) $\log x + \log(x+2) = \log(x+6)$

(5) $\dfrac{\log 49 - (\log 7)^2}{\log 0.07} = \log x$

Solution

(1) $\because 2\log x - \log(x+2) = 0$

$\therefore \log x^2 = \log(x+2)$

$\therefore x^2 = x + 2$

$\therefore x^2 - x - 2 = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$ (refused)

$\therefore$ S.S. $= \{2\}$

Remember that

You must substitute by the values which you obtained in the original equation , then the solution is the value that satisfies the equation , where the logarithm of non-positive number is meaningless.

(2) $\because \log x^2 = \log 4 + \log 9$ $\therefore \log x^2 = \log (4 \times 9)$

$\therefore \log x^2 = \log 36$ $\therefore x^2 = 36$

$\therefore x = \pm 6$ $\therefore$ S.S. $= \{6 , -6\}$

Another solution :

$\because \log x^2 = \log 4 + \log 9$ $\therefore \log x^2 = \log 36 = \log 6^2$

$\therefore 2 \log |x| = 2 \log 6$ $\therefore |x| = 6$

$\therefore x = \pm 6$ $\therefore$ S.S. $= \{6 , -6\}$

(3) $\because \log_2 x + \log_2 (x - 2) = 3$ $\therefore \log_2 x (x - 2) = 3$

$\therefore x^2 - 2x = 2^3 = 8$ $\therefore x^2 - 2x - 8 = 0$

$\therefore (x - 4)(x + 2) = 0$ $\therefore x = 4$ or $x = -2$ (refused)

$\therefore$ S.S. $= \{4\}$

(4) $\because \log x + \log (x + 2) = \log (x + 6)$ $\therefore \log x (x + 2) = \log (x + 6)$

$\therefore x (x + 2) = x + 6$ $\therefore x^2 + 2x - x - 6 = 0$

$\therefore x^2 + x - 6 = 0$ $\therefore (x + 3)(x - 2) = 0$

$\therefore x = -3$ (refused) or $x = 2$ $\therefore$ S.S. $= \{2\}$

(5) $\because \frac{\log 49 - (\log 7)^2}{\log 0.07} = \log x$ $\therefore \frac{\log 7^2 - (\log 7)^2}{\log \frac{7}{100}} = \log x$

$\therefore \frac{2 \log 7 - (\log 7)^2}{\log 7 - \log 100} = \log x$

Remember that

$\log 100 = 2$

$\therefore \frac{\log 7 (2 - \log 7)}{\log 7 - 2} = \log x$

$\therefore -\log 7 = \log x$ $\therefore \log 7^{-1} = \log x$

$\therefore x = 7^{-1} = \frac{1}{7}$ $\therefore$ S.S. $= \left\{\frac{1}{7}\right\}$

Example 7

Find in $\mathbb{R}$ the S.S. of each of the following equations :

(1) $\log_3 (x^2 - 3x + 2) - \log_3 (x - 2) = \log_7 49$

(2) $\log_2 x = \log_4 25$

(3) $\log_3 x \times \log_9 x = 2$

(4) $\log_4 x + \log_x 4 = 2$

(5) $\log x^2 = (\log x)^2$

Solution

(1) $\because \log_3 (x^2 - 3x + 2) - \log_3 (x - 2) = \log_7 49$

$\therefore \log_3 \frac{x^2 - 3x + 2}{x - 2} = \log_7 7^2$ $\qquad$ $\therefore \log_3 \frac{(x-2)(x-1)}{x-2} = 2 \log_7 7$

$\therefore \log_3 (x - 1) = 2$ $\qquad$ $\therefore x - 1 = 3^2$

$\therefore x = 1 + 9 = 10$

$\therefore$ S.S. $= \{10\}$

(2) $\because \log_2 x = \log_4 25$ $\qquad$ $\therefore \frac{\log x}{\log 2} = \frac{\log 25}{\log 4}$

$\therefore \log x = \frac{\log 5^2 \times \log 2}{\log 2^2} = \frac{2 \log 5 \times \log 2}{2 \log 2} = \log 5$

$\therefore x = 5$ (verify) $\qquad$ $\therefore$ S.S. $= \{5\}$

(3) $\because \log_3 x \times \log_9 x = 2$ $\qquad$ $\therefore \log_3 x \times \frac{\log_3 x}{\log_3 9} = 2$

$\therefore (\log_3 x)^2 = 2 \log_3 9 = 2 \log_3 3^2 = 4 \log_3 3 = 4$

$\therefore \log_3 x = \pm 2$ $\qquad$ $\therefore x = 3^2 = 9$ (verify)

or $x = 3^{-2} = \frac{1}{9}$ (verify) $\qquad$ $\therefore$ S.S. $= \{9 , \frac{1}{9}\}$

(4) $\because \log_4 x + \log_x 4 = 2$

$\therefore \log_4 x + \frac{1}{\log_4 x} = 2$ (multiply $\times \log_4 x$)

$\therefore (\log_4 x)^2 + 1 = 2 \log_4 x$

$\therefore (\log_4 x)^2 - 2 \log_4 x + 1 = 0$

$\therefore \left((\log_4 x) - 1\right)^2 = 0$ $\qquad$ $\therefore \log_4 x = 1$

$\therefore x = 4$ (verify) $\qquad$ $\therefore$ S.S. $= \{4\}$

(5) $\because \log x^2 = (\log x)^2$ $\qquad$ $\therefore 2 \log x = (\log x)^2$

$\therefore (\log x)^2 - 2 \log x = 0$ $\qquad$ $\therefore \log x (\log x - 2) = 0$

$\therefore \log x = 0$, then $x = 10^0 = 1$ or $\log x = 2$, then $x = 10^2 = 100$

$\therefore$ S.S. $= \{1 , 100\}$

Example 8

If $x y = 16$, prove that : $3 \log_2 x + 4 \log_2 y - \log_2 x y^2 = 8$

Solution

$$\text{L.H.S.} = \log_2 x^3 + \log_2 y^4 - \log_2 x y^2$$

$$= \log_2 \frac{x^3 y^4}{x y^2} = \log_2 x^2 y^2 = \log_2 (x y)^2$$

$$= 2 \log_2 x y = 2 \log_2 16 = 2 \log_2 2^4 = 2 \times 4 \log_2 2$$

$$= 2 \times 4 \times 1 = 8 = \text{R.H.S.}$$

Example 9

If the magnitude of the intensity M (I) of an earthquake on Richter scale is given by the relation $M(I) = \log \frac{I}{I_0}$ where I is the earthquake intensity , I_0 is the smallest earth movement that can be recorded , called the reference intensity.

(1) Find on Richter scale the magnitude of the earthquake of intensity 1.6×10^8 times the reference intensity.

(2) If the magnitude of the earthquake = 7 on Richter scale find how many times the intensity of this earthquake equals the reference intensity.

Solution

(1) $M(I) = \log \frac{I}{I_0}$, $I = 1.6 \times 10^8 \; I_0$

$$\therefore M = \log \frac{1.6 \times 10^8 \; I_0}{I_0} = \log (1.6 \times 10^8) \simeq 8.2$$

i.e. The intensity of the earthquake on Richter = 8.2

(2) $\because$ The magnitude of the earthquake (M) = 7

$\therefore 7 = \log \frac{I}{I_0}$ $\qquad \therefore \frac{I}{I_0} = 10^7$

$\therefore I = 10^7 I_0$

$\therefore$ The intensity of the earthquake = 10 000 000 times the reference intensity.

Second

Calculus and Trigonometry



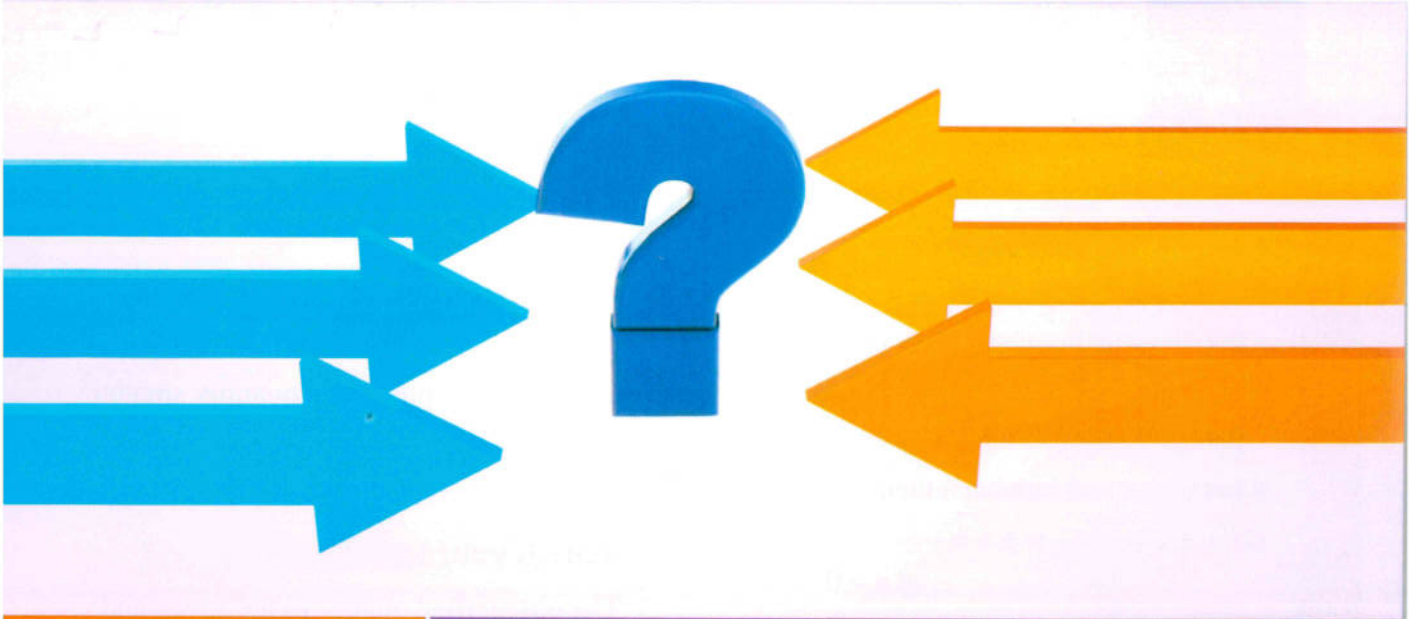
UNIT 3 Limits.

UNIT 4 Trigonometry.

UNIT 3

Limits.

Unit Lessons



Lesson One

Introduction to limits of functions
"Evaluation of the limit numerically and graphically"

Specified , unspecified and undefined quantities

When we do an arithmetic operation on $\mathbb{R}$, we will get one of the following three types of quantities.

1 Specified quantity

It is the quantity which has determined result.

For example : $\frac{8}{5}$ is a specified quantity

i.e. It has a determined result which is 1.6

because : The real number which if multiplied by 5 , the result will be 8 is 1.6

Examples for the specified quantities : $\frac{0}{3}$, 5×0 , 7×3 , ...

2 Unspecified quantity

It is the quantity which has no determined answer.

For example : $\frac{0}{0}$ is an unspecified quantity

i.e. It has an infinite number of answers in $\mathbb{R}$

because : The product of any real number × zero = zero

Noticing that there are other unspecified quantities we shall study later.

3 Undefined quantity

It is the quantity which is meaningless.

For example : $\frac{5}{0}$ is undefined quantity

i.e. It has no meaning to divide by zero.

because : There is no real number if multiplied by zero , the result will be 5

Generally : $\frac{a}{0}$ where $a \in \mathbb{R} - \{0\}$ is undefined quantity.

The symbols ∞ and – ∞

* The symbol ∞ (infinity) is not a real number but it represents a quantity greater than any positive real number can be recognized.
* The symbol $-\infty$ (negative infinity) is not a real number but it represents a quantity smaller than any negative real number can be recognized.
* Let a be a real number , then :

(1) $\infty \pm a = \infty$, $-\infty \pm a = -\infty$

(2) $\infty \times a = \begin{cases} \infty & \text{at } a > 0 \\ -\infty & \text{at } a < 0 \\ \text{unspecified} & \text{at } a = 0 \end{cases}$

, $-\infty \times a = \begin{cases} -\infty & \text{at } a > 0 \\ \infty & \text{at } a < 0 \\ \text{unspecified} & \text{at } a = 0 \end{cases}$

Enrich your knowledge

The unspecified quantities are seven and they are :

$\frac{\text{zero}}{\text{zero}}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $\infty \times$ zero , $(\text{zero})^{\text{zero}}$, $(\infty)^{\text{zero}}$ and $(1)^{\infty}$

For example : $\infty \pm 7 = \infty$, $-\infty \pm 2 = -\infty$, $\infty \times 15 = \infty$, $-\infty \times 7 = -\infty$

, $-\infty \times -2 = \infty$, $\infty + \infty = \infty$

The concept of the limit of a function at a point

Illustrated Example

If we want to find the value of the function $f : f(X) = \frac{X^2 - 1}{X - 1}$ at $X = 1$

We find that : $f(1) = \frac{1^2 - 1}{1 - 1} = \frac{0}{0}$ "unspecified quantity"

that means we can not determine the value of the function at $X = 1$

So , we go to study the approaching of $f(X)$ to a specified quantity , when X approaches to the number 1 , that by one of the following two methods :

1 Evaluation of the limit numerically

Give values for the variable X approaches to one through taking values more than 1 and less than 1 in which X does not take the value 1 , the following table shows the values X takes approaching to «1» and their corresponding values of $f(X)$:

X approaches to 1 from the left ------→ | ←----X approaches to 1 from the right

X	0.5	0.6	0.7	0.8	0.9		1.1	1.2	1.3	1.4	1.5
$f(X)$	1.5	1.6	1.7	1.8	1.9		2.1	2.2	2.3	2.4	2.5

$f(X)$ approaches to 2 from the left ----→ | ←----$f(X)$ approaches to 2 from the right

∗ We find that :

When X approaches to the number 1 (from the right or the left)

which is written mathematically as «$X \longrightarrow 1$» and is read as «X tends to 1»

, then $f(X)$ approaches to the number 2

i.e. $f(X) \longrightarrow 2$

The previous method which we follow to study the approaching of $f(X)$ to 2 when X approaches to 1 is called finding the limit of the function at a point and is written as $\lim\limits_{X \to 1} f(X) = 2$

But this method needs much time and effort.

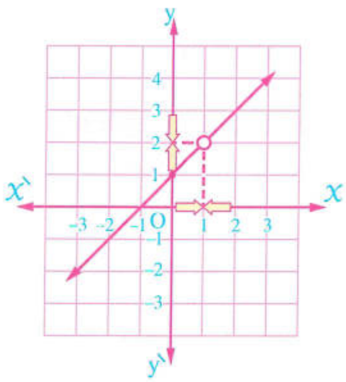
Definition

If the value of the function f approaches to a unique value ℓ when X approaches to a from the two sides right and left , then the limit of $f(X)$ equals ℓ and it is written symbolically $\lim\limits_{X \to a} f(X) = \ell$, and is read as : the limit of $f(X)$ when X approaches to a equals ℓ

2 Evaluation of the limit graphically

$\because f(X) = \dfrac{X^2 - 1}{X - 1}$ is undefined at $X = 1$

$\therefore f(X) = \dfrac{(X-1)(X+1)}{(X-1)} = X + 1$, where $X \neq 1$



i.e. It is represented by a straight line with an open dot at the point whose X-coordinate = 1 as in the opposite figure , and from the figure we notice that :

when $X \xrightarrow{\text{tends to}} 1$ (from the right and the left) ,

then $f(X) \xrightarrow{\text{tends to}} 2$

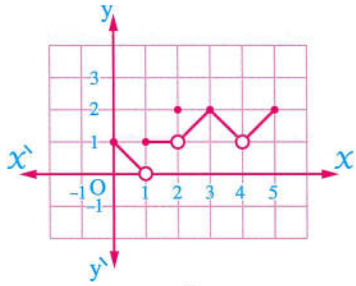
i.e. $\lim\limits_{X \to 1} f(X) = 2$

Remark

At finding $\lim\limits_{X \to a} f(X)$, it is not necessary that the function is defined at $X = a$, and vice versa : If the function is defined at $X = a$, it is not necessary that the limit of the function at $X = a$ exists.

Important remarks at finding the limit of the function graphically

(1) In the opposite figure :



We find that :

First : At $x = 1$: $f(1) = 1$

, $\lim_{x \to 1} f(x)$ does not exist.

«There is a jump at $x = 1$»

[Notice that : Although f is defined at $x = 1$, the limit does not exist]

Second : At $x = 2$: $f(2) = 2$, $\lim_{x \to 2} f(x) = 1$

[Notice that : It is not necessary that the value of the function equals the value of the limit]

Third : At $x = 3$: $f(3) = 2$, $\lim_{x \to 3} f(x) = 2$

Fourth : At $x = 4$: $f(4)$ is undefined , $\lim_{x \to 4} f(x) = 1$

«There is an open dot at $x = 4$»

[Notice that : Although the function is undefined , the limit exists]

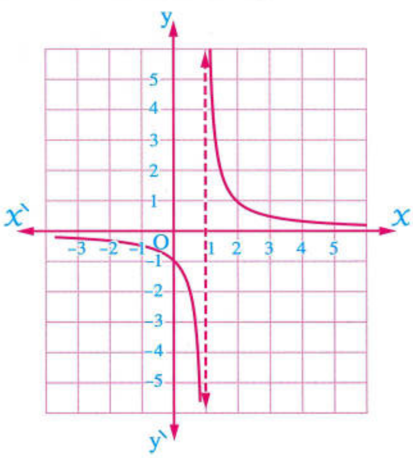
Remark

From the graph of the function in the previous figure, then :

* The point which is represented by an open dot does not affect on the existing of a limit at it.
* The point which has a brupt break (jump) dues to non existence a limit.

(2) The opposite figure represents the function

$f : f(x) = \frac{1}{x - 1}$



and we find that :

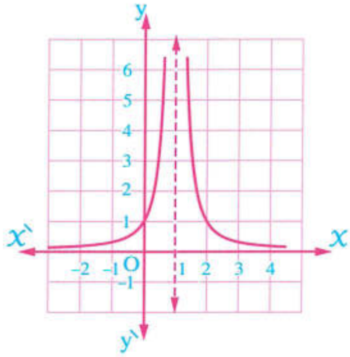
When x approaches to 1 from the right and left ,

then $f(x)$ approaches to ∞ , $-\infty$ respectively.

$\therefore \lim_{x \to 1} f(x)$ does not exist.

(3) The opposite figure represents the function

$f : f(X) = \frac{1}{(X-1)^2}$



and we find that :

When X approaches to 1 from the right

and left , then $f(X)$ approaches to ∞

$\therefore \lim_{X \to 1} f(X) = \infty$

Example 1

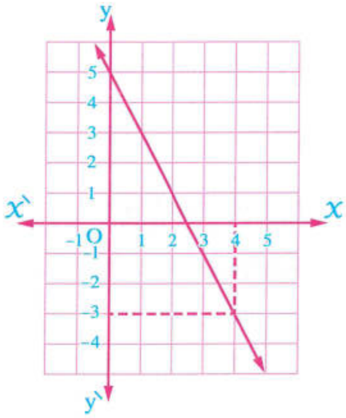
Find : $\lim_{X \to 4} (5 - 2X)$ graphically and numerically.

Solution

*** Graphically :**

We represent the linear function $f : f(X) = 5 - 2X$

as in the opposite figure :



We notice that , when $X \longrightarrow 4$

, then $f(X) \longrightarrow -3$

i.e. $\lim_{X \to 4} (5 - 2X) = -3$

*** Numerically :**

We form a table for the values of $f(X)$ and this by choosing values of X approaches to the number 4 from the right and the left as follows :

X	3.9	3.99	3.999	..... 4	4.001	4.01	4.1
$f(X)$	– 2.8	– 2.98	– 2.998	..... – 3	– 3.002	– 3.02	– 3.2

From the table , we notice that , when X approaches to the number 4 from the right or the left , the values of $f(X)$ approaches to the number – 3

$\therefore \lim_{X \to 4} (5 - 2X) = -3$

Example 2

Study each of the following figures , then find the value of :

(1) $f(2)$ (2) $\lim\limits_{x \to 2} f(x)$

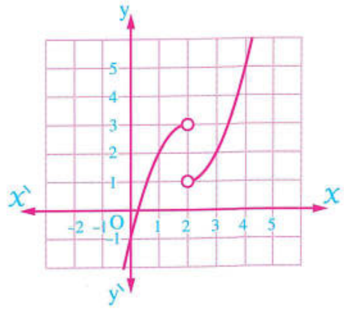


Fig. (1)

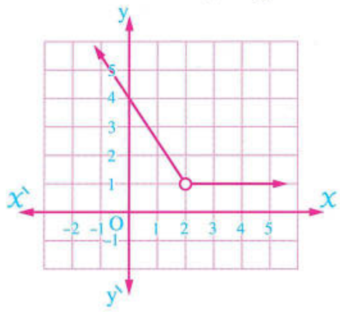


Fig. (2)

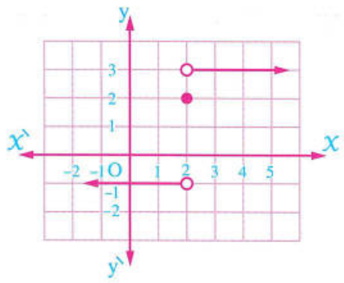


Fig. (3)

Solution

In fig. (1) : $f(2)$ is undefined , $\lim\limits_{x \to 2} f(x)$ does not exist.

In fig. (2) : $f(2)$ is undefined , $\lim\limits_{x \to 2} f(x) = 1$

In fig. (3) : $f(2) = 2$, $\lim\limits_{x \to 2} f(x)$ does not exist.

Example 3

Study each of the following figures , then find the value of :

(1) $f(0)$ (2) $\lim\limits_{x \to 0} f(x)$

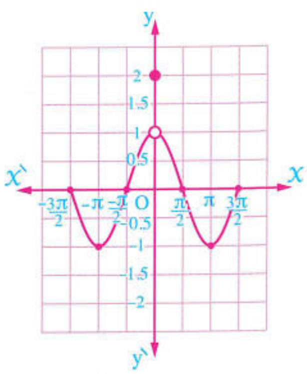


Fig. (1)

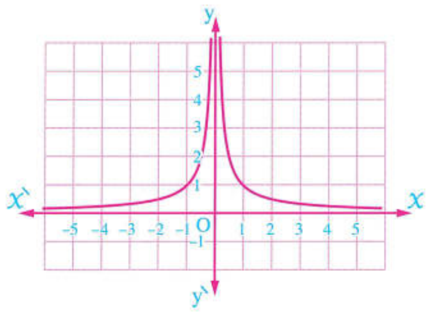


Fig. (2)

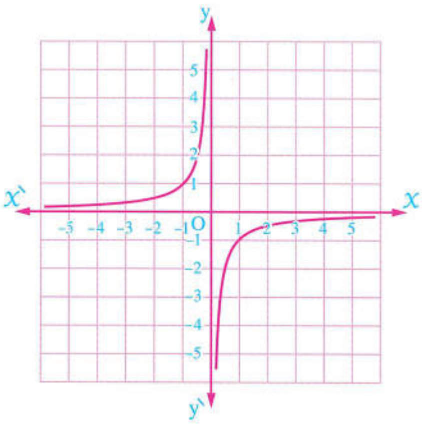


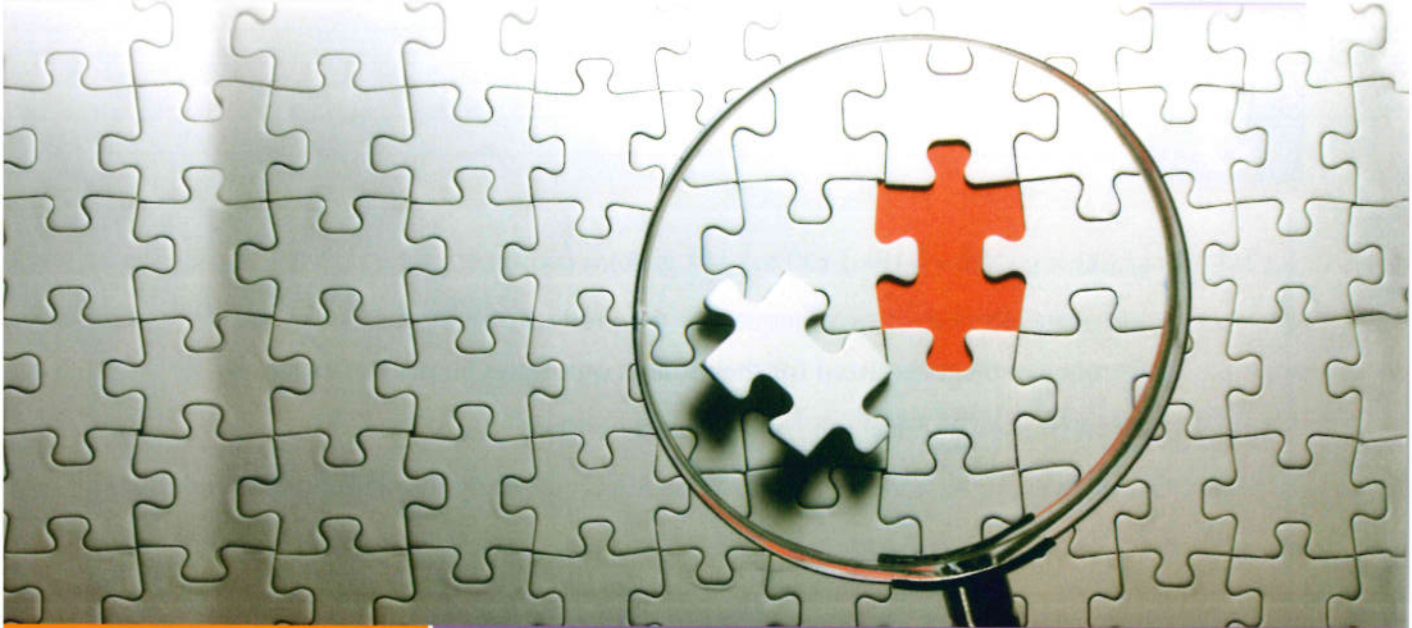
Fig. (3)

Solution

In fig. (1) : $f(0) = 2$, $\lim\limits_{x \to 0} f(x) = 1$

In fig. (2) : $f(0)$ is undefined , $\lim\limits_{x \to 0} f(x) = \infty$

In fig. (3) : $f(0)$ is undefined , $\lim\limits_{x \to 0} f(x)$ does not exist.



Lesson Two Finding the limit of a function algebraically

The following are some fundamental theorems and corollaries which help for finding the limit of a function without restoring to the graphing or studying the values of the function.

Theorem (1) ***(Limit of a polynomial function) :***

If $f(X)$ is a polynomial function in X , then $\lim\limits_{X \to a} f(X) = f(a)$

For example :

$\lim\limits_{X \to 2} (2X + 5) = f(2) = 2(2) + 5 = 9$

, $\lim\limits_{X \to 1} (X^2 - 3X + 2) = f(1) = 1 - 3 + 2 = \text{zero}$

Corollary

Limit of the constant function.

If $f(X) = k$ where k is constant , then $\lim\limits_{X \to a} f(X) = \lim\limits_{X \to a} k = k$

For example : $\lim\limits_{X \to 3} 4 = 4$, $\lim\limits_{X \to -1} -5 = -5$

Theorem (2)

If f , g are two real functions in X , $\lim\limits_{X \to a} f(X) = \ell$, $\lim\limits_{X \to a} g(X) = m$ where ℓ and $m \in \mathbb{R}$, then :

(1) $\lim\limits_{X \to a} [f(X) \pm g(X)] = \lim\limits_{X \to a} f(X) \pm \lim\limits_{X \to a} g(X) = \ell \pm m$

i.e. Limit of the algebraic sum of two functions = the algebraic sum of their limits.

This rule can be generalized for the sum of a finite number of functions.

(2) $\lim\limits_{x \to a} [f(x) \times g(x)] = \lim\limits_{x \to a} f(x) \times \lim\limits_{x \to a} g(x) = \ell \times m$

i.e. Limit of the product of two functions = the product of their limits.

This rule can be generalized for the product of a finite number of functions.

(3) $\lim\limits_{x \to a} k f(x) = k \lim\limits_{x \to a} f(x) = k \ell$, where k is constant.

i.e. Limit of the product of the constant × function = the constant × limit of this function.

(4) $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{\ell}{m}$ where $m \neq 0$

i.e. Limit of the quotient of two functions = the quotient of their limits regarding that the denominator ≠ 0

This rule can be generalized for the product of a finite number of functions divided by the product of a finite number of functions under condition that the denominator ≠ 0

(5) $\lim\limits_{x \to a} [f(x)]^n = [\lim\limits_{x \to a} f(x)]^n = \ell^n$, $n \in \mathbb{Z}^+$

Example 1

Find each of the following limits :

(1) $\lim\limits_{x \to 0} x^2 + 3x - 2 + \frac{2x}{x-1}$

(2) $\lim\limits_{x \to -2} (x^2 + 3)(x^3 - 2x^2 + 4x - 1)$

(3) $\lim\limits_{x \to 1} \frac{(2x^2 - 5x + 6)(x+1)}{(x^2+2)(3x-1)}$

Solution

(1) $\lim\limits_{x \to 0} x^2 + 3x - 2 + \frac{2x}{x-1} = \lim\limits_{x \to 0} (x^2 + 3x - 2) + \lim\limits_{x \to 0} \frac{2x}{x-1}$

$= \lim\limits_{x \to 0} (x^2 + 3x - 2) + \frac{\lim\limits_{x \to 0} 2x}{\lim\limits_{x \to 0} (x-1)} = -2 + \frac{0}{-1} = -2$

(2) $\lim\limits_{x \to -2} (x^2 + 3)(x^3 - 2x^2 + 4x - 1)$

$= \lim\limits_{x \to -2} (x^2 + 3) \times \lim\limits_{x \to -2} (x^3 - 2x^2 + 4x - 1)$

$= (4 + 3)(-8 - 8 - 8 - 1) = 7 \times (-25) = -175$

(3) $\lim\limits_{x \to 1} \frac{(2x^2 - 5x + 6)(x+1)}{(x^2+2)(3x-1)} = \frac{\lim\limits_{x \to 1} (2x^2 - 5x + 6) \times \lim\limits_{x \to 1} (x+1)}{\lim\limits_{x \to 1} (x^2+2) \times \lim\limits_{x \to 1} (3x-1)}$

$= \frac{(2-5+6)(1+1)}{(1+2)(3-1)} = \frac{3 \times 2}{3 \times 2} = 1$

Notice that :

We can solve the previous example by using direct substitution without separating limits.

Remark

We can use the direct substitution :

$\lim_{x \to a} f(x) = f(a)$ if the function is polynomial or rational with denominator $\neq 0$

Theorem (3)

If f , g are two functions in the variable x , $f(x) = g(x)$ for all the values of $x \in \mathbb{R} - \{a\}$ and $\lim_{x \to a} g(x) = \ell$, then : $\lim_{x \to a} f(x) = \ell$

The use of the previous theorem :

This theorem is used to find the limit of a rational function (fraction each of its numerator and denominator is polynomial) say $f(x)$ at $x \longrightarrow a$ where $f(a) = \frac{\text{zero}}{\text{zero}}$

This means that $(x - a)$ is a common factor between the numerator and the denominator.

Notice that :

$x \longrightarrow a$ means $(x - a) \longrightarrow 0$

i.e. $(x - a) \neq 0$ and because that the simplifying is done.

In this case , to find $\lim_{x \to a} f(x)$, we cancel the factor $(x - a)$ using factorization or long division to get a new function $g(x)$ equal to $f(x)$ when $x \neq a$,

then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$ and the next example illustrates this process.

Example 2

Find each of the following :

(1) $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$ (2) $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 5x + 6}$ (3) $\lim_{x \to -1} \frac{(2x + 3)^2 - 1}{x^2 + x}$

Solution

(1) Let $f(x) = \frac{x^2 - 16}{x - 4}$ $\therefore f(4) = \frac{4^2 - 16}{4 - 4} = \frac{\text{zero}}{\text{zero}}$

$$\therefore \lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} \frac{(x - 4)(x + 4)}{x - 4} = \lim_{x \to 4} x + 4 = 4 + 4 = 8$$

(2) Let $f(x) = \frac{x^3 - 8}{x^2 - 5x + 6}$ $\therefore f(2) = \frac{2^3 - 8}{2^2 - 5(2) + 6} = \frac{\text{zero}}{\text{zero}}$

$$\therefore \lim_{x \to 2} \frac{x^3 - 8}{x^2 - 5x + 6} = \lim_{x \to 2} \frac{(x - 2)(x^2 + 2x + 4)}{(x - 2)(x - 3)}$$

$$= \lim_{x \to 2} \frac{x^2 + 2x + 4}{x - 3} = \frac{2^2 + 2(2) + 4}{2 - 3} = -12$$

(3) Let $f(x) = \frac{(2x+3)^2 - 1}{x^2 + x}$ $\qquad \therefore f(-1) = \frac{(-2+3)^2 - 1}{(-1)^2 - 1} = \frac{\text{zero}}{\text{zero}}$

$$\therefore \lim_{x \to -1} \frac{(2x+3)^2 - 1}{x^2 + x} = \lim_{x \to -1} \frac{(2x+3-1)(2x+3+1)}{x(x+1)}$$

$$= \lim_{x \to -1} \frac{(2x+2)(2x+4)}{x(x+1)} = \lim_{x \to -1} \frac{2(x+1) \times 2(x+2)}{x(x+1)}$$

$$= \lim_{x \to -1} \frac{4(x+2)}{x} = \frac{4(-1+2)}{-1} = -4$$

Example 3

Find : $\lim_{x \to 2} \frac{x^3 - 7x + 6}{3x^2 - 8x + 4}$

Solution

Let $f(x) = \frac{x^3 - 7x + 6}{3x^2 - 8x + 4}$

$\therefore f(2) = \frac{(2)^3 - 7(2) + 6}{3(2)^2 - 8(2) + 4} = \frac{\text{zero}}{\text{zero}}$

$\therefore (x-2)$ is a common factor between the numerator and the denominator , , then divide the numerator by the factor $(x-2)$

$$\therefore \begin{array}{r|rrr} x-2 & x^3 & & -7x+6 \\ x^2+2x-3 & \ominus x^3 & \oplus -2x^2 & \\ \hline & & 2x^2 & -7x+6 \\ & & \ominus 2x^2 & \oplus -4x \\ \hline & & & -3x+6 \\ & & & \oplus -3x \ominus +6 \\ \hline & & & 00 \quad 00 \end{array}$$

Remember that

For the long division operation :

(1) Arrange the terms of each dividend and divisor according to the powers of x ascendingly or descendingly by the same way with leave an empty place for the powers which do not exist.

(2) Divide the first term of the dividend by the first term of the divisor , then write the quotient.

(3) Multiply the quotient by the divisor , and subtract the result from the dividend to get the left.

(4) Continue with the same way until the division operation finished.

$\therefore$ The numerator $= (x-2)(x^2 + 2x - 3)$

$$\therefore \lim_{x \to 2} \frac{x^3 - 7x + 6}{3x^2 - 8x + 4} = \lim_{x \to 2} \frac{(x-2)(x^2 + 2x - 3)}{(x-2)(3x-2)} = \lim_{x \to 2} \frac{x^2 + 2x - 3}{3x - 2}$$

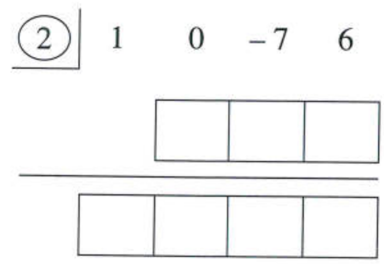
$$= \frac{4 + 4 - 3}{6 - 2} = \frac{5}{4}$$

Enrich your knowledge

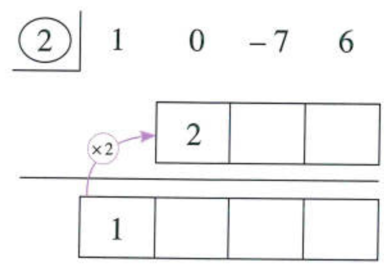
In the case of dividing by an expression of the first degree and the coefficient of $x = 1$

i.e. In the form of $(x - a)$, you can use the synthetic division method to make the long division easier , and you can use it in the previous example as follows to divide $(x^3 - 7x + 6)$ by $(x - 2)$

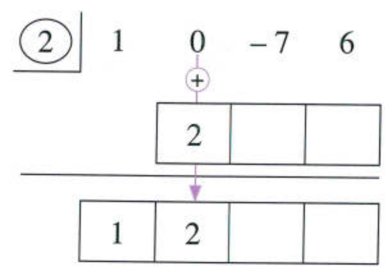
(1) Arrange the coefficients according to the ascendingly or descendingly powers of x and put (0) as a coefficient for the powers which do not exist and put the number (2) (Zero of the divisor) in the place of divisor.



(2) The coefficient of the greatest power is brought down to the third row , then multiply it by 2 and put the product in the second row place of the neighboring column directly.

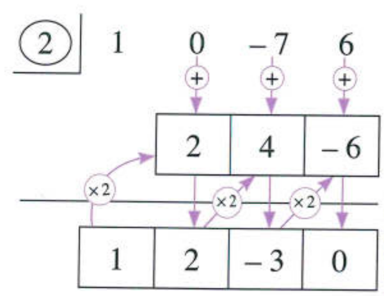


(3) Add the coefficient of the next power to the product which you got immediately.



(4) Repeat multiplying and adding to get the factors of the quotient 1 , 2 and – 3

$\therefore$ The quotient is : $x^2 + 2x - 3$



Important remark

In case of existence of a difference of two square roots of algebraic expressions (in numerator or denominator or both) , we multiply each of the numerator and denominator by the conjugate of (the numerator or the denominator or both) when the result of the direct substitution equals $\frac{\text{zero}}{\text{zero}}$ and the next example illustrates this.

Example 4

Find each of the following :

(1) $\lim\limits_{x \to 0} \dfrac{x^2 + 2x}{\sqrt{x+9} - 3}$ (2) $\lim\limits_{x \to 0} \dfrac{\sqrt{4+x} - 2}{\sqrt{9+x} - \sqrt{9-x}}$

WATCH VIDEO

Solution

By substitution in each of the two functions by $x = 0$, we find that the value of each $= \dfrac{\text{zero}}{\text{zero}}$

(1) $\lim\limits_{x \to 0} \dfrac{x^2 + 2x}{\sqrt{x+9} - 3} = \lim\limits_{x \to 0} \dfrac{x(x+2)}{\sqrt{x+9} - 3} \times \dfrac{\sqrt{x+9} + 3}{\sqrt{x+9} + 3}$

(multiplying by the conjugate of the denominator)

$$= \lim_{x \to 0} \frac{x(x+2)\left[\sqrt{x+9} + 3\right]}{x + 9 - 9}$$

$$= \lim_{x \to 0} (x+2)\left[\sqrt{x+9} + 3\right] = 2 \times 6 = 12$$

(2) $\lim\limits_{x \to 0} \dfrac{\sqrt{4+x} - 2}{\sqrt{9+x} - \sqrt{9-x}} = \lim\limits_{x \to 0} \dfrac{\sqrt{4+x} - 2}{\sqrt{9+x} - \sqrt{9-x}} \times \dfrac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2} \times \dfrac{\sqrt{9+x} + \sqrt{9-x}}{\sqrt{9+x} + \sqrt{9-x}}$

(multiplying by the conjugates of numerator and denominator)

$$= \lim_{x \to 0} \frac{4 + x - 4}{(9+x) - (9-x)} \times \frac{\sqrt{9+x} + \sqrt{9-x}}{\sqrt{4+x} + 2}$$

$$= \lim_{x \to 0} \frac{x\left(\sqrt{9+x} + \sqrt{9-x}\right)}{2x\left(\sqrt{4+x} + 2\right)} = \lim_{x \to 0} \frac{\sqrt{9+x} + \sqrt{9-x}}{2\left(\sqrt{4+x} + 2\right)}$$

$$= \frac{\sqrt{9+0} + \sqrt{9-0}}{2\left(\sqrt{4+0} + 2\right)} = \frac{3}{4}$$

Example 5

Find each of the following :

(1) $\lim\limits_{x \to -2} \dfrac{x - 3}{x^2 + 1}$

(2) $\lim\limits_{x \to 2} \left(\dfrac{x^2 - x}{x - 2} - \dfrac{2}{x - 2} \right)$

(3) $\lim\limits_{x \to -1} \dfrac{3x + 4}{x + 1}$

Solution

(1) $\because f(-2) = \frac{-2-3}{(-2)^2+1} = -1$

$\therefore \lim_{x \to -2} \frac{x-3}{x^2+1} = -1$

(2) Let $f(x) = \frac{x^2-x}{x-2} - \frac{2}{x-2}$

$\therefore f(x) = \frac{x^2-x-2}{x-2}$

$\therefore f(2) = \frac{4-2-2}{2-2} = \frac{\text{zero}}{\text{zero}}$

$\therefore \lim_{x \to 2} \left(\frac{x^2-x}{x-2} - \frac{2}{x-2} \right) = \lim_{x \to 2} \frac{x^2-x-2}{x-2}$

$= \lim_{x \to 2} \frac{(x-2)(x+1)}{x-2} = \lim_{x \to 2} (x+1) = 2+1 = 3$

(3) $\because f(-1) = \frac{-3+4}{-1+1} = \frac{1}{0}$ "undefined quantity"

$\therefore \lim_{x \to -1} \frac{3x+4}{x+1}$ does not exist.

Example 6

If $\lim_{x \to 3} \frac{f(x)-7}{x-3} = 4$ where $f(x)$ is a polynomial function , find $\lim_{x \to 3} f(x)$

Solution

$\because \lim_{x \to 3} \frac{f(x)-7}{x-3}$ exists and equals 4

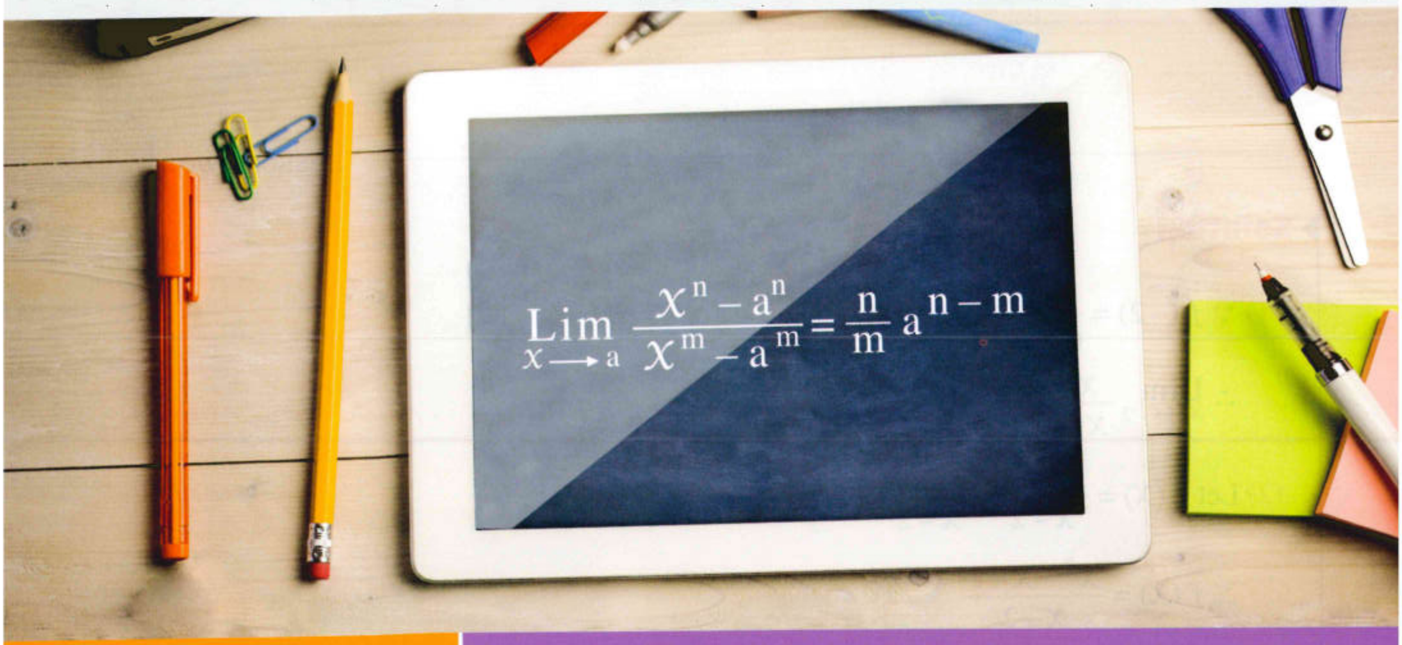
, $\because$ the denominator = zero at $x = 3$

$\therefore$ The numerator = zero at $x = 3$

$\therefore f(3) - 7 = \text{zero}$ $\qquad \therefore f(3) = 7$

, $\because f(x)$ is a polynomial function

$\therefore \lim_{x \to 3} f(x) = f(3) = 7$



Lesson Three — Theorem (4) "The law"

Theorem (4)

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\, a^{n-1} \quad \text{for every } n \in \mathbb{R} - \{0\}$$

To use this theorem , we must note that :

(1) The function must be in the form (or we can put it in the form) $\frac{x^n - a^n}{x - a}$

(2) The required is finding the limit when $x \longrightarrow a$

Example 1

Find each of the following :

(1) $\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$

(2) $\lim_{x \to -2} \frac{x^5 + 32}{x + 2}$

(3) $\lim_{x \to \frac{1}{2}} \frac{32\,x^5 - 1}{2\,x - 1}$

Solution

(1) $\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = \lim_{x \to 3} \frac{x^4 - 3^4}{x - 3} = 4 \times (3)^3 = 108$

Notice that : by direct substitution , we get $f(3) = \frac{\text{zero}}{\text{zero}}$

(2) $\lim_{x \to -2} \frac{x^5 + 32}{x + 2} = \lim_{x \to -2} \frac{x^5 - (-32)}{x - (-2)} = \lim_{x \to -2} \frac{x^5 - (-2)^5}{x - (-2)} = 5 \times (-2)^4 = 80$

(3) $\lim\limits_{x \to \frac{1}{2}} \dfrac{32x^5 - 1}{2x - 1} = \lim\limits_{x \to \frac{1}{2}} \dfrac{32\left[x^5 - \frac{1}{32}\right]}{2\left[x - \frac{1}{2}\right]}$

$$= \lim_{x \to \frac{1}{2}} 16 \times \frac{x^5 - \left(\frac{1}{2}\right)^5}{x - \frac{1}{2}} = 16 \lim_{x \to \frac{1}{2}} \frac{x^5 - \left(\frac{1}{2}\right)^5}{x - \frac{1}{2}} = 16 \times 5 \times \left(\frac{1}{2}\right)^4 = 5$$

Another solution :

As $x \longrightarrow \frac{1}{2}$, then $2x \longrightarrow 1$

$\therefore \lim\limits_{x \to \frac{1}{2}} \dfrac{32x^5 - 1}{2x - 1} = \lim\limits_{2x \to 1} \dfrac{(2x)^5 - 1^5}{(2x) - 1} = 5 \times (1)^4 = 5$

Corollaries

(1) $\lim\limits_{x \to 0} \dfrac{(x + a)^n - a^n}{x} = n\, a^{n-1}$

(2) $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x^m - a^m} = \dfrac{n}{m} (a)^{n-m}$, where $n \in \mathbb{R} - \{0\}$, $m \in \mathbb{R} - \{0\}$

Example 2

Find each of the following :

(1) $\lim\limits_{x \to 1} \dfrac{x^5 - 1}{x^3 - 1}$

(2) $\lim\limits_{x \to -3} \dfrac{x^5 + 243}{x^4 - 81}$

(3) $\lim\limits_{x \to 3} \dfrac{x^4 - 27x}{3x^4 - 243}$

(4) $\lim\limits_{x \to 1} \dfrac{(x + 1)^6 - 64}{(x + 1)^3 - 8}$

Solution

(1) $\lim\limits_{x \to 1} \dfrac{x^5 - 1}{x^3 - 1} = \lim\limits_{x \to 1} \dfrac{x^5 - 1^5}{x^3 - 1^3} = \dfrac{5}{3} \times (1)^{5-3} = \dfrac{5}{3}$

(2) $\lim\limits_{x \to -3} \dfrac{x^5 + 243}{x^4 - 81} = \lim\limits_{x \to -3} \dfrac{x^5 - (-243)}{x^4 - 81} = \lim\limits_{x \to -3} \dfrac{x^5 - (-3)^5}{x^4 - (-3)^4} = \dfrac{5}{4} \times (-3)^{5-4} = \dfrac{-15}{4}$

(3) $\lim\limits_{x \to 3} \dfrac{x^4 - 27x}{3x^4 - 243} = \lim\limits_{x \to 3} \dfrac{x(x^3 - 27)}{3(x^4 - 81)} = \lim\limits_{x \to 3} \dfrac{x}{3} \times \lim\limits_{x \to 3} \dfrac{x^3 - 3^3}{x^4 - 3^4} = \dfrac{3}{3} \times \dfrac{3}{4} \times 3^{3-4} = \dfrac{1}{4}$

(4) $\because x \longrightarrow 1$ $\quad \therefore x + 1 \longrightarrow 2$

$\therefore \lim\limits_{x \to 1} \dfrac{(x + 1)^6 - 64}{(x + 1)^3 - 8} = \lim\limits_{x + 1 \to 2} \dfrac{(x + 1)^6 - 2^6}{(x + 1)^3 - 2^3} = \dfrac{6}{3} (2)^{6-3} = 16$

Another solution by using factorization :

$$\lim_{x \to 1} \frac{(x + 1)^6 - 64}{(x + 1)^3 - 8} = \lim_{x \to 1} \frac{[(x + 1)^3 - 8]\,[(x + 1)^3 + 8]}{(x + 1)^3 - 8} = \lim_{x \to 1} \left((x + 1)^3 + 8\right) = 8 + 8 = 16$$

Example 3

Find each of the following :

(1) $\lim\limits_{x \to 0} \frac{(x+5)^4 - 625}{x}$ (2) $\lim\limits_{x \to 6} \frac{(x-5)^7 - 1}{x-6}$ (3) $\lim\limits_{h \to 0} \frac{(a+2h)^6 - a^6}{5h}$

Solution

(1) $\lim\limits_{x \to 0} \frac{(x+5)^4 - 625}{x} = \lim\limits_{x \to 0} \frac{(x+5)^4 - (5)^4}{x} = 4 \times 5^3 = 500$

(2) $\lim\limits_{x \to 6} \frac{(x-5)^7 - 1}{x-6} = \lim\limits_{x-5 \to 1} \frac{(x-5)^7 - 1^7}{(x-5) - 1} = 7 \times (1)^6 = 7$

(3) $\lim\limits_{h \to 0} \frac{(a+2h)^6 - a^6}{5h} = \lim\limits_{h \to 0} \frac{[(a+2h)^6 - a^6] \times \frac{2}{5}}{5h \times \frac{2}{5}} = \lim\limits_{2h \to 0} \frac{\frac{2}{5}[(a+2h)^6 - a^6]}{2h}$

$= \frac{2}{5} \times 6 \times a^5 = \frac{12}{5} a^5$

Remark

$\sqrt[n]{a} = a^{\frac{1}{n}}$ where $n \in \mathbb{Z}^+ - \{1\}$, $a \in \mathbb{R}^+$

For example :

$\sqrt{x} = x^{\frac{1}{2}}$, $\sqrt[3]{8} = 8^{\frac{1}{3}} = 2$

Example 4

Find each of the following :

(1) $\lim\limits_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$ (2) $\lim\limits_{x \to 1} \frac{\sqrt[5]{x} - 1}{\sqrt[3]{x} - 1}$

Solution

(1) $\lim\limits_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \lim\limits_{x \to 9} \frac{\sqrt{x} - \sqrt{9}}{x - 9} = \lim\limits_{x \to 9} \frac{x^{\frac{1}{2}} - 9^{\frac{1}{2}}}{x - 9} = \frac{1}{2} \times 9^{\frac{1}{2} - 1} = \frac{1}{2} \times 9^{-\frac{1}{2}} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

Another solution by using factorization :

$\lim\limits_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \lim\limits_{x \to 9} \frac{\sqrt{x} - 3}{(\sqrt{x} - 3)(\sqrt{x} + 3)} = \lim\limits_{x \to 9} \frac{1}{\sqrt{x} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$

(2) $\lim\limits_{x \to 1} \frac{\sqrt[5]{x} - 1}{\sqrt[3]{x} - 1} = \lim\limits_{x \to 1} \frac{x^{\frac{1}{5}} - 1^{\frac{1}{5}}}{x^{\frac{1}{3}} - 1^{\frac{1}{3}}} = \frac{\frac{1}{5}}{\frac{1}{3}} \times 1^{\frac{1}{5} - \frac{1}{3}} = \frac{3}{5}$

Example 5

Find each of the following :

(1) $\lim\limits_{x \to 0} \dfrac{\sqrt[4]{x+1}-1}{x}$ (2) $\lim\limits_{x \to 3} \dfrac{x^3+x^2-36}{x^4-81}$

Solution

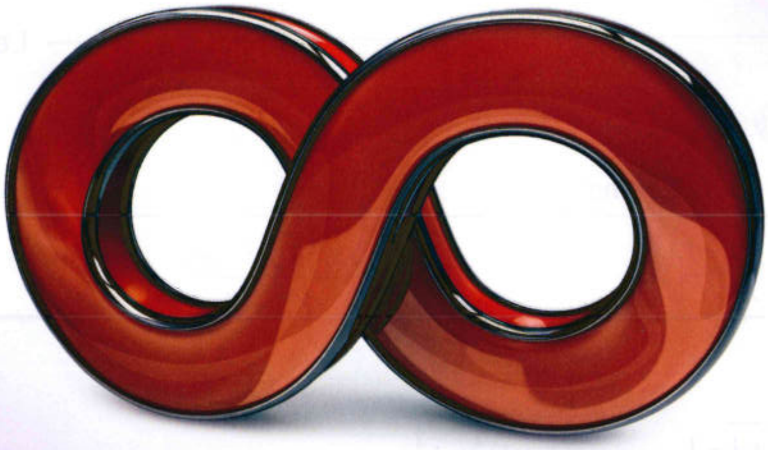
(1) $\lim\limits_{x \to 0} \dfrac{\sqrt[4]{x+1}-1}{x} = \lim\limits_{x \to 0} \dfrac{(x+1)^{\frac{1}{4}}-1^{\frac{1}{4}}}{x}$

$= \dfrac{1}{4} \times 1^{\frac{1}{4}-1} = \dfrac{1}{4}$

(2) $\lim\limits_{x \to 3} \dfrac{x^3+x^2-36}{x^4-81} = \lim\limits_{x \to 3} \dfrac{x^3-27+x^2-9}{x^4-81}$

$= \lim\limits_{x \to 3} \dfrac{x^3-3^3}{x^4-3^4} + \lim\limits_{x \to 3} \dfrac{x^2-3^2}{x^4-3^4}$

$= \dfrac{3}{4}(3)^{3-4} + \dfrac{2}{4}(3)^{2-4} = \dfrac{1}{4} + \dfrac{1}{18} = \dfrac{11}{36}$



Lesson Four — The limit of the function at infinity

WATCH VIDEO

The meaning of finding the limit of a function at infinity , is studying the behavior of this function when X (the independent variable) takes very large values.

If $f(X)$ approaches a real number ℓ as X tends to infinity , then we say that $f(X)$ has a limit «ℓ» at infinity , and we write it as : $\lim\limits_{X \to \infty} f(X) = \ell$

WATCH VIDEO

Illustrated Example 1

If $f : f(X) = \frac{2X+1}{X}$ and we want to study the behavior of the function f when X takes a very large values tend to ∞ let X takes the values : 1 , 10 , 100 , 1000 , 10000 , ...

We get the following table :

X	1	10	100	1000	10000	.....
$f(X) = \frac{2X+1}{X}$	3	2.1	2.01	2.001	2.0001	.....

From this table :

It's clear that when X takes values gradually increasing , we note that $f(X)$ approaches to 2 , then $f(X) \longrightarrow 2$ as $X \longrightarrow \infty$

and we write $\lim\limits_{X \to \infty} f(X) = 2$

Notice that :

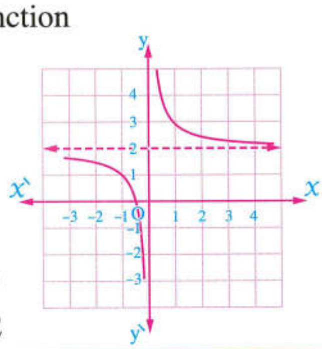
We can't get this result by the direct substitution which gives $f(X) = \frac{\infty}{\infty}$ (unspecified)

The graphical solution

At drawing the function

$f : f(X) = \frac{2X+1}{X}$

$= 2 + \frac{1}{X}$



From the graph , we notice that : when $X \longrightarrow \infty$, then $f(X) \longrightarrow 2$

Illustrated Example 2

If $f : f(x) = \frac{1}{x}$ and we want to study the behavior of this function as $x \longrightarrow \infty$

Form the table :

x	1	10	100	1000	10000	.....
$f(x) = \frac{1}{x}$	1	0.1	0.01	0.001	0.0001	.....

From this table we notice that :

as $x \longrightarrow \infty$, then $f(x) \longrightarrow 0$

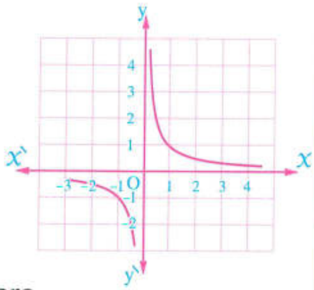
, then we can write $\lim\limits_{x \to \infty} \frac{1}{x} = 0$

This example leads us to the following theorem.

The graphical solution

At drawing the function $f : f(x) = \frac{1}{x}$

From the graph , we notice that : when $x \longrightarrow \infty$, then $f(x) \longrightarrow$ zero



Theorem (5)

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

Corollaries

If $a \in \mathbb{R}$, then :

(1) $\lim\limits_{x \to \infty} \frac{a}{x} =$ zero

(2) $\lim\limits_{x \to \infty} \frac{a}{x^n} =$ zero , $n \in \mathbb{R}^+$

Basic rules

* $\lim\limits_{x \to \infty} c = c$ where c is a constant
* $\lim\limits_{x \to \infty} x^n = \infty$ where n is a positive number
* Theorem (2) which is related by the limit of sum , difference , multiplying or dividing two functions at $x = a$ that we studied before is satisfied also when we put $x \longrightarrow \infty$ instead of $x \longrightarrow a$

Example 1

Find each of the following :

(1) $\lim\limits_{x \to \infty} \left(\frac{1}{x} + 2\right)$ (2) $\lim\limits_{x \to \infty} \left(3 - \frac{1}{x^2}\right)$ (3) $\lim\limits_{x \to \infty} \left(x^3 - 5x + 3\right)$

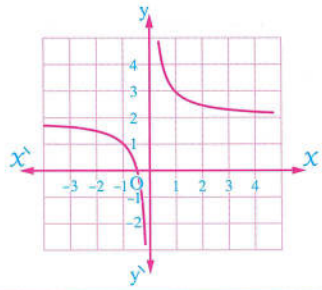
Solution

(1) $\lim\limits_{x \to \infty} \left(\frac{1}{x} + 2\right)$

$= \lim\limits_{x \to \infty} \frac{1}{x} + \lim\limits_{x \to \infty} 2$

$= 0 + 2 = 2$

The graph is for one's guidance only

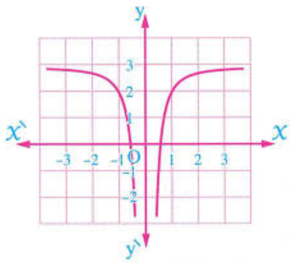


(2) $\lim\limits_{x \to \infty} \left(3 - \frac{1}{x^2}\right)$

$= \lim\limits_{x \to \infty} 3 - \lim\limits_{x \to \infty} \frac{1}{x^2}$

$= 3 - 0 = 3$

The graph is for one's guidance only



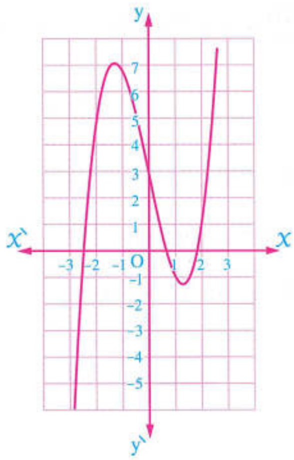
(3) Notice that , at direct substitution , the limit gives $(\infty - \infty)$ and it is an unspecified quantity , so we use the method of taking a common factor by the greatest power , then :

$\lim\limits_{x \to \infty} x^3 \left(1 - \frac{5}{x^2} + \frac{3}{x^3}\right)$

$= \lim\limits_{x \to \infty} x^3 \times \lim\limits_{x \to \infty} \left(1 - \frac{5}{x^2} + \frac{3}{x^3}\right)$

$= \infty \times 1 = \infty$

The graph is for one's guidance only



Finding the limit of a rational function at infinity

If the direct substitution by $x = \infty$ gives $\frac{\infty}{\infty}$, we divide each of numerator and denominator by x raised to the higher power in the denominator (degree of denominator) , then we use the theorem and its corollaries to get the limit (if it exists).

Example 2

Find each of the following :

(1) $\lim\limits_{x \to \infty} \frac{2x - 5}{3x - 7}$

(2) $\lim\limits_{x \to \infty} \frac{5x^2 - 3x + 6}{2x - 7x^2}$

(3) $\lim\limits_{x \to \infty} \frac{3x^2 - 5x}{2x^3 - 6x^2 + 4x - 1}$

(4) $\lim\limits_{x \to \infty} \frac{x^5 - 2x^2}{x^4 + 3x^3 - 1}$

Solution

(1) Dividing both numerator and denominator by x

$$\therefore \lim_{x \to \infty} \frac{2x - 5}{3x - 7} = \lim_{x \to \infty} \frac{2 - \frac{5}{x}}{3 - \frac{7}{x}} = \frac{2 - 0}{3 - 0} = \frac{2}{3}$$

(2) Dividing both numerator and denominator by x^2

$$\therefore \lim_{x \to \infty} \frac{5x^2 - 3x + 6}{2x - 7x^2} = \lim_{x \to \infty} \frac{5 - \frac{3}{x} + \frac{6}{x^2}}{\frac{2}{x} - 7} = -\frac{5}{7}$$

(3) Dividing both numerator and denominator by x^3

$$\therefore \lim_{x \to \infty} \frac{3x^2 - 5x}{2x^3 - 6x^2 + 4x - 1} = \lim_{x \to \infty} \frac{\frac{3}{x} - \frac{5}{x^2}}{2 - \frac{6}{x} + \frac{4}{x^2} - \frac{1}{x^3}} = \frac{\text{zero}}{2} = \text{zero}$$

(4) Dividing both numerator and denominator by x^4

$$\therefore \lim_{x \to \infty} \frac{x^5 - 2x^2}{x^4 + 3x^3 - 1} = \lim_{x \to \infty} \frac{x - \frac{2}{x^2}}{1 + \frac{3}{x} - \frac{1}{x^4}} = \frac{\infty - 0}{1 + 0 - 0} = \infty$$

Important remark

At finding $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are polynomial functions , then :

(1) The limit = a real number not equal to zero "if the degree of the numerator is equal to the degree of the denominator"

(2) The limit = zero "if the degree of the numerator is less than the degree of the denominator"

(3) The limit = $\pm \infty$ "if the degree of the numerator is greater than the degree of the denominator"

Example 3

Find each of the following :

(1) $\lim_{x \to \infty} \frac{(x-1)(x^2+1)}{x^2(5x-1)}$

(2) $\lim_{x \to \infty} \frac{(3x^3+2)^2(x^2-1)^3}{x^5(x+1)^7}$

Solution

(1) Dividing both numerator and denominator by x^3

$$\therefore \lim_{x \to \infty} \frac{(x-1)(x^2+1)}{x^2(5x-1)} = \lim_{x \to \infty} \frac{\left(1-\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right)}{1\left(5-\frac{1}{x}\right)} = \frac{1 \times 1}{1 \times 5} = \frac{1}{5}$$

(2) Dividing both numerator and denominator by x^{12}

$$\therefore \lim_{x \to \infty} \frac{(3x^3+2)^2(x^2-1)^3}{x^5(x+1)^7} = \lim_{x \to \infty} \frac{\left(3+\frac{2}{x^3}\right)^2\left(1-\frac{1}{x^2}\right)^3}{1\left(1+\frac{1}{x}\right)^7} = \frac{9 \times 1}{1} = 9$$

Example 4

Find each of the following :

(1) $\lim_{x \to \infty} \frac{2x^3-9}{|3x|^3+7}$

(2) $\lim_{x \to \infty} \frac{5x-6}{\sqrt{9x^2+7}}$

(3) $\lim_{x \to \infty} \frac{\sqrt[3]{8x^3-5x+1}}{3x-2}$

(4) $\lim_{x \to \infty} \left(\sqrt{x^2-x+1} - \sqrt{x^2+x+1}\right)$

Solution

(1) $\because x \longrightarrow \infty \qquad \therefore |x| = x$

$\therefore$ The limit $= \lim_{x \to \infty} \frac{2x^3-9}{27x^3+7}$

Dividing both numerator and denominator by x^3

$$\therefore \lim_{x \to \infty} \frac{2x^3-9}{27x^3+7} = \lim_{x \to \infty} \frac{2-\frac{9}{x^3}}{27+\frac{7}{x^3}} = \frac{2}{27}$$

(2) Dividing both numerator and denominator by $x = \sqrt{x^2}$

$$\therefore \lim_{x \to \infty} \frac{5x-6}{\sqrt{9x^2+7}} = \lim_{x \to \infty} \frac{5-\frac{6}{x}}{\sqrt{9+\frac{7}{x^2}}} = \frac{5}{\sqrt{9}} = \frac{5}{3}$$

Notice that :

When $x \longrightarrow \infty$

, then : $x = |x| = \sqrt{x^2}$

$= \sqrt[3]{x^3} = \sqrt[4]{x^4} = \ldots\ldots$

(3) Dividing both numerator and denominator by $x = \sqrt[3]{x^3}$

$$\therefore \lim_{x \to \infty} \frac{\sqrt[3]{8x^3 - 5x + 1}}{3x - 2} = \lim_{x \to \infty} \frac{\sqrt[3]{8 - \frac{5}{x^2} + \frac{1}{x^3}}}{3 - \frac{2}{x}} = \frac{\sqrt[3]{8}}{3} = \frac{2}{3}$$

(4) $\lim_{x \to \infty} \left(\sqrt{x^2 - x + 1} - \sqrt{x^2 + x + 1}\right)$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2 - x + 1} - \sqrt{x^2 + x + 1}}{1} \times \frac{\sqrt{x^2 - x + 1} + \sqrt{x^2 + x + 1}}{\sqrt{x^2 - x + 1} + \sqrt{x^2 + x + 1}}$$

$$= \lim_{x \to \infty} \frac{(x^2 - x + 1) - (x^2 + x + 1)}{\sqrt{x^2 - x + 1} + \sqrt{x^2 + x + 1}} = \lim_{x \to \infty} \frac{-2x}{\sqrt{x^2 - x + 1} + \sqrt{x^2 + x + 1}}$$

"dividing both numerator and denominator by $x = \sqrt{x^2}$"

$$= \lim_{x \to \infty} \frac{-2}{\sqrt{1 - \frac{1}{x} + \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x} + \frac{1}{x^2}}} = \frac{-2}{1 + 1} = -1$$

UNIT 4

Trigonometry

Unit Lessons

* Revision on the most important rules have been studied before.



Revision on the most important rules have been studied before

Radian and degree measures of an angle

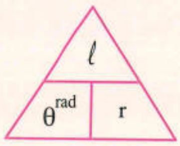
- The radian measure of a central angle in a circle

$$= \frac{\text{The length of the arc which the central angle subtends}}{\text{The length of the radius of this circle}}$$

i.e.

$\theta^{rad} = \frac{\ell}{r}$ and from it

$\ell = \theta^{rad}\, r$, $r = \frac{\ell}{\theta^{rad}}$



- The converting between the radian measure and the degree measure :

$\frac{x^\circ}{180^\circ} = \frac{\theta^{rad}}{\pi}$ and from it $\theta^{rad} = x^\circ \times \frac{\pi}{180^\circ}$, $x^\circ = \theta^{rad} \times \frac{180^\circ}{\pi}$

Notice that :

π in radians is equivalent to 180° in degrees.

The basic trigonometric identities

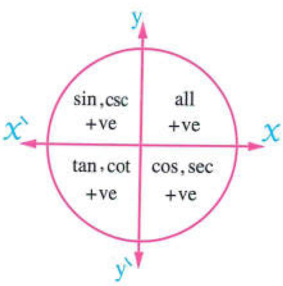
(1) $\cos^2\theta + \sin^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $1 + \cot^2\theta = \csc^2\theta$

(4) $\sin\theta \csc\theta = 1$, $\cos\theta \sec\theta = 1$, $\tan\theta \cot\theta = 1$

(5) $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$



UNIT

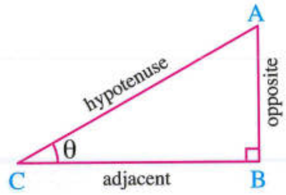
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Remember the following relations

(1) $\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{AB}{AC}$

(2) $\cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{BC}{AC}$

(3) $\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{AB}{BC}$



(4) If the terminal side of the directed angle of measure θ in the standard position intersects the unit circle at the point (x , y) , then :

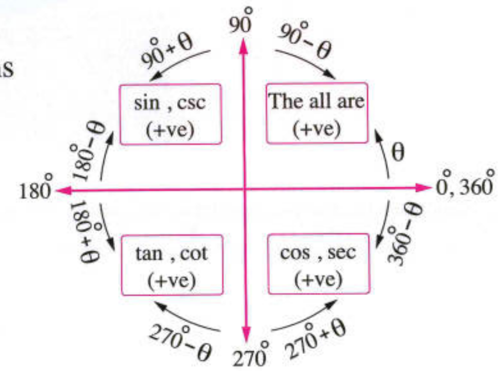
$x = \cos\theta$, $y = \sin\theta$ and $x^2 + y^2 = 1$

(5) The relations between the trigonometric functions of the related angles are identities :

For example :

$\sin(90° + \theta) = \cos\theta$

, $\tan(360° - \theta) = -\tan\theta$, ... each one of them is a trigonometric identity.



Areas of some geometric figures

* The area of $\Delta ABC = \frac{1}{2}\, a\, b \sin C = \frac{1}{2}\, b\, c \sin A = \frac{1}{2}\, a\, c \sin B$

* The area of $\Delta ABC = \sqrt{S(S-a)(S-b)(S-c)}$

  Where $S = \frac{a+b+c}{2}$

* The area of the quadrilateral = $\frac{1}{2}$ the product of the lengths of its diagonals × sine of the included angle between them.

* The area of the regular polygon whose number of its sides is n sides and the length of its side is $x = \frac{1}{4}\, n\, x^2 \cot\frac{\pi}{n}$

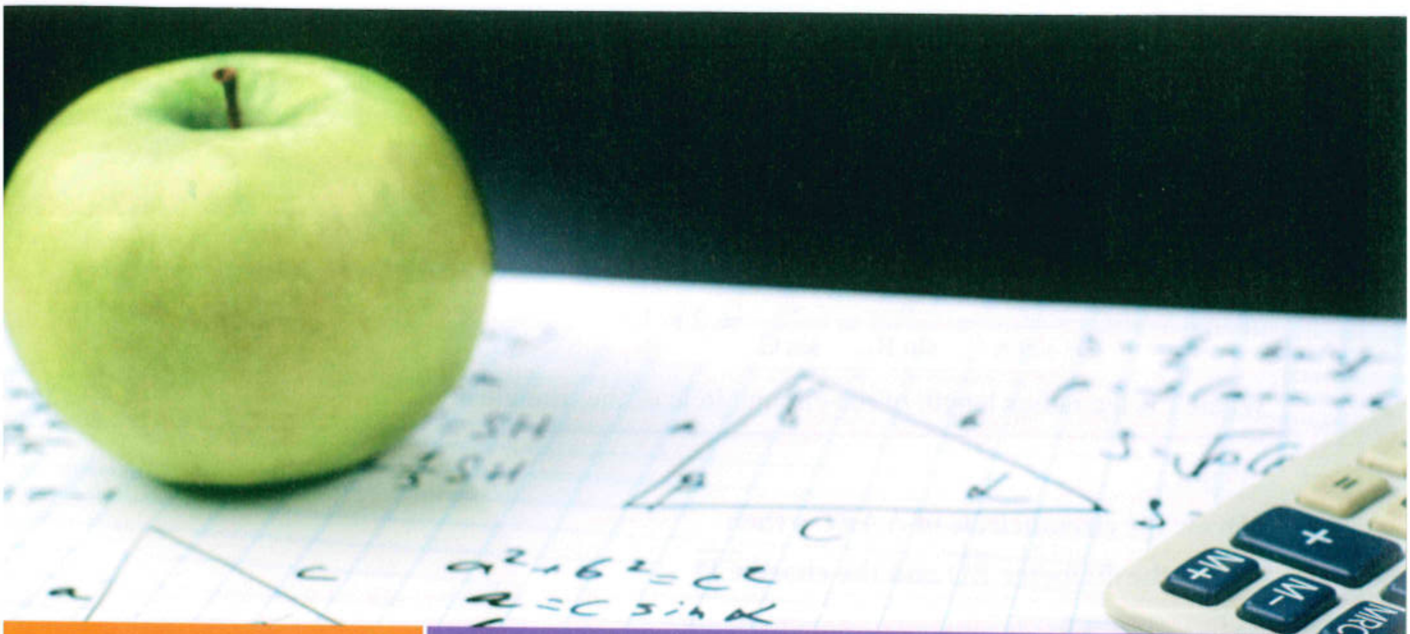
* The area of the circle $= \pi r^2$

  , the circumference of the circle $= 2\pi r$

* The area of the circular sector $= \frac{1}{2}\, \ell\, r = \frac{1}{2}\, \theta^{rad}\, r^2$

  , the perimeter of the circular sector $= 2r + \ell$

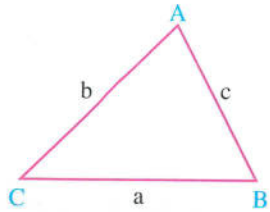
* The area of the circular segment $= \frac{1}{2}\, r^2 (\theta^{rad} - \sin\theta)$



Lesson One — The sine rule

> **"In any triangle the lengths of the sides are proportional to the sines of the opposite angles"**

i.e. In Δ ABC : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Where the symbols A , B and C represent the measures of the angles of Δ ABC and the symbols a , b and c represent the lengths of the sides $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$ respectively.

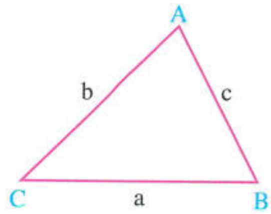
Proof :

$\because$ The area of the triangle = $\frac{1}{2}$ the product of the lengths of any two sides × sine of their included angle.

$\therefore$ The area of Δ ABC = $\frac{1}{2}$ ac sin B (1)

= $\frac{1}{2}$ cb sin A (2)

= $\frac{1}{2}$ ab sin C (3)



From (1) , (2) , (3) : $\therefore$ cb sin A = ac sin B = ab sin C

, dividing by abc : $\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (Q.E.D.)

Well known problem

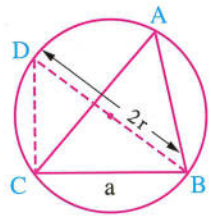
In any Δ ABC : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$

Where r is the radius length of the circumcircle of the triangle ABC

Proof :

Draw the circumcircle of Δ ABC , then draw the diameter $\overline{BD}$ and the chord $\overline{CD}$

First : If Δ ABC is an acute-angled triangle :



$\therefore m(\angle BCD) = 90°$ (drawn in a semicircle)

, $m(\angle A) = m(\angle D)$ (subtended by $\overset{\frown}{BC}$)

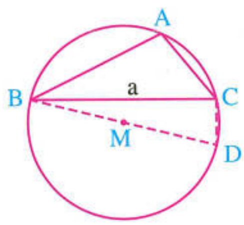
In Δ DBC : $\sin D = \frac{a}{BD} = \frac{a}{2r}$

$\therefore \sin A = \frac{a}{2r}$ $\qquad \therefore \frac{a}{\sin A} = 2r$

Similarly : $\frac{b}{\sin B} = 2r$ $\qquad , \frac{c}{\sin C} = 2r$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$ (Q.E.D.)

Second : If Δ ABC is an obtuse-angled triangle :



$\therefore m(\angle BDC) = 180° - m(\angle A)$

$\therefore \sin(180° - A) = \frac{BC}{BD}$

$\therefore \sin A = \frac{a}{2r}$ $\qquad \therefore \frac{a}{\sin A} = 2r$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$ (Q.E.D.)

Example 1

In Δ ABC , if a = 10 cm. , m ($\angle$ A) = 30° , m ($\angle$ B) = 45° , find using the calculator each of b and c to the nearest one decimal , find also the area of Δ ABC to the nearest whole number.

Solution

$\because m(\angle C) = 180° - (30° + 45°) = 105°$

, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad \therefore \frac{10}{\sin 30°} = \frac{b}{\sin 45°} = \frac{c}{\sin 105°}$

$\therefore b = \frac{10 \sin 45°}{\sin 30°} \approx 14.1$ cm. , $c = \frac{10 \sin 105°}{\sin 30°} \approx 19.3$ cm.

, the area of Δ ABC $= \frac{1}{2}$ bc sin A $= \frac{1}{2} \times 14.1 \times 19.3 \sin 30° \approx 68$ cm^2

Example 2

In Δ ABC : m ($\angle$ A) = $25^\circ 42'$, m ($\angle$ B) = $118^\circ 48'$, AB = 20 cm.

Find the length of each of : $\overline{BC}$, $\overline{AC}$

Solution

$\because$ m ($\angle$ C) = $180^\circ - (25^\circ 42' + 118^\circ 48') = 35^\circ 30'$

, $\frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C}$

$\therefore \frac{BC}{\sin 25^\circ 42'} = \frac{AC}{\sin 118^\circ 48'} = \frac{20}{\sin 35^\circ 30'}$

$\therefore BC = \frac{20 \sin 25^\circ 42'}{\sin 35^\circ 30'} \approx 14.9$ cm. , $AC = \frac{20 \sin 118^\circ 48'}{\sin 35^\circ 30'} \approx 30.2$ cm.

Example 3

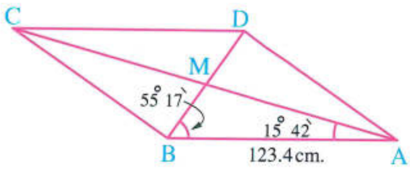
ABCD is a parallelogram in which :

AB = 123.4 cm. , m ($\angle$ CAB) = $15^\circ 42'$, m ($\angle$ DBA) = $55^\circ 17'$ Find :

(1) The length of each of the two diagonals $\overline{BD}$, $\overline{AC}$

(2) The area of ▱ ABCD

Solution



Let $\overline{AC} \cap \overline{BD} = \{M\}$

$\therefore$ In Δ MAB :

m ($\angle$ AMB) = $180^\circ - (15^\circ 42' + 55^\circ 17') = 109^\circ 1'$

$\therefore \frac{123.4}{\sin 109^\circ 1'} = \frac{BM}{\sin 15^\circ 42'} = \frac{AM}{\sin 55^\circ 17'}$

$\therefore BM = \frac{123.4 \sin 15^\circ 42'}{\sin 109^\circ 1'} \approx 35.3$ cm.

$\therefore BD = 2\ BM = 70.6$ cm.

$, AM = \frac{123.4 \sin 55^\circ 17'}{\sin 109^\circ 1'} \approx 107.3$ cm.

$\therefore AC = 2\ AM = 214.6$ cm.

, the area of ▱ ABCD = 4 the area of Δ MAB

$= 4 \times \frac{1}{2} \times BM \times AM \sin (\angle AMB)$

$= 4 \times \frac{1}{2} \times 35.3 \times 107.3 \sin 109^\circ 1' \approx 7162\ cm^2$

Example 4

ABC is a triangle in which : $\frac{1}{2}\sin A = \frac{1}{3}\sin B = \frac{1}{4}\sin C$

Find the lengths of its sides , given that its perimeter = 18 cm.

Solution

$\because \frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{4}$ $\quad\therefore \frac{2}{\sin A} = \frac{3}{\sin B} = \frac{4}{\sin C}$

$\therefore a : b : c = 2 : 3 : 4$

Let $a = 2k$, $b = 3k$, $c = 4k$

, $\because$ the perimeter of $\Delta ABC = 18$ cm. $\quad\therefore 2k + 3k + 4k = 18$

$\therefore 9k = 18$ $\quad\therefore k = 2$

$\therefore a = 4$ cm. , $b = 6$ cm. , $c = 8$ cm.

Example 5

If the perimeter of $\Delta ABC = 24$ cm. , $m(\angle B) = 30°$, $m(\angle C) = 48°$, find b

Solution

$\because m(\angle A) = 180° - (30° + 48°) = 102°$

$\therefore \frac{a}{\sin 102°} = \frac{b}{\sin 30°} = \frac{c}{\sin 48°}$

, $\because \frac{\text{the sum of antecedents}}{\text{the sum of consequents}} = \text{one of the ratios}$

$\therefore \frac{a + b + c}{\sin 102° + \sin 30° + \sin 48°} = \frac{b}{\sin 30°}$

$\therefore \frac{24}{\sin 102° + \sin 30° + \sin 48°} = \frac{b}{\sin 30°}$

$\therefore b = \frac{24 \sin 30°}{\sin 102° + \sin 30° + \sin 48°} \approx 5.4$ cm.

Example 6

In ΔABC , if $b = 7$ cm. , $m(\angle B) = 30°$, $c = 9$ cm. , calculate the radius length of the circumcircle of ΔABC , calculate also $m(\angle A)$ to the nearest degree.

Solution

$\because \frac{b}{\sin B} = 2r$ $\quad\quad$ $\therefore \frac{7}{\sin 30°} = 2r$

$\therefore r = \frac{7}{2 \sin 30°} = 7$ cm.

, $\because \frac{c}{\sin C} = 2r$ $\quad\quad$ $\therefore \frac{9}{\sin C} = 14$

$\therefore \sin C = \frac{9}{14}$ $\quad\quad$ $\therefore m(\angle C) \approx 40°$

$\therefore m(\angle A) = 180° - (30° + 40°) = 110°$

or $m(\angle C) \approx 140°$

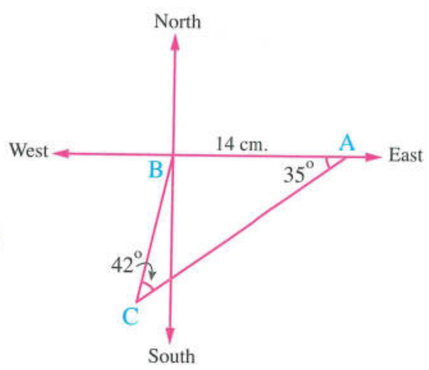
$\therefore m(\angle A) = 180° - (30° + 140°) = 10°$

Notice that :

There are two triangles satisfy these givens and this case is called ambiguous case and we will study it at the lesson "solution of the triangle"

Example 7

The opposite figure represents three positions of cities A , B and C. If the distance between A and B in the drawing is 14 cm. , $m(\angle A) = 35°$, $m(\angle C) = 42°$, find to the nearest km. the distance between the two cities B and C , if each 1 cm. in the drawing represents 20 km. in real.



Solution

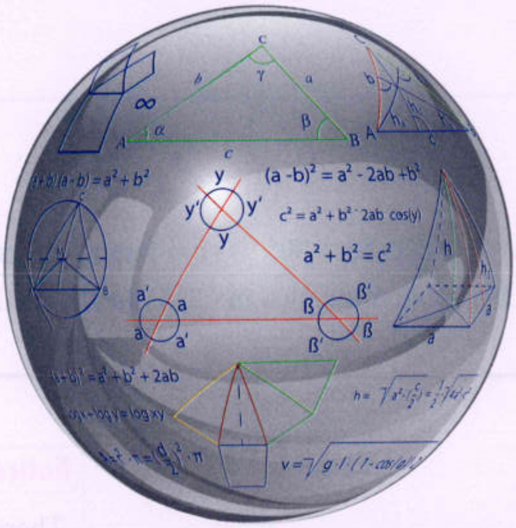
$\because \frac{14}{\sin 42°} = \frac{BC}{\sin 35°}$

$\therefore BC = \frac{14 \sin 35°}{\sin 42°} \approx 12$ cm.

, $\because$ each 1 cm. in the drawing represents 20 km. in real

$\therefore$ 12 cm. in the drawing represent $20 \times 12 = 240$ km.

i.e. The distance between the two cities B and C is 240 km.



Lesson Two — The cosine rule

In any triangle ABC :

$a^2 = b^2 + c^2 - 2\,bc\cos A$	, hence	$\cos A = \dfrac{b^2 + c^2 - a^2}{2\,bc}$
$b^2 = c^2 + a^2 - 2\,ca\cos B$	, hence	$\cos B = \dfrac{c^2 + a^2 - b^2}{2\,ca}$
$c^2 = a^2 + b^2 - 2\,ab\cos C$	, hence	$\cos C = \dfrac{a^2 + b^2 - c^2}{2\,ab}$

This rule is used if :
The lengths of two sides in Δ ABC and the measure of their included angle are given.

This rule is used if :
The lengths of the sides of Δ ABC or the ratio among these lengths are given.

Proof : To prove that : $a^2 = b^2 + c^2 - 2\,bc\cos A$

First : If Δ ABC is an acute-angled triangle :

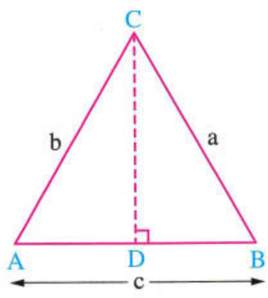
Draw $\overrightarrow{CD} \perp \overline{AB}$ to intersect it at D

In Δ CDB :

$\because (BC)^2 = (CD)^2 + (DB)^2$

$\therefore (BC)^2 = (CD)^2 + (AB - AD)^2$

$\qquad = (CD)^2 + (AD)^2 + (AB)^2 - 2\,(AB)\,(AD)$



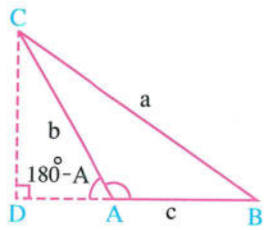
, ∵ $(CD)^2 + (AD)^2 = (AC)^2$, $AD = AC \cos A$

∴ $(BC)^2 = (AC)^2 + (AB)^2 - 2\,(AB)\,(AC)\cos A$

∴ $a^2 = b^2 + c^2 - 2\,bc\cos A$

∴ $\cos A = \dfrac{b^2 + c^2 - a^2}{2\,bc}$ (Q.E.D)

Second : If Δ ABC is an obtuse-angled triangle at A :



Draw $\overrightarrow{CD} \perp \overrightarrow{BA}$ to intersect it at D

In Δ CDB : ∵ $(BC)^2 = (CD)^2 + (DB)^2$

∴ $(BC)^2 = (CD)^2 + (AB + AD)^2$

$= (CD)^2 + (AD)^2 + (AB)^2 + 2\,(AB)\,(AD)$

, ∵ $(CD)^2 + (AD)^2 = (AC)^2$

, $AD = AC\cos(180° - A) = -AC\cos A$

∴ $(BC)^2 = (AC)^2 + (AB)^2 - 2\,(AB)\,(AC)\cos A$

∴ $a^2 = b^2 + c^2 - 2\,bc\cos A$

∴ $\cos A = \dfrac{b^2 + c^2 - a^2}{2\,bc}$ (Q.E.D.)

* **Notice that :** The cosine rule is also true in case of the right-angled triangle

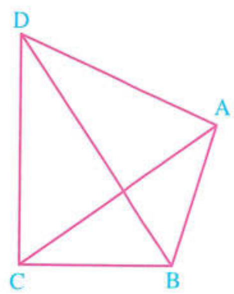
[putting : $\cos A = \cos 90° = 0$]

Remarks

* To find the measure of an angle of a triangle , it is better to use the cosine rule , because it determines the type of the angle whether it is acute or obtuse.

* If a : b : c = 2 : 3 : 4 , then we suppose : $a = 2\,k$, $b = 3\,k$, $c = 4\,k$ where $k \in \mathbb{R}^*$, then we substitute in the cosine rule to find the measures of the angles of Δ ABC

* To prove that ABCD is a cyclic quadrilateral :



– We prove that there are two opposite supplementary angles :

m (∠ A) + m (∠ C) = 180° *i.e.* cos A + cos C = zero

or m (∠ B) + m (∠ D) = 180° *i.e.* cos B + cos D = zero

– We prove that the measures of two angles drawn on one base and on one side of it are equal : m (∠ BAC) = m (∠ BDC)

i.e. cos (∠ BAC) = cos (∠ BDC)

UNIT 4

Example 1

In Δ ABC : If b = 30 cm. , c = 14 cm. , m ($\angle$ A) = 60° , find a

Solution

$\because a^2 = b^2 + c^2 - 2\,bc\cos A$

$\therefore a^2 = (30)^2 + (14)^2 - 2 \times 30 \times 14 \times \cos 60° = 676$

$\therefore a = \sqrt{676} = 26$ cm.

Example 2

XYZ is a triangle in which : $x = 4$ cm. , y = 5 cm. , z = 6 cm.

Calculate the measure of its greatest angle and its area.

Solution

$\because \angle Z$ is the greatest angle.

$\therefore \cos Z = \dfrac{x^2 + y^2 - z^2}{2\,x\,y} = \dfrac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5} = 0.125$

$\therefore m(\angle Z) \approx 82° \; 49' \; 9''$

, the area of Δ XYZ $= \dfrac{1}{2}\, x y \sin Z = \dfrac{1}{2} \times 4 \times 5 \times \sin 82° \; 49' \; 9'' \approx 9.9\ \text{cm.}^2$

Example 3

In Δ ABC : $\frac{1}{2}\sin A = \frac{1}{3}\sin B = \frac{1}{4}\sin C$, calculate m ($\angle$ C)

Solution

$\because \dfrac{\sin A}{2} = \dfrac{\sin B}{3} = \dfrac{\sin C}{4}$

$\therefore \dfrac{2}{\sin A} = \dfrac{3}{\sin B} = \dfrac{4}{\sin C}$

$\therefore a : b : c = 2 : 3 : 4$

Let a = 2 k , b = 3 k , c = 4 k

$\therefore \cos C = \dfrac{a^2 + b^2 - c^2}{2\,a\,b} = \dfrac{4k^2 + 9k^2 - 16k^2}{2 \times 2k \times 3k} = \dfrac{-3k^2}{12k^2} = -\dfrac{1}{4}$

$\therefore m(\angle C) = 104° \; 28' \; 39''$

Example 4

In Δ ABC : a = 13 cm. , b = 14 cm. , c = 15 cm. Find the radius length of its circumcircle.

Solution

$\because \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{196 + 225 - 169}{2 \times 14 \times 15} = \frac{3}{5}$

$\therefore \sin A = \frac{4}{5}$, $\because \frac{a}{\sin A} = 2r$

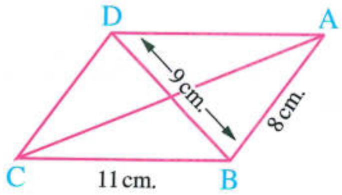
$\therefore \frac{13}{\frac{4}{5}} = 2r$ $\therefore r = \frac{13}{2 \times \frac{4}{5}} = 8\frac{1}{8}$ cm.

Example 5

ABCD is a parallelogram in which : AB = 8 cm. , BC = 11 cm. , BD = 9 cm.
Find the length of $\overline{AC}$

Solution

In Δ ABD : $\cos A = \frac{(8)^2 + (11)^2 - (9)^2}{2 \times 8 \times 11} = \frac{13}{22}$



, $\because m(\angle A) + m(\angle B) = 180°$ (two consecutive angles in ▱ ABCD)

$\therefore \cos B = -\cos A = \frac{-13}{22}$

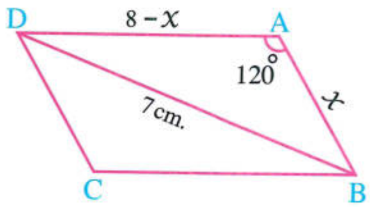
$\therefore$ In Δ ABC : $(AC)^2 = (8)^2 + (11)^2 - 2 \times 8 \times 11 \times \frac{-13}{22} = 289$

$\therefore$ AC = 17 cm.

Example 6

ABCD is a parallelogram in which : m ($\angle$ A) = 120° , its perimeter = 16 cm.
, the length of its greater diagonal = 7 cm.
Find the area of ▱ ABCD , given that : AB < BC

Solution



$\because \frac{1}{2}$ perimeter $= \frac{16}{2} = 8$ cm.

Let AB = X cm.

$\therefore$ AD = $(8 - X)$ cm.

In Δ ABD : $\because (BD)^2 = (AB)^2 + (AD)^2 - 2\,(AB)\,(AD)\cos 120°$

$\therefore 49 = x^2 + (8-x)^2 - 2x(8-x) \times \left(-\frac{1}{2}\right)$

$\therefore 49 = x^2 + 64 - 16x + x^2 + 8x - x^2$

$\therefore x^2 - 8x + 15 = 0$

$\therefore (x-3)(x-5) = 0$

$\therefore x = 3$ or $x = 5$

, $\because$ AB < BC

$\therefore$ AB = 3 cm. , AD = 5 cm.

$\therefore$ The area of (▱ ABCD) = 2 the area of (Δ ABD)

$$= 2 \times \frac{1}{2} \times 3 \times 5 \sin 120° = \frac{15\sqrt{3}}{2} \text{ cm}^2$$

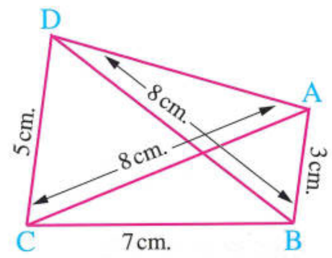
Example 7

ABCD is a quadrilateral in which :

AB = 3 cm. , BC = 7 cm. , CD = 5 cm. , AC = BD = 8 cm.

Prove that : ABCD is a cyclic quadrilateral.

Solution



In Δ BAC : $\cos(\angle BAC) = \frac{3^2 + 8^2 - 7^2}{2 \times 3 \times 8} = \frac{1}{2}$

, in Δ BDC : $\cos(\angle BDC) = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8} = \frac{1}{2}$

$\therefore \cos(\angle BAC) = \cos(\angle BDC)$

$\therefore m(\angle BAC) = m(\angle BDC)$ (and they are drawn on $\overline{BC}$ and on one side of it)

$\therefore$ ABCD is a cyclic quadrilateral.



Lesson Three — Solution of the triangle

WATCH VIDEO

Solution of the triangle means to find the lengths of its sides and the measures of its angles , if it is given three of these six elements (one of them at least is the length of one side).

WATCH VIDEO

There are four cases for solving a triangle :

First case: Solving the triangle given the length of one side and the measures of two angles :

In ΔABC , if $m(\angle A)$, $m(\angle B)$, a are given :

(1) Use the relation : $m(\angle C) = 180^\circ - [m(\angle A) + m(\angle B)]$ to find $m(\angle C)$

(2) Use the law : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ to find b , c

Example 1

Solve the triangle ABC in which : $m(\angle A) = 38^\circ 52'$, $m(\angle B) = 96^\circ 51'$, a = 22.3 cm.

Solution

$m(\angle C) = 180^\circ - (38^\circ 52' + 96^\circ 51') = 44^\circ 17'$

, $\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\therefore \frac{22.3}{\sin 38^\circ 52'} = \frac{b}{\sin 96^\circ 51'} = \frac{c}{\sin 44^\circ 17'}$

$\therefore b = \frac{22.3 \sin 96^\circ 51'}{\sin 38^\circ 52'} \approx 35.3$ cm.

, $c = \frac{22.3 \sin 44^\circ 17'}{\sin 38^\circ 52'} \approx 24.8$ cm.

Second case — Solving the triangle given the lengths of two sides and the measure of the included angle :

In ΔABC , if a , b , m ($\angle$ C) are given :

(1) Use the law : $c^2 = a^2 + b^2 - 2\,ab\cos C$ to find c

(2) It's better to use the law : $\cos A = \frac{b^2 + c^2 - a^2}{2\,b\,c}$ to find m ($\angle$ A) because it determines the acute or obtuse angle (or you can use the sine law to find the measure of the angle opposite to the smaller of the two given sides)

(3) Use the relation : $m(\angle B) = 180^\circ - [m(\angle A) + m(\angle C)]$ to find m ($\angle$ B)

Example 2

Solve the triangle ABC in which : a = 8 cm. , b = 5 cm. , m ($\angle$ C) = $60^\circ\, 2'$

Solution

$\because c^2 = a^2 + b^2 - 2\,a\,b\cos C = 64 + 25 - 2 \times 8 \times 5 \cos 60^\circ\, 2' \approx 49.04$

$\therefore c \approx$ 7 cm.

$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2\,b\,c} = \frac{25 + 49 - 64}{2 \times 5 \times 7} = \frac{1}{7}$

$\therefore m(\angle A) \approx 81^\circ\, 47'$

$\therefore m(\angle B) = 180^\circ - (60^\circ\, 2' + 81^\circ\, 47') = 38^\circ\, 11'$

Another solution :

After finding c , you can find m ($\angle$ B) using the sine law because $\angle$ B is opposite to the smaller given side.

$\because \frac{c}{\sin C} = \frac{b}{\sin B}$ $\qquad \therefore \frac{7}{\sin 60^\circ\, 2'} = \frac{5}{\sin B}$

$\therefore \sin B = \frac{5 \sin 60^\circ\, 2'}{7}$ $\qquad \therefore m(\angle B) \approx 38^\circ\, 14'$ or $141^\circ\, 46'$

, $\because$ b is not the length of the longest side.

$\therefore \angle$ B cannot be obtuse. $\qquad \therefore m(\angle B) = 38^\circ\, 14'$

$\therefore m(\angle A) = 180^\circ - (60^\circ\, 2' + 38^\circ\, 14') = 81^\circ\, 44'$

* **Notice that :** The differences in the measures of angles between the two solutions is due to approximation in calculators.

Third case — Solving the triangle given the lengths of the three sides :

In Δ ABC , if a , b , c are given :

(1) Use the law : $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ to find m ($\angle$ A)

(2) Use the law : $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$ to find m ($\angle$ B)

(3) Use the relation : m ($\angle$ C) = 180° – [m ($\angle$ A) + m ($\angle$ B)] to find m ($\angle$ C)

Example 3

Solve the triangle ABC in which : a = 5 cm. , b = 7 cm. , c = 11 cm.

Solution

$\because \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{49 + 121 - 25}{2 \times 7 \times 11} = \frac{145}{154}$

$\therefore$ m ($\angle$ A) $\approx$ 19° 41`

, $\because \cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{121 + 25 - 49}{2 \times 11 \times 5} = \frac{97}{110}$

$\therefore$ m ($\angle$ B) $\approx$ 28° 8`

$\therefore$ m ($\angle$ C) = 180° – (19° 41` + 28° 8`) = 132° 11`

Remember that

The sum of any two side lengths in a triangle is greater than the length of the third side.

For example :

If a = 2 cm. , b = 5 cm. and c = 8 cm. , then these lengths cannot be side lengths of a triangle.

Example 4

Solve the triangle ABC in which : m ($\angle$ A) = 40° , m ($\angle$ C) = 35° , the radius length of its circumcircle = 6 cm.

Solution

m ($\angle$ B) = 180° – (40° + 35°) = 105°

$\therefore \frac{a}{\sin 40°} = \frac{b}{\sin 105°} = \frac{c}{\sin 35°} = 12$

$\therefore$ a $\approx$ 7.7 cm. , b $\approx$ 11.6 cm. , c $\approx$ 6.9 cm.

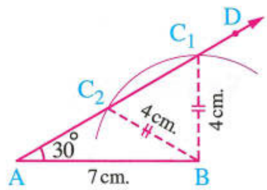
UNIT 4

Activity **Solving the triangle given the lengths of two sides and the measure of the opposite angle to one of them «Ambiguous case»**

Illustrated Example

Using the geometric tools , draw Δ ABC in which AB = 7 cm. , m (∠ A) = 30° and BC = 4 cm. , then verify your answer using the sine rule.

Solution



* We draw a line segment $\overline{AB}$ of length 7 cm.
* We draw ∠ A of measure 30° with $\overline{AB}$ and it is ∠ BAD
* We place the sharp point of the compasses at the point B and adjust it with length 4 cm. , and draw an arc intersecting the straight line $\overleftrightarrow{AD}$ at C
* We notice that the point C has two positions , i.e. we can draw two triangles having the same previous conditions and they are ABC_1 and ABC_2 , by measuring we find that :

  m (∠ C) ≈ 61° in Δ ABC_1 or m (∠ C) ≈ 119° in Δ ABC_2

Verifying the answer by using the sine rule :

$\because \frac{a}{\sin A} = \frac{c}{\sin C}$ $\qquad \therefore \frac{4}{\sin 30^\circ} = \frac{7}{\sin C}$

$\therefore \sin C = \frac{7 \sin 30^\circ}{4} = \frac{7}{8}$ (positive)

∴ ∠ C lies in the first quadrant (acute) or in the second quadrant (obtuse)

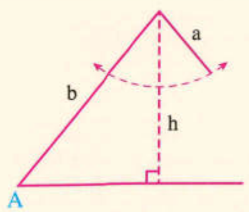
∴ m (∠ C) ≈ 61° or m (∠ C) ≈ 119°

Generally , by using the geometric solution , we can reach to the following :

In Δ ABC , if a , b and m (∠ A) are given , then we find h = b sin A , and to find the possible solutions of the triangle , we compare between the values a , b and h as follows :

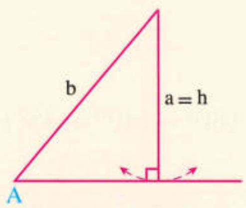
First : If ∠ A is acute and :

(1)



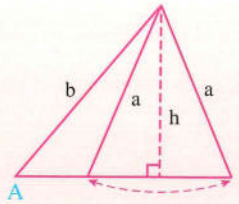
a < h , then we cannot draw the triangle.

(2)



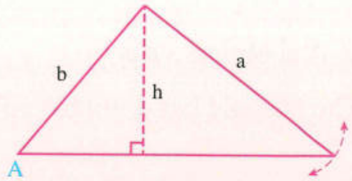
a = h , then we can draw a unique right-angled triangle.

(3)



$h < a < b$, then we can draw two triangles.

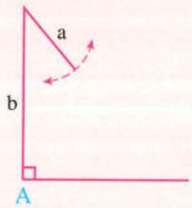
(4)



$a \geq b$, then we can draw a unique triangle.

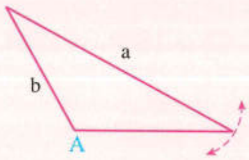
Second : If ∠ A is right or obtuse and :

(1)



$a \leq b$, then we cannot draw a triangle.

(2)



$a > b$, then we can draw a unique triangle

In this case , we can solve the triangle using the sine rule directly without determining the number of possible triangles with considering the following :

(1) ∠ A lies in the first quadrant (if it is acute) and lies in the second quadrant (if it is obtuse)

(2) The range of the sine function is $[-1 , 1]$

(3) If the triangle has an obtuse angle , then the other two angles must be acute angles.

Example 5

Show if the following conditions satisfy the existence of one triangle or more , or don't satisfy the existence of any triangle at all , then find the possible solutions :

(1) ABC is a triangle in which m (∠ A) = 112° , a = 7 cm. and b = 4 cm.

(2) ABC is a triangle in which m (∠ A) = 112° , a = 4 cm. and b = 7 cm.

(3) LMN is a triangle in which m (∠ L) = 50° , ℓ = 4 cm. and m = 7 cm.

(4) DEF is a triangle in which m (∠ D) = 60° , d = 7.5 cm. and e = $5\sqrt{3}$ cm.

(5) LMN is a triangle in which m (∠ L) = 30° , ℓ = 6 cm. and m = 9 cm.

UNIT 4

Solution

(1) $\because \angle A$ is obtuse , $a > b$

$\therefore$ The triangle has a unique solution.

, $\because \frac{a}{\sin A} = \frac{b}{\sin B}$

$\therefore \frac{7}{\sin 112^\circ} = \frac{4}{\sin B}$

$\therefore \sin B = \frac{4 \sin 112^\circ}{7} \approx 0.5298$

$\therefore m(\angle B) \approx \boxed{32^\circ}$

$\therefore m(\angle C) = 180^\circ - (112^\circ + 32^\circ) = \boxed{36^\circ}$

, $\because \frac{a}{\sin A} = \frac{c}{\sin C}$ $\therefore \frac{7}{\sin 112^\circ} = \frac{c}{\sin 36^\circ}$

$\therefore c \approx \boxed{4.4 \text{ cm.}}$

Notice that :

The triangle has one obtuse angle at most.

, $\because \angle A$ is obtuse.

$\therefore \angle B$ must be acute.

$\therefore \angle B$ lies in the first quadrant only.

(2) $\because \angle A$ is obtuse , $a < b$

$\therefore$ The conditions do not satisfy the existence of any triangle at all.

Notice that :

$\because \frac{4}{\sin 112^\circ} = \frac{7}{\sin B}$ $\therefore \sin B = \frac{7 \sin 112^\circ}{4} \approx 1.6$

and this is impossible because $\sin B \notin [-1, 1]$

(3) $\because \angle L$ is acute , $h = m \sin L = 7 \sin 50^\circ \approx 5.4$ cm.

, $\because \ell < h$

$\therefore$ The conditions do not satisfy the existence of any triangle at all.

(4) $\because \angle D$ is acute , $h = e \sin D = 5\sqrt{3} \sin 60^\circ = 7.5$ cm.

, $\because d = h$

$\therefore$ There is a unique solution to the triangle which is right-angled at E

$\therefore m(\angle F) = 180^\circ - (60^\circ + 90^\circ) = \boxed{30^\circ}$

, $f = \sqrt{(5\sqrt{3})^2 - (7.5)^2} = \boxed{\frac{5\sqrt{3}}{2} \text{ cm.}}$

(5) $\because \angle L$ is acute , $h = m \sin L = 9 \sin 30^\circ = 4.5$ cm.

, $\because 4.5 < 6 < 9$ *i.e.* $h < \ell < m$

$\therefore$ There are two solutions to the triangle

$\because \dfrac{\ell}{\sin L} = \dfrac{m}{\sin M}$

$\therefore \dfrac{6}{\sin 30^\circ} = \dfrac{9}{\sin M}$

$\therefore \sin M = \dfrac{3}{4}$

$\therefore \angle M$ lies in the first or the second quadrant.

$\therefore m(\angle M) \approx \boxed{48^\circ\ 35'\ 25''}$

$\therefore m(\angle N)$

$= 180^\circ - (30^\circ + 48^\circ\ 35'\ 25'')$

$= \boxed{101^\circ\ 24'\ 35''}$

$, \because \dfrac{\ell}{\sin L} = \dfrac{n}{\sin N}$

$\therefore \dfrac{6}{\sin 30^\circ} = \dfrac{n}{\sin 101^\circ\ 24'\ 35''}$

$\therefore n \approx \boxed{11.76 \text{ cm.}}$

or

$\therefore m(\angle M) = 180^\circ - 48^\circ\ 35'\ 25''$

$= \boxed{131^\circ\ 24'\ 35''}$

$\therefore m(\angle N)$

$= 180^\circ - (30^\circ + 131^\circ\ 24'\ 35'')$

$= \boxed{18^\circ\ 35'\ 25''}$

$, \because \dfrac{6}{\sin 30^\circ} = \dfrac{n}{\sin 18^\circ\ 35'\ 25''}$

$\therefore n \approx \boxed{3.83 \text{ cm.}}$

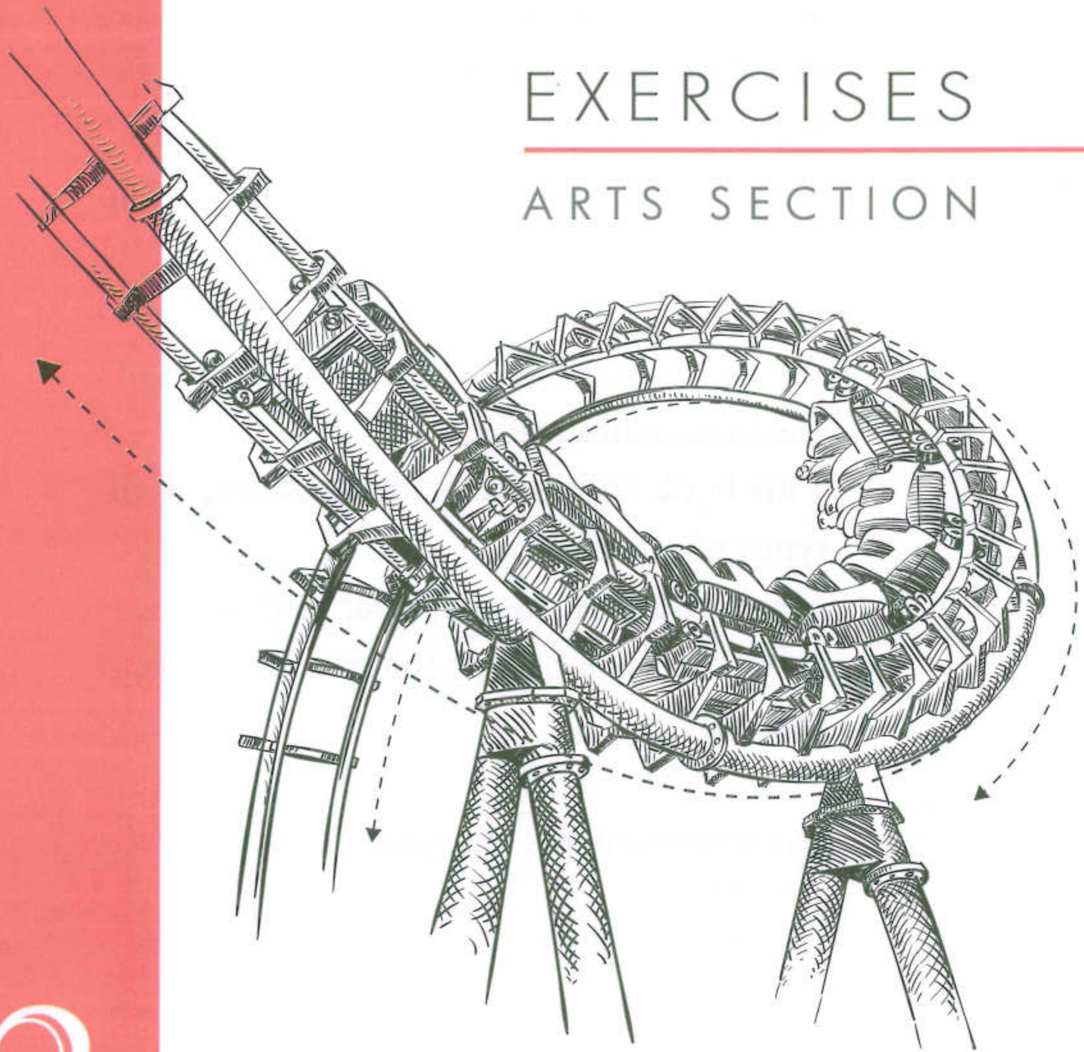
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General Mathematics

By a group of supervisors

EXERCISES

ARTS SECTION



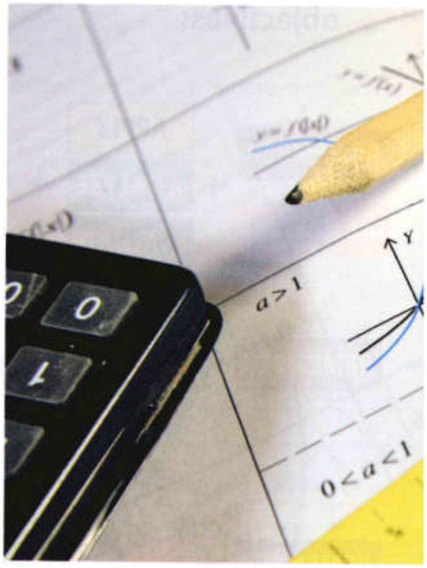
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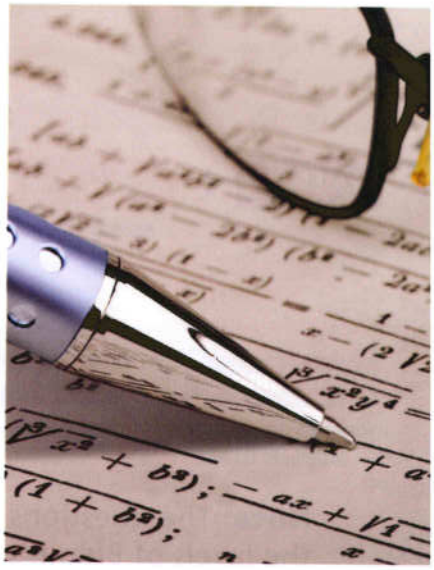
CONTENTS

First Algebra

UNIT 1 Functions of a real variable and drawing curves.



UNIT 2 Exponents , logarithms and their applications.



Second Calculus and Trigonometry

UNIT 3

Limits.

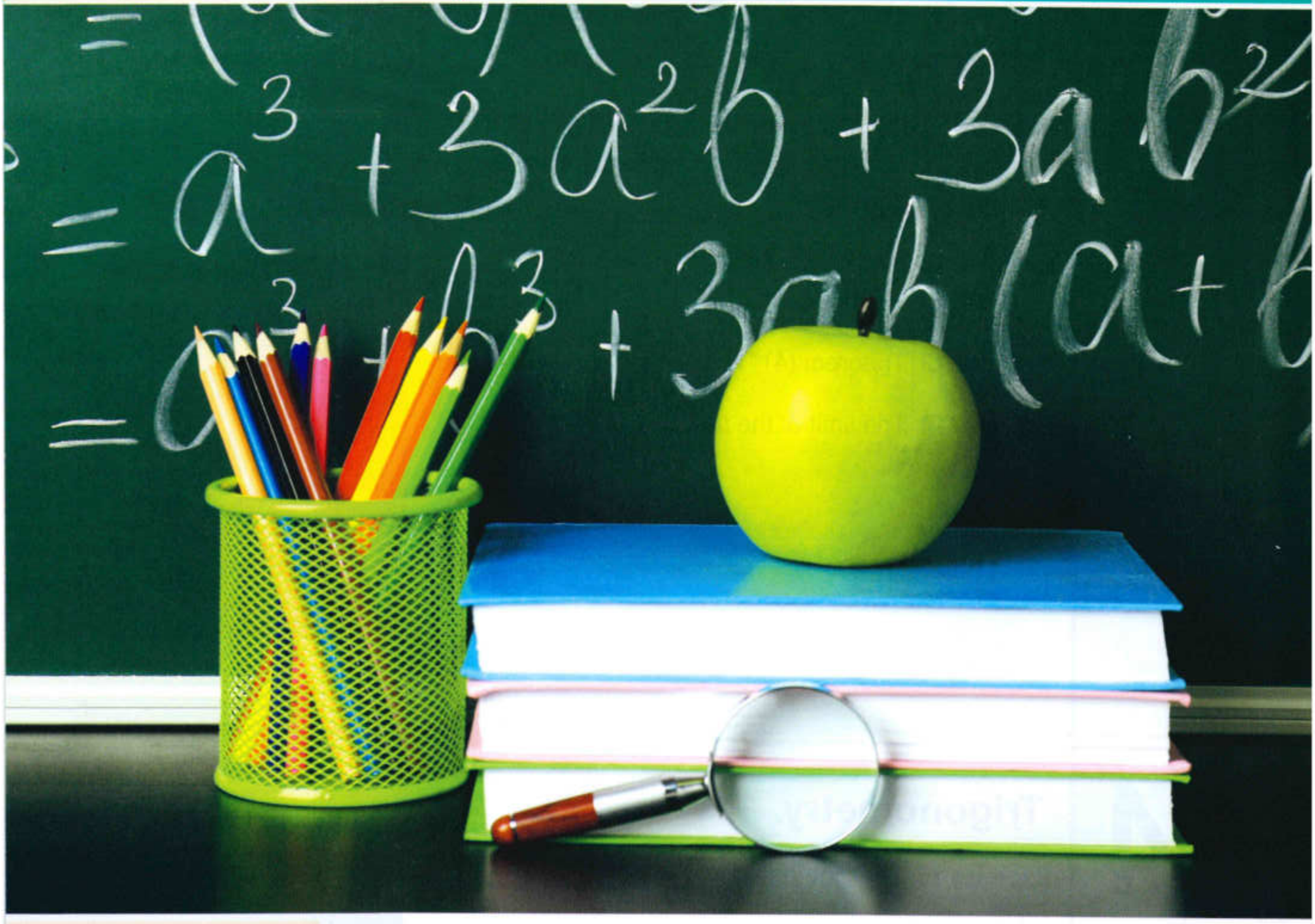


UNIT 4

Trigonometry.



First Algebra



UNIT 1 Functions of a real variable and drawing curves.

UNIT 2 Exponents , logarithms and their applications.

UNIT 1

Functions of a real variable and drawing curves.

Unit Exercises

* Exercise on pre-requirements for unit one.

Exercise 1 Real functions.
(Determination the domain and range – Discuss the monotony).

Exercise 2 Even and odd functions.

Exercise 3 Graphical representation of basic functions and graphing piecewise functions.

Exercise 4 Geometrical transformations of basic function curves.

Exercise 5 Solving absolute value equations.

Exercise 6 Solving absolute value inequalities.

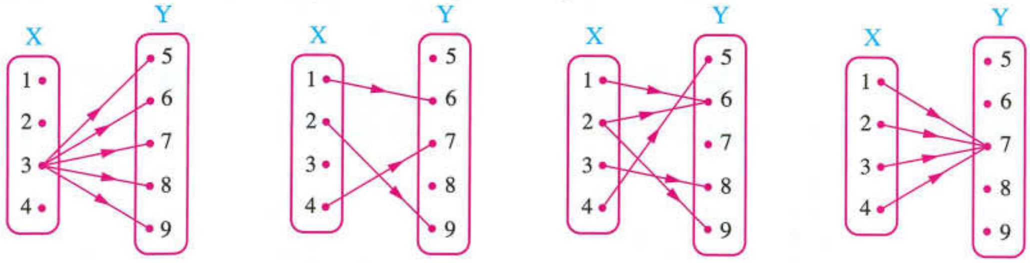
• At the end of the unit :
Life applications on unit one.

Exercise on pre-requirements

📖 From the school book

Choose the correct answer from the given ones :

(1) 📖 Which of the following arrow diagrams represents a function from X to Y ?



(a) (b) (c) (d)

(2) 📖 The relation shown by the set of the ordered pairs and does not represent a function is ....

(a) $\{(1, 3), (3, 5), (5, 7), (7, 9)\}$

(b) $\{(2, 3), (3, 4), (2, 1), (3, 5)\}$

(c) $\{(0, 3), (1, 3), (2, 3), (3, 3)\}$

(d) $\{(-3, 5), (-1, 5), (0, 5), (2, 5)\}$

(3) If $f : \mathbb{R} \longrightarrow \mathbb{R}$ and f maps a number to half its square added to 3, then $f(2) =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) 3 (d) 5

(4) If $\mathbb{N}$ is the set of the natural numbers, which of the following represents a function from $\mathbb{N} \longrightarrow \mathbb{N}$?

(a) $f(X) = \frac{2X}{3}$ (b) $g(X) = 1 - X$

(c) $h(X) = 2X + 3$ (d) $n(X) = \frac{1}{X - 2}$

(5) If $f : \{1, 2, 3, 4, 5\} \longrightarrow \mathbb{R}$ where $f(X + 2) = 3X + 1$, then $f(3) =$ ....

(a) 10 (b) 9 (c) 4 (d) 1

(6) The domain of the function f where $f(X) = \frac{X^3 - 8}{4}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{8\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{4\}$

(7) If $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$, $f(X) = \frac{X^2 + 1}{X}$, then the domain of the function is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{-1\}$



Exercise 1 Real functions

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) In all the following relations , y is a function in x except ....

(a) $y = 3x + 1$ (b) $y = x^2 - 4$ (c) $x = y^2 - 2$ (d) $y = \sin x$

(2) In all the following relations , y is a function except ....

(a) $y = \cos x$ (b) $y = 2$ (c) $y = x^2 - 1$ (d) $y^2 = x^2 + 1$

(3) The domain of the function $f : f(x) = 5$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\{0 , 5\}$

(4) The domain of the function $f : f(x) = \frac{2x + 1}{x - 2}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-\frac{1}{2}\}$ (c) $\mathbb{R} - \{-\frac{1}{2} , 2\}$ (d) $\mathbb{R} - \{2\}$

(5) The domain of the function $f : f(x) = \frac{x + 5}{(x + 5)(x - 5)}$ is ....

(a) $\mathbb{R}$ (b) $\{5 , -5\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{5 , -5\}$

(6) The domain of the function $f : f(x) = \frac{x^2 + 1}{x^2 + 4x}$ is ....

(a) $\mathbb{R} - \{1 , -1\}$ (b) $\mathbb{R} - \{0 , -4\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{0 , 4\}$

(7) The domain of the function f where $f(x) = \frac{5 + 2x}{x^2 + x + 1}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{-2 , -5\}$

(8) The domain of the function f where $f(x) = \frac{7}{x^3 - x}$ is ....

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{7\}$ (c) $\mathbb{R} - \{0, 1\}$ (d) $\mathbb{R} - \{0, 1, -1\}$

(9) The domain of the function f where $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$, $f(x) = \frac{x-1}{4x}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{1\}$

(10) If the domain of the function $f : f(x) = \frac{2}{x^2 - 6x + k}$ is $\mathbb{R} - \{3\}$, then k = ...

(a) 3 (b) 9 (c) ± 9 (d) 18

(11) The domain of the function $f : f(x) = \sqrt{x - 3}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $[3, \infty[$ (d) $]-\infty, 3[$

(12) The domain of the function f where $f(x) = \sqrt{4 - x}$ is ....

(a) $[4, \infty[$ (b) $]-\infty, 4[$ (c) $]4, \infty[$ (d) $]-\infty, 4]$

(13) The domain of the function $f : f(x) = \sqrt[3]{x - 5}$ is ....

(a) $[5, \infty[$ (b) $]-\infty, 5[$ (c) $\mathbb{R}$ (d) $\mathbb{R}^+$

(14) The domain of the function $f : f(x) = \sqrt[3]{9 - x^2}$ is ....

(a) $]-3, 3[$ (b) $\mathbb{R}$ (c) $\mathbb{R} -]-3, 3[$ (d) $[-3, 3]$

(15) The domain of the function $f : f(x) = \sqrt{x + 3}$ is ....

(a) $]-3, \infty[$ (b) $[-3, \infty[$ (c) $\{-3\}$ (d) $\{0\}$

(16) The domain of the function $f : f(x) = \sqrt{x^2 + 9}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-3, 3\}$ (c) $\mathbb{R} - [-3, 3]$ (d) $]-3, 3[$

(17) The domain of the function $f : f(x) = \frac{5}{\sqrt{x - 4}}$ is ....

(a) $[4, \infty[$ (b) $]4, \infty[$ (c) $]-\infty, 4]$ (d) $]-\infty, 4[$

(18) The domain of the function f where $f(x) = \sqrt[4]{x^2 + 4}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{4\}$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{-2, 2\}$

(19) The domain of the function f where $f(x) = \frac{1}{\sqrt[3]{x^2 - 5x - 6}}$ is ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{6\}$ (c) $\mathbb{R} - \{1, -6\}$ (d) $\mathbb{R} - \{-1, 6\}$

(20) The domain of the function $f : f(x) = \frac{1}{\sqrt[3]{x^2 + 9x + 8}}$ is ....

(a) $\mathbb{R} - \{-8, -1\}$ (b) $\mathbb{R} - [-8, -1]$

(c) $]-8, -1[$ (d) $\{-8, -1\}$

(21) If the domain of the function $f : f(x) = \frac{1}{\sqrt{x - a}}$ is $]-3, \infty[$, then a = ...

(a) 3 (b) – 3 (c) ± 3 (d) $\sqrt{3}$

(22) If the domain of the function $f : f(x) = \frac{1}{\sqrt{x^2 + a}}$ is $\mathbb{R}$, then a can not be equal ....

(a) 5 (b) $\sqrt{4}$ (c) zero. (d) 9

(23) If : $f(x) = \begin{cases} -4x + 3 & , \quad x < 3 \\ -x^3 & , \quad 3 \leq x \leq 8 \\ 3x^2 + 1 & , \quad x > 8 \end{cases}$, then $f(10)$ = ...

(a) – 37 (b) – 1000 (c) 301 (d) 43

(24) The domain of the function f where $f(x) = \begin{cases} -2 & , \quad x < 2 \\ 3 & , \quad x > 2 \end{cases}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{2\}$

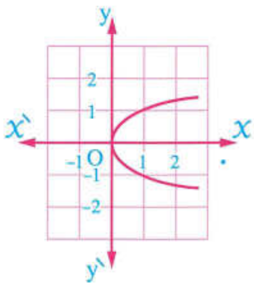
(25) The domain of the function f where $f(x) = \begin{cases} x & , \quad 0 \leq x \leq 1 \\ 2 - x & , \quad 1 < x \leq 2 \end{cases}$ is ....

(a) $\mathbb{R} - \{1\}$ (b) $[0, 2]$ (c) $\mathbb{R} - \{0, 2\}$ (d) $]0, 2[$

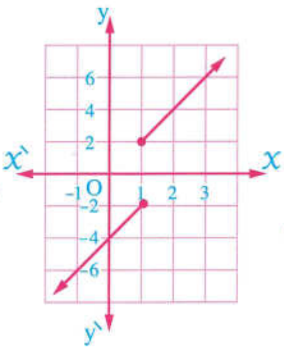
(26) The range of the function $f : f(x) = \begin{cases} 0 & , \quad x \leq 0 \\ 1 & , \quad x > 0 \end{cases}$ is ....

(a) $\{1\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\{0, 1\}$

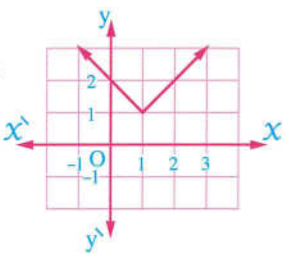
(27) The figure which represents y as a function in x is ....



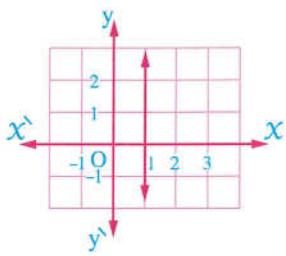
(a)



(b)

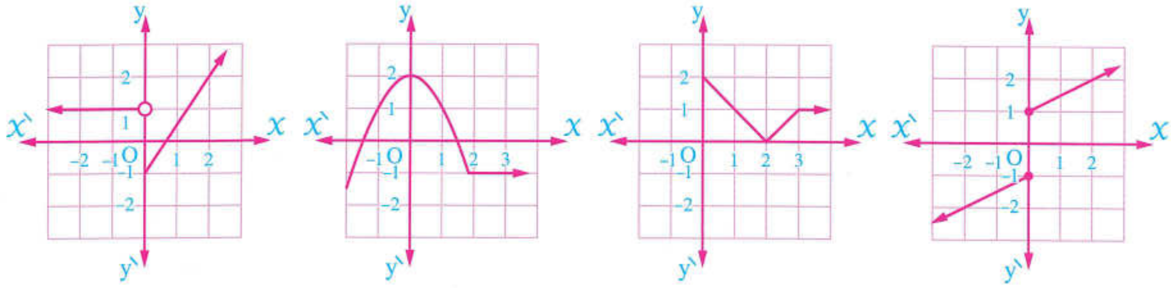


(c)



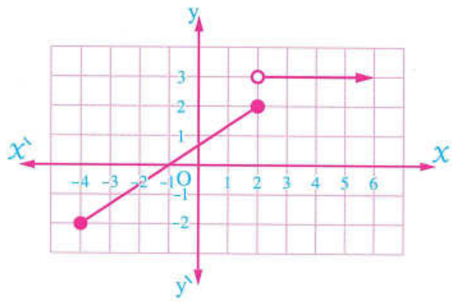
(d)

(28) Which of the following graphs does not represent a function ?



(a) (b) (c) (d)

(29) The opposite figure represents ....



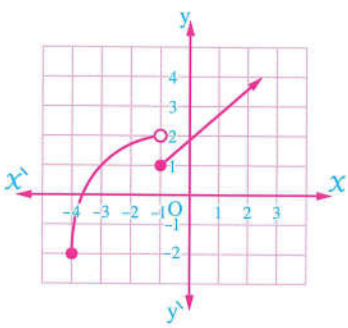
(a) function $f : [-4 , 2] \longrightarrow \mathbb{R}$

(b) function $f : [-4 , \infty[\longrightarrow \mathbb{R}$

(c) function $f : [-4 , 2] \longrightarrow [-2 , 3]$

(d) relation between x , y but not a function.

(30) The opposite figure represents the curve of function f , then its domain is ....



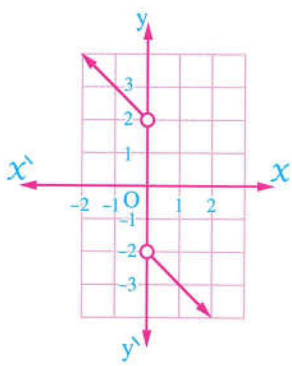
(a) $\mathbb{R} - \{-4 , -1\}$

(b) $]-4 , -1[$

(c) $[-4 , \infty[$

(d) $[-4 , \infty[- \{-1\}$

(31) The opposite figure represents function of x , its domain is ....



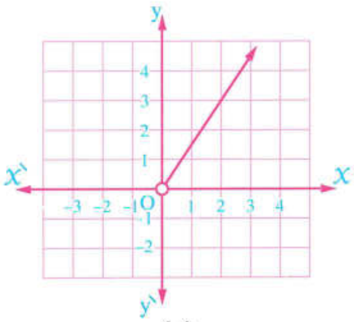
(a) $\mathbb{R}$

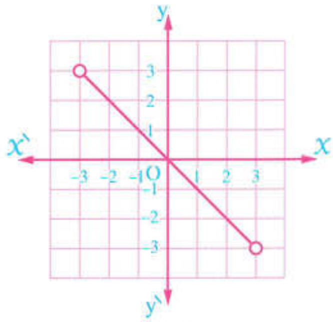
(b) $\mathbb{R} -]-2 , 2[$

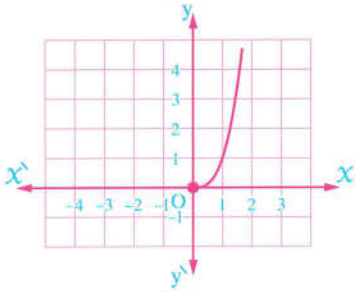
(c) $\mathbb{R} - [-2 , 2]$

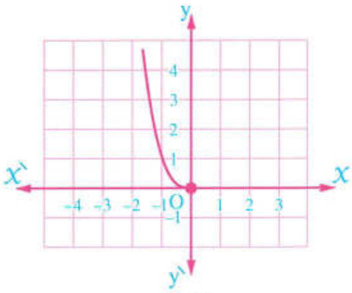
(d) $\mathbb{R} - \{0\}$

(32) Which of the following figures represents the curve of a function in which its range ≠ its domain ?

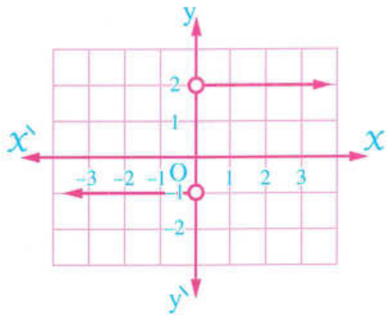

(a)


(b)


(c)


(d)

(33) The range of the function shown in the opposite figure is ....



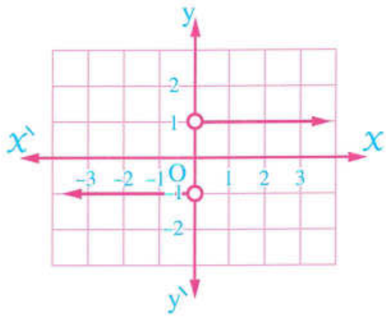
(a) $\{-1 , 2\}$

(b) $\{2\}$

(c) $\{-3 , 3\}$

(d) $\{-1\}$

(34) The range of the function shown in the opposite figure is ....



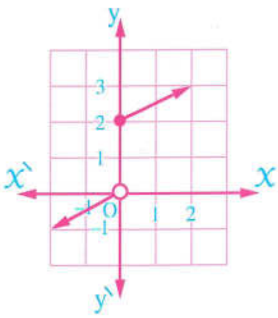
(a) $\{1\}$

(b) $\{1 , -1\}$

(c) $\{-1\}$

(d) $\mathbb{R} - \{0\}$

(35) The opposite figure represents a function of x , its range is ....



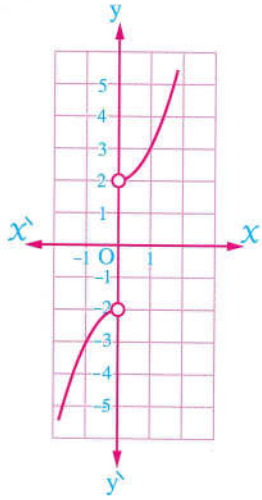
(a) $\mathbb{R} - [0 , 2]$

(b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - [0 , 2[$

(d) $\mathbb{R} -]0 , 2]$

(36) In the opposite figure :



First : The range of the function is ....

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - [-2, 2]$

(c) $\mathbb{R}$ (d) $[-2, 2]$

Second : The function is increasing in ....

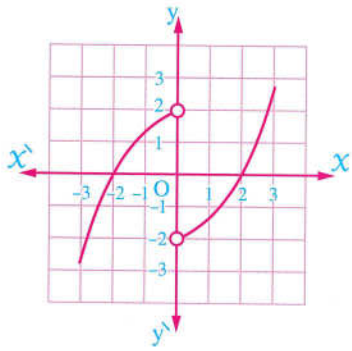
(a) $]-\infty, 0[$ only

(b) $]0, \infty[$ only

(c) $]-\infty, 0[,]0, \infty[$

(d) $\mathbb{R} - [-2, 2]$

(37) In the opposite figure : If the drawn curve shows the function f, which of the following statements is true ?



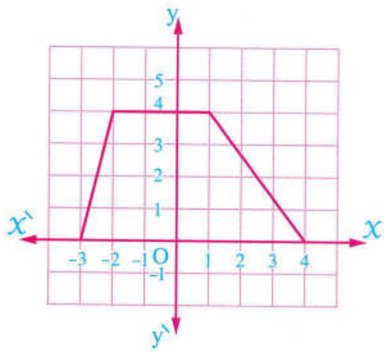
(a) The function increases on its domain.

(b) The function decreases on $]-\infty, -2[$ and increases on $]0, \infty[$

(c) The function increases on each $]-\infty, 2[,]-2, \infty[$

(d) The function increases on each $]-\infty, 0[,]0, \infty[$

(38) The opposite figure represents the curve of the function f which of the following statements is false ?



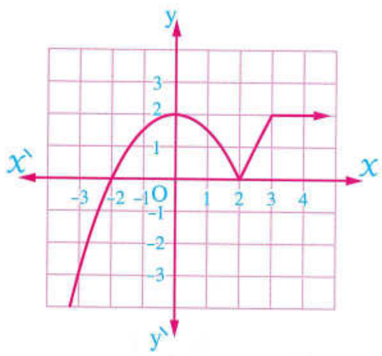
(a) f is constant on $]-2, 1[$

(b) f is decreasing on $]1, 4[$

(c) f is increasing on $]-3, -2[$

(d) f is constant on $]-3, 4[$

(39) In the opposite figure : If the function decreases on $]0, a[$ and constant on $]b, \infty[$, then $a - b =$ ....



(a) 5

(b) 1

(c) -1

(d) 3

Second — Essay questions

1 If x and y are two real variables , then determine which of the following relations represents a rule of a function in x :

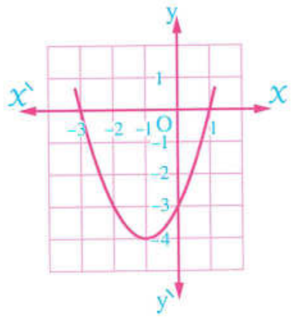
(1) $y = 2x + 5$ (2) $y^2 = x + 4$ (3) $y = \sqrt{x^2 + 4}$

(4) $(x - y)^2 = 5$ (5) $y = 2$ (6) $x = 3$

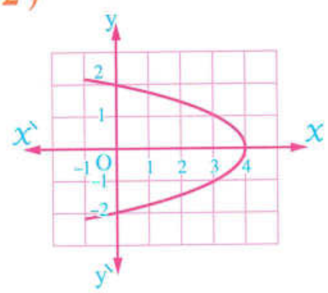
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2 In each of the following graphs , show if y is a function in x or not :

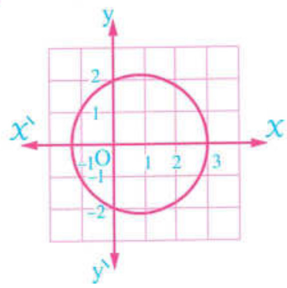
(1)



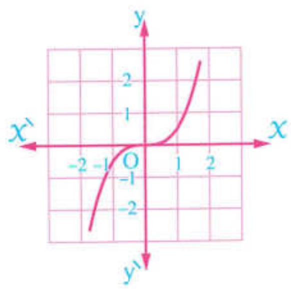
(2)



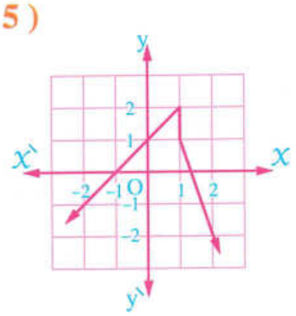
(3)



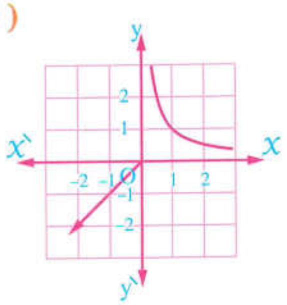
(4)



(5)



(6)



---

3 Determine the domain of each of the real functions defined by the following rules :

(1) $f(x) = \dfrac{x + 3}{x^2 - 9}$ (2) $f(x) = \dfrac{2x + 3}{x^2 - 3x + 2}$

(3) $f(x) = \dfrac{8}{x^2 - 6x + 9}$ (4) $f(x) = \dfrac{x + 3}{x^2 - 5x + 6}$

(5) $f(x) = \dfrac{x + 3}{3x^2 - x - 2}$ (6) $f(x) = \dfrac{x + 1}{x^3 + 1}$

---

4 Determine the domain of each of the real functions defined by the following rules :

(1) $f(x) = \sqrt{x}$ (2) $f(x) = \dfrac{4}{\sqrt[3]{2x - 5}}$

(3) $f(x) = \dfrac{5}{\sqrt{x + 4}}$ (4) $f(x) = \dfrac{1}{\sqrt{3 - x}}$

UNIT 1

● Understand ● Apply ● Higher Order Thinking Skills

5 Determine the domain of each of the real functions defined by the following rules :

(1) $f(x) = \begin{cases} -3 & , \quad x < 3 \\ 5 - x & , \quad x \geq 3 \end{cases}$

(2) $f(x) = \begin{cases} x^2 - 1 & , \quad x \leq 2 \\ -5 & , \quad 2 < x < 4 \end{cases}$

(3) $f(x) = \begin{cases} 3x & , \quad x \in [0, 2] \\ 6 & , \quad x \in]2, 4[\\ x + 2 & , \quad x \in [4, 6] \end{cases}$

6 If $f : X \longrightarrow \mathbb{R}$ and $X = \{1, 2, -2, -3\}$

, find the range of the function if $f(x) = 5x - 3$

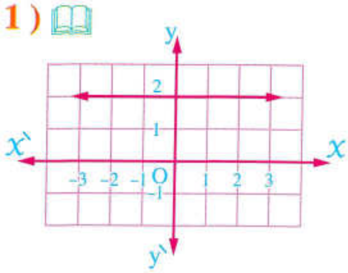
7 If $g : \{1, 2, 3, 4, 5\} \longrightarrow \mathbb{Z}^+$ where $g(x) = 4x - 3$

(1) Write down the range of the function.

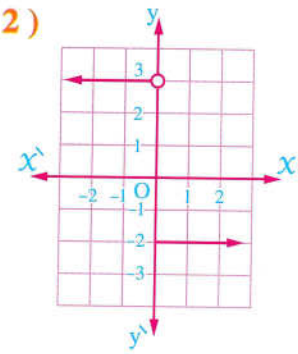
(2) If $g(k) = 17$, find the value of k

8 Determine the domain and range , then discuss the monotony of each of the functions represented by the following graphs :

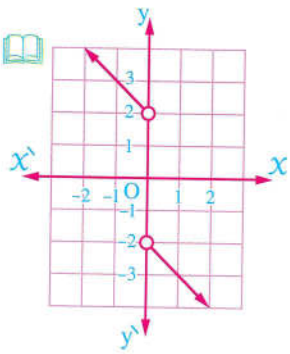
(1)



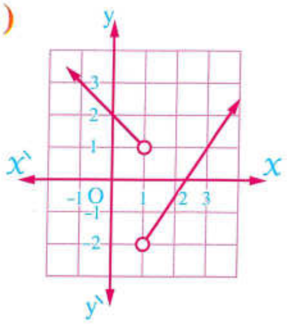
(2)



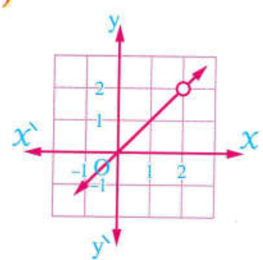
(3)



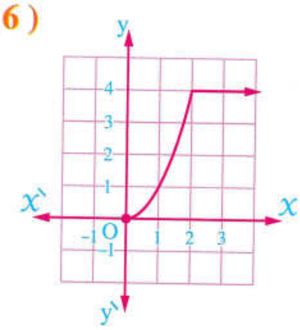
(4)



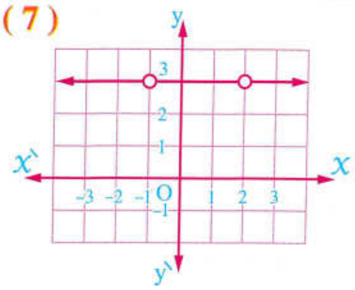
(5)



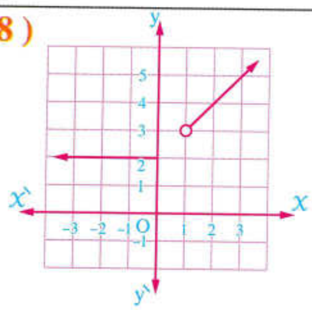
(6)



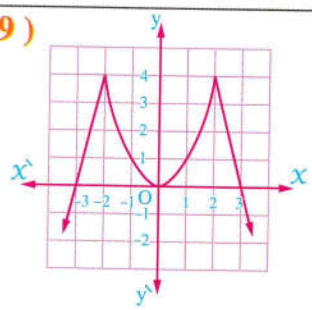
(7)



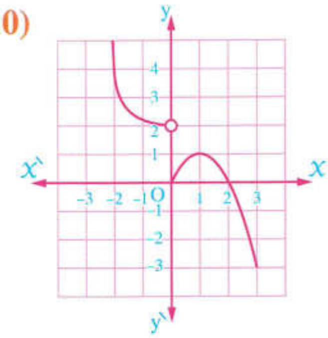
(8)



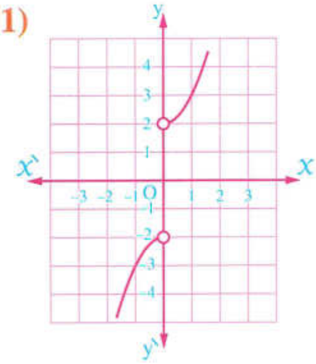
(9)



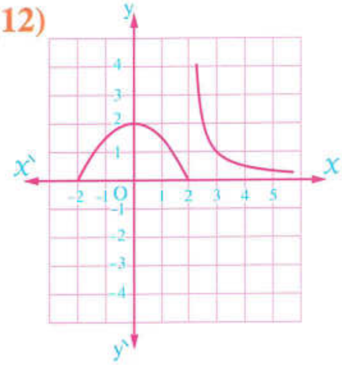
(10)



(11)



(12)



Third **Higher skills**

Choose the correct answer from those given :

(1) If the relation between the sum of the interior angle measures of a polygon (y) and the number of its sides (X) is $y = \pi (X - 2)$, then the domain of this function is ……………

(a) $\mathbb{R}^+$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{Z}^+$ (d) $\mathbb{Z}^+ - \{1 , 2\}$

(2) The domain of the function $f : f(X) = \frac{X}{\sqrt[3]{X} - 2}$ is ……………

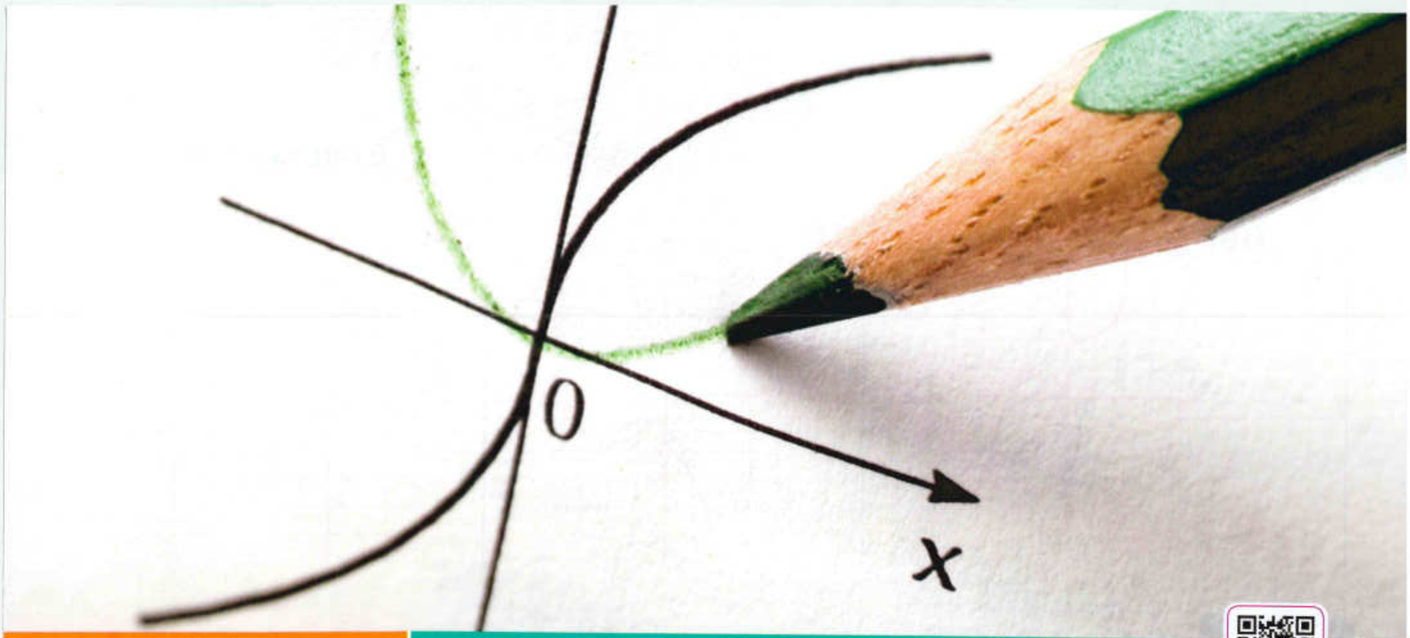
(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{0 , 2\}$ (d) $\mathbb{R} - \{8\}$

(3) The domain of the function $f : f(X) = \frac{X}{\sqrt{3X} - X}$ is ……………

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[- \{1\}$ (d) $]0 , \infty[- \{3\}$

(4) The domain of the function $f : f(X) = \frac{5}{\sqrt{X - 1} - 3}$ is ……………

(a) $[1 , \infty[$ (b) $[1 , \infty[- \{3\}$ (c) $[1 , \infty[- \{10\}$ (d) $[-3 , \infty[$



Exercise 2 Even and odd functions

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The even function from the functions that are defined by the following rules is ....

(a) $f(x) = x^3$ (b) $f(x) = \sin x$

(c) $f(x) = x \cos x$ (d) $f(x) = x \sin x$

(2) The odd function from the functions that are defined by the following rules is ....

(a) $f(x) = x^2 \sin x$ (b) $f(x) = \tan^2 x$

(c) $f(x) = \cos x$ (d) $f(x) = 1$

(3) Which of the following rules is defined as odd function ?

(1) $f(x) = \frac{1}{x} + x + \frac{1}{x^3}$

(2) $f(x) = x^2 \tan x$

(3) $f(x) = x \sin x$

(a) both (1) , (2) (b) only (1)

(c) both (2) , (3) (d) All the previous.

(4) The type of the function $f : f(x) = \frac{\sin x}{x}$ is ....

(a) even (b) odd

(c) neither even nor odd. (d) linear

(5) The function $f : f(x) = x \cos x$ is ....

(a) even. (b) odd.

(c) neither even nor odd. (d) linear.

(6) The following functions are even except ....

(a) $f(x) = \sin x$ (b) $f(x) = \cos x$ (c) $f(x) = x^2 - 1$ (d) $f(x) = 1$

(7) Which of the following functions is not even ?

(a) $y = \frac{1}{x^2}$ (b) $y = \sec x$

(c) $y = x^2 + \sin x$ (d) $y = 3x^4 - 2x^2 + 27$

(8) If $f(x) = \frac{1}{\sin x}$, then ....

(a) $f(x) = \frac{1}{f(x)}$ (b) $f(x) = -f(-x)$

(c) $f(x) = f(-x)$ (d) $f(-x) = f\left(\frac{1}{x}\right)$

(9) If f is an odd function , $f(1) = 2$, then which of the following points lies on the curve of f ?

(a) $(-1, 2)$ (b) $(-1, -2)$ (c) $(1, -2)$ (d) $(-1, 0)$

(10) If f is an odd function , $a \in$ the domain of f , then $f(a) + f(-a) =$ ....

(a) zero (b) $2f(a)$ (c) $2a$ (d) $f(a)$

(11) If f is an odd function , then $f(a) - f(-a) =$ ....

(a) zero. (b) $f(a)$ (c) $2f(a)$ (d) $f(2a)$

(12) If f is an even function , then $f(a) - f(-a) =$ ....

(a) zero. (b) $f(a)$ (c) $2f(a)$ (d) $f(2a)$

(13) If f is an even function , $2 \in$ the domain of f , then $f(2) + f(-2) =$ ....

(a) zero. (b) 4 (c) 2 (d) $2f(2)$

(14) 📖 If the function f is an even over $[a, b]$, then $b =$ ....

(a) a (b) $-a$ (c) $2a$ (d) a^3

(15) If f is an odd function in $[a, b]$, $a < 0$, then $b =$ ....

(a) a (b) $-a$ (c) $2a$ (d) a^2

(16) If f is a function where $f :]-5, 5] \longrightarrow \mathbb{R}$, $f(x) = x^2$, then the function f is ....

(a) even. (b) odd.

(c) linear. (d) neither odd nor even.

(17) If $f : f(x) = a x^3 + b x + c$ is an odd function , then $c =$ ....

(a) 2 (b) 1 (c) zero. (d) -1

(18) If $f : f(X) = X^2 + a X + 9$ is an even function , then a = ...

(a) 6 (b) 3 (c) zero. (d) – 6

(19) If $f(X) = X^3 - X$, then $|f(X) + f(-X)|$ = ...

(a) zero. (b) 1 (c) 2 (d) 4

(20) If $f : f(X) = a X^3 + b$ is an odd function and the curve of the function passes through the point (2 , 8) , then $a + b^2$ = ...

(a) zero. (b) – 1 (c) 1 (d) 5

(21) The function f where $f(X) = 5 X$ is symmetric about ....

(a) the point (2 , 1) (b) the straight line y = 1
(c) the point (5 , 0) (d) the point (0 , 0)

(22) The function $f : f(X) = X^3 + 5 X$ is symmetric about ....

(a) the X-axis. (b) the y-axis.
(c) the origin. (d) can not be determined.

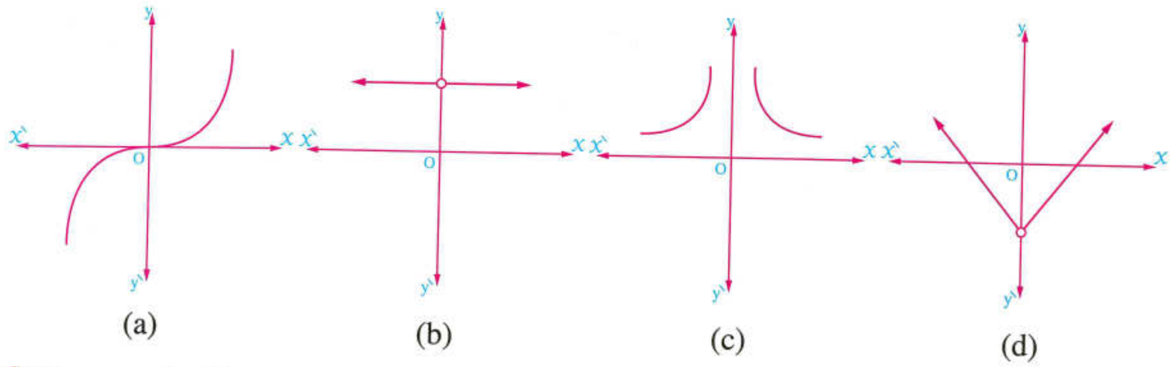
(23) The function $f : f(X) = X^2 + X^4 + 1$ is symmetric about ....

(a) the origin. (b) the X-axis.
(c) the y-axis. (d) it has neither symmetric point nor symmetric line.

(24) The function $f : f(X) = \sin 3X$ is symmetric about the point ....

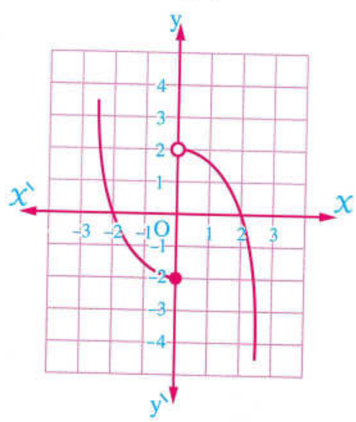
(a) (0 , 0) (b) (3 , 0) (c) (– 3 , 0) (d) (– 3 , 3)

(25) Which of the following functions is not even ?

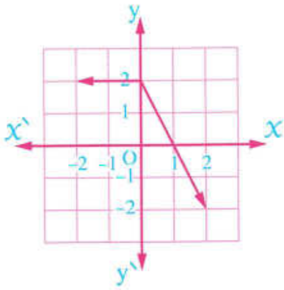


(26) The opposite figure represents the curve of the function f , then f is ....

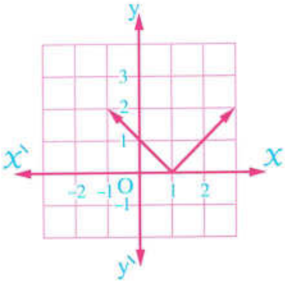
(a) linear.
(b) an even function.
(c) an odd function.
(d) neither odd nor even.



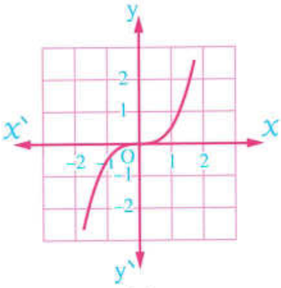
(27) Which of the following functions is even ?



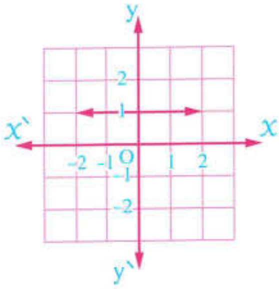
(a)



(b)



(c)

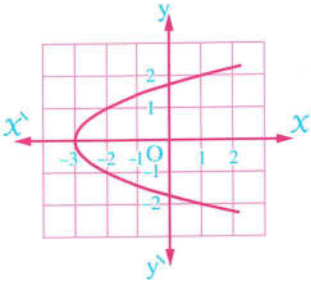


(d)

(28) The curve represented in the opposite figure is symmetric about the straight line whose equation is ....

(a) $x = 0$ (b) $y = 0$

(c) $y = -2$ (d) $x = 2$



Second Essay questions

1 In each of the following figures , mention the curve which is symmetric about the x-axis , the y-axis or the origin point :

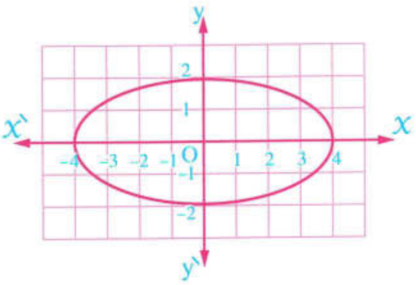


Fig. (1)

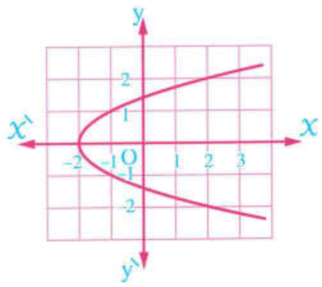


Fig. (2)

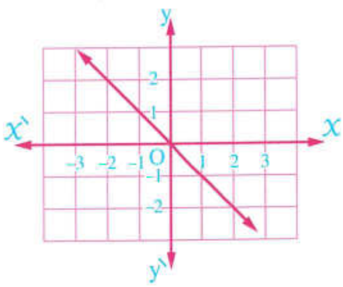


Fig. (3)

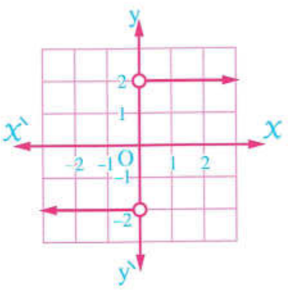


Fig. (4)

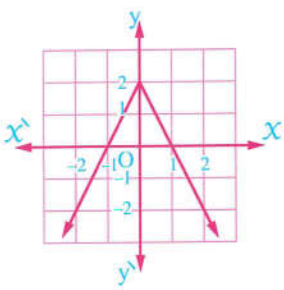


Fig. (5)

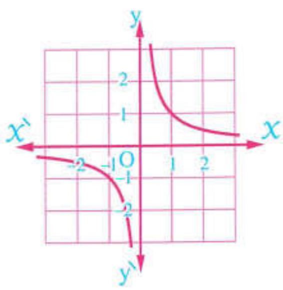
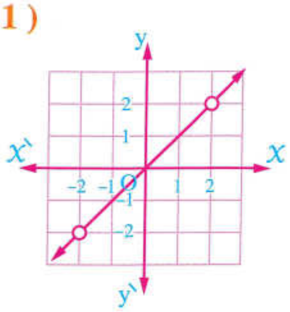


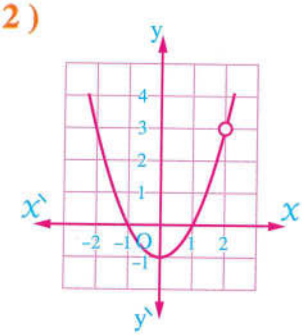
Fig. (6)

2 Determine which of the functions represented by the following graphs is even , odd or neither even nor odd :

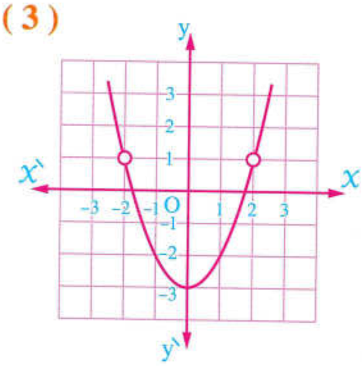
(1)



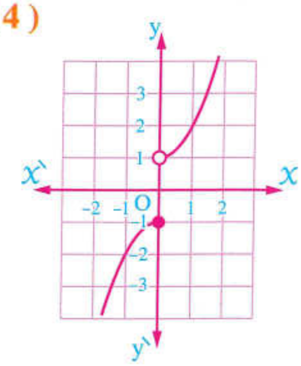
(2)



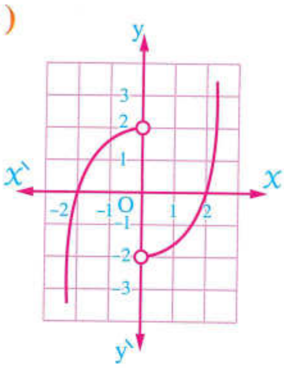
(3)



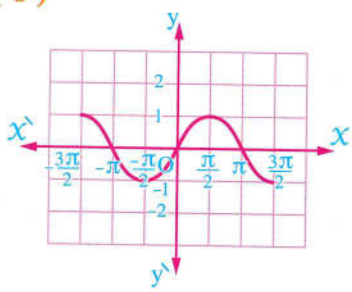
(4)



(5)



(6)



3 Find the range of each function and mention its type whether it is even , odd or otherwise :

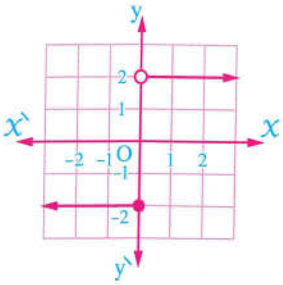


Fig. (1)

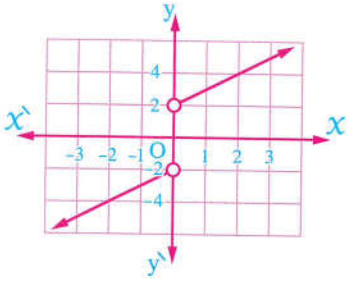


Fig. (2)

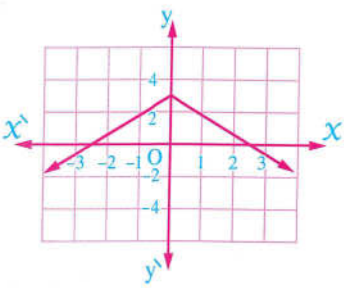


Fig. (3)

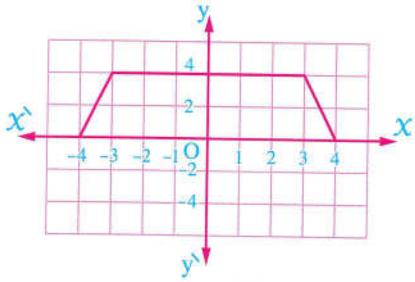


Fig. (4)

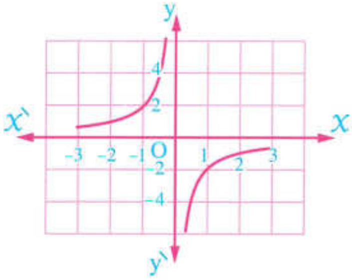


Fig. (5)

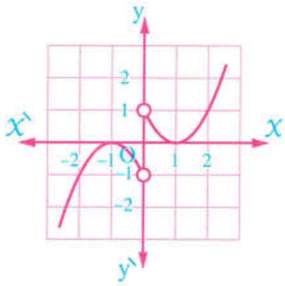


Fig. (6)

4 **Each of the following graphs represents the curve of the function f , determine whether the function f is even , odd or otherwise verifying your answers algebraically :**

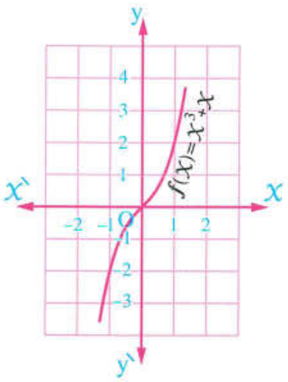


Fig. (1)

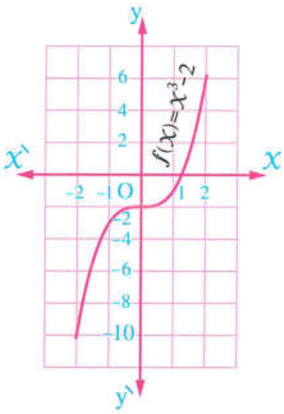


Fig. (2)

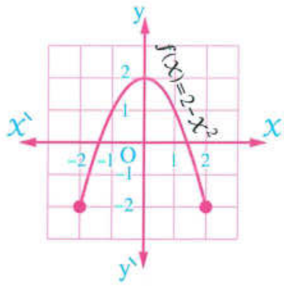


Fig. (3)

5 **Use the following figures to answer the following questions :**

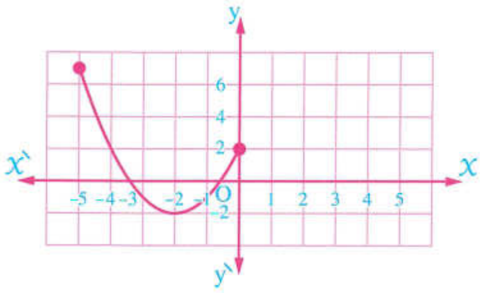


Fig. (1)

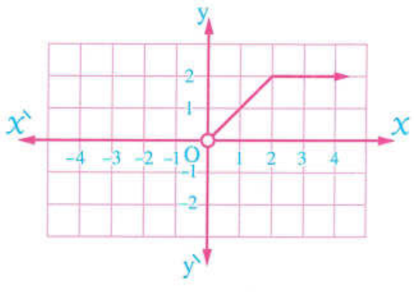


Fig. (2)

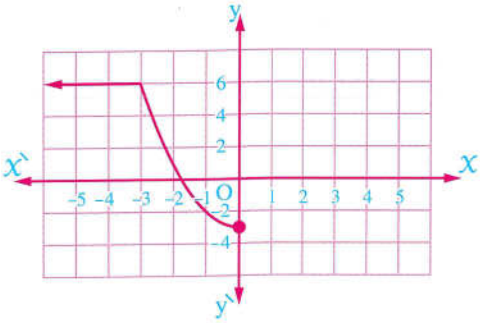


Fig. (3)

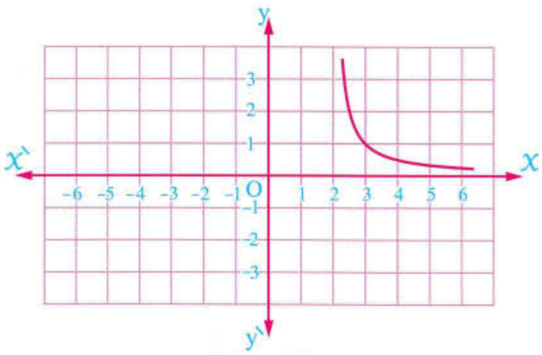


Fig. (4)

First : Complete the curve in each of fig. (1) and fig. (3) in your notebook to get an even function over its domain.

Second : Complete the curve in each of fig. (2) and fig. (4) in your notebook to get an odd function over its domain.

Third : Determine the domain and the range of the function in each case , then investigate its monotony.

---

6 **Determine which of the functions defined by the following rules is even , which is odd and which is neither even nor odd :**

(1) $f(x) = 5$

(2) $f(x) = x^4 + x^2 - 1$

(3) $f(x) = 3x - 4x^3$

(4) $f(x) = x^2 - 3x + 4$

(5) $f(x) = x^3(x^2 - 1)$

(6) $f(x) = (x - 3)^2 - 7$

(7) $f(x) = \dfrac{x^3 + 2}{x - 3}$

(8) $f(x) = \dfrac{2x^3 - x^5}{x}$

(9) $f(x) = \dfrac{x^2}{1 + x}$

(10) $f(x) = \sqrt{x + 3}$

(11) $f(x) = \sqrt[3]{x^3 + x}$

(12) $f(x) = x^3 - \dfrac{1}{x}$

(13) $f(x) = \left(x - \dfrac{2}{x}\right)^3$

(14) $f(x) = x \cos x$

(15) $f(x) = \dfrac{3x}{\tan x}$

(16) $f(x) = \dfrac{x^3 \sin 3x}{1 + x^4}$

(17) $f(x) = x^2 \sin^3 x$

(18) $f(x) = x \sin x^3$

(19) $f(x) = \dfrac{x^2 + \tan x}{x^4 + \sin x}$

(20) $f(x) = \dfrac{\sin 3x \cos 2x}{\sec x}$

7 If f_1 , f_2 and f_3 are three real functions where $f_1(x) = x^5$, $f_2(x) = \sin x$ and $f_3(x) = 5x^2$, tell which of the following functions is even , odd or otherwise :

(1) $f_1 + f_2$ (2) $f_1 + f_3$ (3) $f_1 \times f_2$ (4) $f_3 \times f_2$

8 Let f_1 , f_2 , g_1 and g_2 be real functions such that :

$f_1(x) = x^4$, $f_2(x) = \cos^5 x$, $g_1(x) = 2x^3$ and $g_2(x) = \sin^3 x$

Determine which of the following functions is even , odd or otherwise :

(1) $f_1 + g_2$ (2) $f_1 - f_2$ (3) $g_1 + g_2$

(4) $f_1 \cdot g_2$ (5) $g_1 \cdot g_2$ (6) $\frac{f_2}{f_1}$

9 Determine which of the following functions is even , odd or otherwise :

(1) $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x + 2$

(2) $f(x) = x^2$, $f : \mathbb{Z}^+ \longrightarrow \mathbb{Z}$

(3) $f : [-3 , 3[\longrightarrow \mathbb{R}$, $f(x) = 3x^2$

(4) $f(x) = x^2$, $f : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$

(5) $f : f(x) = x^2$, $x \in \mathbb{R} - \{3\}$

Third Higher skills

Choose the correct answer from those given :

(1) If f is an odd function whose domain is $\mathbb{R}$, then $\frac{7f(-5) + 3f(5)}{2f(-5)} = \dots\dots$

(a) 5 (b) – 5 (c) 2 (d) – 2

(2) If f is an even function whose domain is $\mathbb{R}$, then $\frac{7f(-5) + 3f(5)}{2f(-5)} = \dots\dots$

(a) 5 (b) – 5 (c) 2 (d) – 2

(3) If f is an even function and $f(x) + x^2 f(-x) = 3$, then $f(1) = \dots\dots$

(a) $\frac{1}{4}$ (b) 1 (c) $1\frac{1}{2}$ (d) 2

(4) If f is an odd function and $f(1) = k$ and $f(x + 2) = f(x) + f(2)$

, then $f(3) = \dots\dots$

(a) zero (b) 3 k (c) 6 k (d) 9 k



Exercise 3

Graphical representation of basic functions and graphing piecewise functions

Test yourself

From the school book | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) If $f(X) = 5$, then the domain of the function f is ……………

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\mathbb{R} - \{5\}$

(2) If $f(X) = 7$, then the range of the function f is ……………

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{7\}$ (d) $\mathbb{R} - \{7\}$

(3) The range of the function $f : f(X) = \begin{cases} 0 & \text{when } X \le 0 \\ 1 & \text{when } X > 0 \end{cases}$ is ……………

(a) $\{1\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\{0 , 1\}$

(4) In the opposite figure :

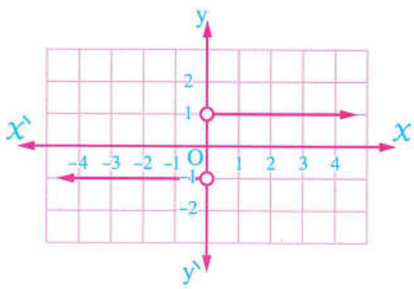
The range of the function is ……………

(a) $\{1\}$

(b) $\{1 , -1\}$

(c) $\{-1\}$

(d) $\mathbb{R}$



(5) The range of the function $f : f(X) = \begin{cases} X & , \quad X > 0 \\ -2 & , \quad X \le 0 \end{cases}$ is ……………

(a) $\mathbb{R}^+$ (b) $\mathbb{R}^+ - \{-2\}$ (c) $\mathbb{R}^+ \cup \{-2\}$ (d) $\mathbb{R}$

(6) The function f where $f(x) = \begin{cases} 2 & , \quad x > 0 \\ -2 & , \quad x < 0 \end{cases}$ is symmetric about the point ....

(a) (2 , 0) (b) (– 2 , 0) (c) (0 , 0) (d) (2 , – 2)

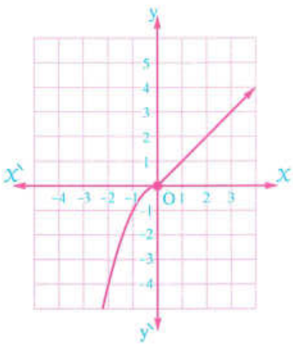
(7) The axis of symmetry for the function $f : f(x) = x^2$ is the straight line ....

(a) y = 0 (b) y = x (c) y = – x (d) $x = 0$

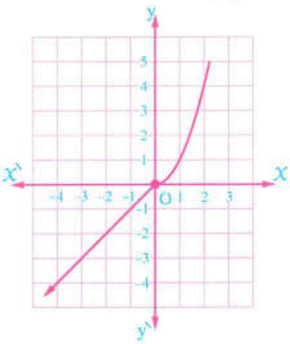
(8) The function $f : f(x) = \begin{cases} -x^2 & , \quad x < 0 \\ \frac{1}{x} & , \quad x > 0 \end{cases}$ is increasing on ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^-$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{0\}$

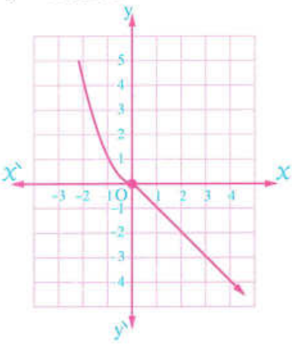
(9) The curve of the function $f : f(x) = \begin{cases} x^2 & , \quad x > 0 \\ x & , \quad x \leq 0 \end{cases}$ is ....



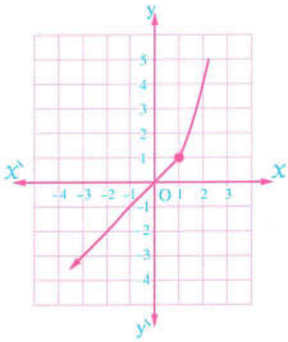
(a)



(b)



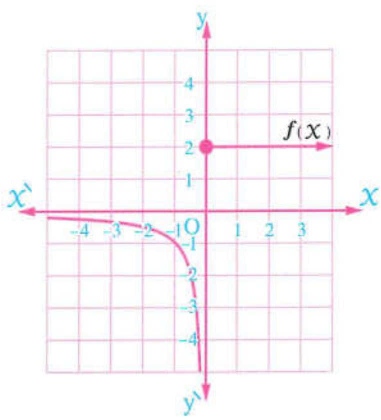
(c)



(d)

(10) In the opposite figure :

The curve of the function f is defined by the rule $f(x)$ = ...



(a) $\begin{cases} 2 & , \quad x > 0 \\ \frac{1}{x} & , \quad x < 0 \end{cases}$

(b) $\begin{cases} 2 & , \quad x \geq 0 \\ \frac{1}{x} & , \quad x < 0 \end{cases}$

(c) $\begin{cases} 2 & , \quad x < 0 \\ \frac{1}{x} & , \quad x > 0 \end{cases}$

(d) $\begin{cases} 2 & , \quad x \geq 2 \\ \frac{1}{x} & , \quad x < 2 \end{cases}$

Second Essay questions

1 Graph each of the following functions and determine its range :

(1) $f : \{-3 , -1 , 1 , 2\} \longrightarrow [-3 , 7]$, $f(x) = 2x + 3$

(2) $g : [1 , 5[\longrightarrow \mathbb{R}$, $g(x) = x + 1$

(3) $g :]-\infty , -1[\longrightarrow \mathbb{R}$, $g(x) = 1 - x$

(4) $f : f(x) = -3x + 7$ for every $x \in \mathbb{R}$

2 If $f : [-2 , 6] \longrightarrow \mathbb{R}$ where $f(x) = \begin{cases} 4 - x & , -2 \le x < 1 \\ x & , 1 \le x \le 6 \end{cases}$

, graph the function f and from the graph deduce its range , and discuss its monotonicity.

3 Graph each of the functions defined by the following rules and from the graph , find the domain and the range of each function and discuss its monotonicity and its type whether the function is even , odd or otherwise showing its symmetry :

(1) $f(x) = \dfrac{3x^2 - 3}{x^2 - 1}$

(2) $g(x) = \dfrac{4 - x^2}{x + 2}$

4 Represent graphically each of the functions that are defined by the following rules , from the graph find the domain and the range of each function and discuss its monotonicity and its type whether it is even , odd or otherwise and show its symmetry :

(1) $f :]-\infty , 3[\longrightarrow \mathbb{R}$ where $f(x) = 2$

(2) $f(x) = \begin{cases} 2 & , x \le 0 \\ -3 & , x > 0 \end{cases}$

(3) $f(x) = \begin{cases} 2 & , x > 1 \\ x - 2 & , x \le 1 \end{cases}$

(4) $f(x) = \begin{cases} x + 2 & , x \in [-2 , 1] \\ -x + 4 & , x \in]1 , 4] \end{cases}$

(5) $f(x) = \begin{cases} 4 & , x < -2 \\ x^2 & , x \ge -2 \end{cases}$

(6) $f(x) = \begin{cases} x^2 & , x < 0 \\ x & , x \ge 0 \end{cases}$

(7) $f(x) = \begin{cases} x^3 & , x < 1 \\ 1 & , x > 1 \end{cases}$

(8) $f(x) = \begin{cases} x^3 & , x < 1 \\ 2 - x & , x \ge 1 \end{cases}$

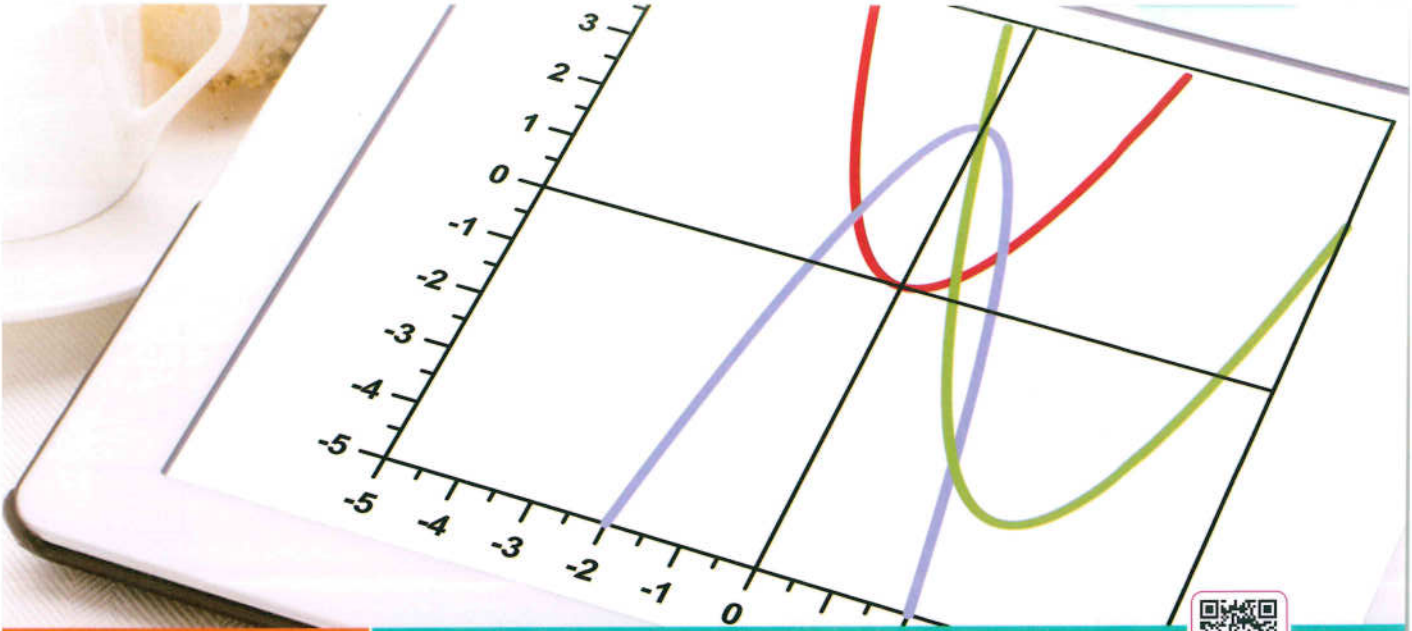
(9) $f(x) = \begin{cases} |x| & , x \le 0 \\ \dfrac{1}{x} & , x > 0 \end{cases}$

(10) $f(x) = \begin{cases} |x| & , x \le 0 \\ x^2 & , x > 0 \end{cases}$

(11) $f(x) = \begin{cases} 3 & , x \le -3 \\ |x| & , -3 < x < 3 \\ 3 & , x \ge 3 \end{cases}$

(12) $f(x) = \begin{cases} 2 & , -3 \le x \le -1 \\ 0 & , -1 < x < 1 \\ 2 & , 1 \le x \le 3 \end{cases}$

(13) $f(x) = \begin{cases} -x - 1 & , -4 \le x < -2 \\ 1 & , -2 \le x \le 2 \\ x - 1 & , 2 < x \le 4 \end{cases}$



Exercise 4 Geometrical transformations of basic function curves

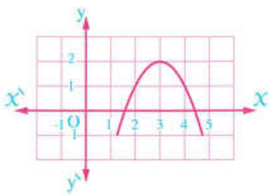
Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

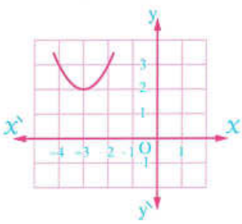
First Multiple choice questions

Choose the correct answer from the given ones :

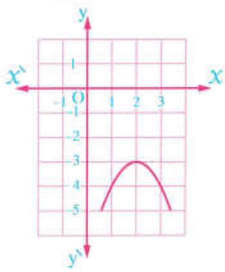
(1) If $f(X) = -(X-3)^2 + 2$, then the graph that represents the function f is ....



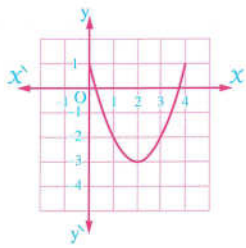
(a)



(b)

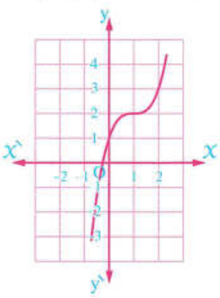


(c)

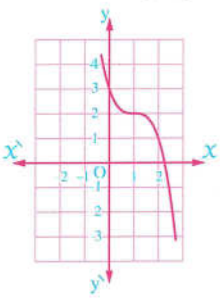


(d)

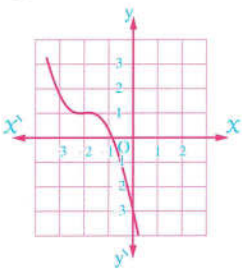
(2) If $f(X) = 2 - (X-1)^3$, then the graph that represents the function f is ....



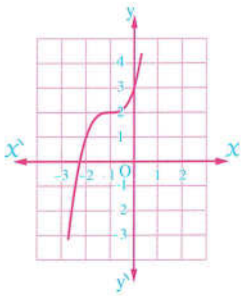
(a)



(b)



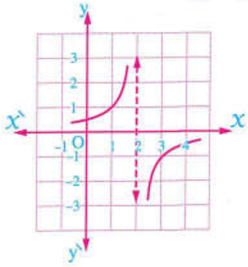
(c)



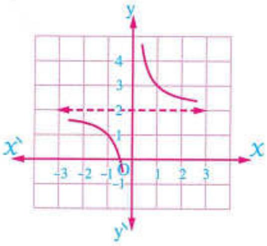
(d)

● Understand ○ Apply Higher Order Thinking Skills

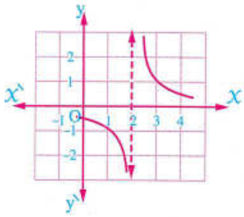
(3) If $f(X) = \dfrac{1}{X - 2}$, then the graph that represents the function f is ....



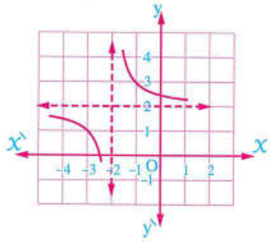
(a)



(b)

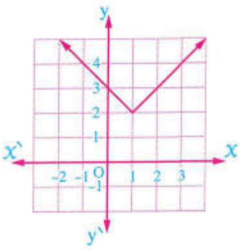


(c)

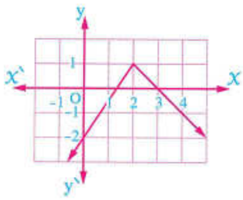


(d)

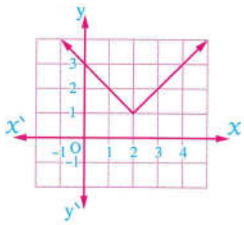
(4) If $f : f(X) = 1 - |X - 2|$, then the figure which represents the function f is ....



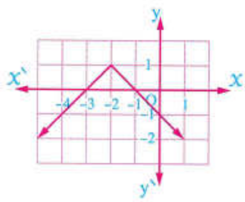
(a)



(b)

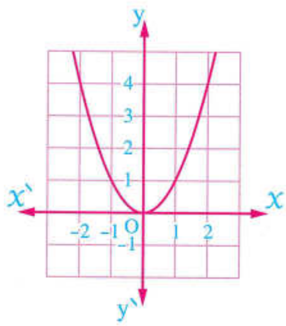


(c)



(d)

(5) The curve of the function in the given figure is translated two units in the positive directions of the two axes then the function represents this translation is ....



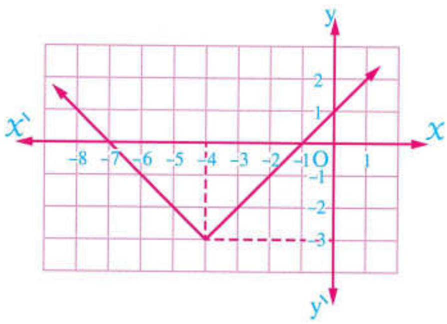
(a) $f(X) = (X + 2)^2 + 2$

(b) $f(X) = (X + 2)^2 - 2$

(c) $f(X) = (X - 2)^2 - 2$

(d) $f(X) = (X - 2)^2 + 2$

(6) Which of the following functions represents the curve in the given figure ?



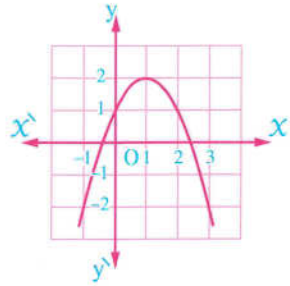
(a) $f(X) = |X - 4| - 3$

(b) $f(X) = |X - 4| + 3$

(c) $f(X) = |X + 4| - 3$

(d) $f(X) = |X + 4| + 3$

(7) Which of the following functions is represented in the given figure ?



(a) $f(x) = (x-1)^2 + 2$

(b) $f(x) = 1 - (x-2)^2$

(c) $f(x) = 2 - (x-1)^2$

(d) $f(x) = (x+1)^2 - 2$

(8) The point of the vertex of the curve of the function $f : f(x) = (2-x)^2 + 3$ is ....

(a) (2 , 3) (b) (2 , – 3) (c) (– 2 , 3) (d) (– 2 , – 3)

(9) The vertex of the curve of the function f where $f(x) = (1+x)^2 - 3$ is ....

(a) (1 , 3) (b) (1 , – 3) (c) (– 1 , 3) (d) (– 1 , – 3)

(10) The symmetric point of the function $f : f(x) = x^3 - 2$ is ....

(a) (0 , 2) (b) (0 , – 2) (c) (2 , 0) (d) (– 2 , 0)

(11) The symmetric point of the function $f : f(x) = 3 - (x+2)^2$ is ....

(a) (3 , 2) (b) (2 , 3) (c) (– 2 , 3) (d) (– 2 , – 3)

(12) The symmetric point of the function $f : f(x) = \frac{1}{x} + 2$ is ....

(a) (2 , 0) (b) (1 , 2) (c) (0 , 2) (d) (0 , 0)

(13) The point of symmetry of the function f where $f(x) = \frac{1}{x-5}$ is ....

(a) (5 , 0) (b) (– 5 , 0) (c) (0 , – 5) (d) (0 , 5)

(14) The point of symmetry of the curve of the function $f : f(x) = \frac{1}{x-3} + 4$ is ....

(a) (3 , – 4) (b) (– 3 , – 4) (c) (3 , 4) (d) (– 3 , 4)

(15) The symmetric point of the function $f : f(x) = \frac{x+1}{x}$ is ....

(a) (1 , 0) (b) (0 , 1) (c) (0 , 0) (d) (1 , – 1)

(16) If f is a function where $f(x) = \frac{1}{x}$, then the symmetric point of the function g : $g(x) = f(x+1)$ is ....

(a) (1 , 0) (b) (0 , 1) (c) (– 1 , 0) (d) (– 1 , 1)

(17) The vertex of the curve of the function $f : f(x) = |x+3| - 2$ is ....

(a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (3 , – 2)

(18) The curve of the function $f : f(x) = |x-2|$ is symmetric about the straight line ....

(a) $x = 2$ (b) $x = -2$ (c) y = 2 (d) y = – 2

(19) The axis of symmetry of the function $f : f(X) = X^2 - 1$ is the straight line ....

(a) $X = 1$ (b) $X = 0$ (c) $y = 1$ (d) $y = 0$

(20) If $f(X) = \frac{1}{|X|}$, then the equation of the axis of symmetry of the curve of the function f is ....

(a) $y = 0$ (b) $X = 0$ (c) $y = X$ (d) $y = -X$

(21) $f(X) = (X - 1)^2 + 2$ is increasing on the interval ....

(a) $\mathbb{R}$ (b) $]1 , \infty[$ (c) $]-\infty , 1[$ (d) $]-1 , 1[$

(22) The function f where $f(X) = \frac{2X - 1}{X - 1}$ is decreasing on the interval ....

(a) $]-\infty , 1]$ (b) $]-\infty , 1[,]1 , \infty[$

(c) $[1 , \infty[$ (d) $]-\infty , 2[,]2 , \infty[$

(23) The range of the function f where $f(X) = (X - 3)^2 + 4$ is ....

(a) $]-\infty , 3[$ (b) $[-3 , 4]$ (c) $[4 , \infty[$ (d) $]-\infty , 4]$

(24) The range of the function $f : f(X) = 3 - (2 - X)^2$ is ....

(a) $]-\infty , 2]$ (b) $[2 , \infty[$ (c) $]-\infty , 3]$ (d) $[3 , \infty[$

(25) The range of the function $f : f(X) = 2 - \frac{3}{X - 1}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{3\}$

(26) The range of the function $f : f(X) = |X - 2|$ is ....

(a) $]0 , \infty[$ (b) $[2 , \infty[$ (c) $[0 , \infty[$ (d) $]2 , \infty[$

(27) The range of the function $f : f(X) = 2 - |3 - 2X|$ is ....

(a) $]-\infty , 2]$ (b) $[-2 , \infty[$ (c) $]\frac{3}{2} , \infty[$ (d) $]-\infty , -2]$

(28) The range of the function $f : f(X) = \frac{|X|}{X}$ is ....

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $\mathbb{R} - \{0\}$ (d) $\{1 , -1\}$

(29) The curve of the function $f : f(X) = \frac{1}{X - 3} + 4$ does not intersect the line ....

(a) $X = -3$ (b) $X = 3$ (c) $y = -4$ (d) $y = 3$

(30) If $y = f(X)$ is a real function , then its image by translation 3 units vertically upwards is $g(X) =$ ....

(a) $f(X - 3)$ (b) $f(X + 3)$ (c) $f(X) + 3$ (d) $(X) - 3$

(31) If the curve $y = f(X)$ represents a real function then its image by translation 5 units vertically downward is the same as $g(X) =$ ....

(a) $f(X - 5)$ (b) $f(X + 5)$ (c) $f(X) + 5$ (d) $f(X) - 5$

(32) The curve of the function $g : g(X) = X^2 + 4$ is the same curve of the function $f : f(X) = X^2$ by a translation of magnitude 4 units in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overleftarrow{OX}$ (c) $\overrightarrow{Oy}$ (d) $\overleftarrow{Oy}$

(33) The curve of the function g where $g(X) = |X| - 2$ is the same as the curve of the function $f : f(X) = |X|$ by translation two units in direction of ....

(a) $\overrightarrow{OX}$ (b) $\overleftarrow{OX}$ (c) $\overrightarrow{Oy}$ (d) $\overleftarrow{Oy}$

(34) If f is a real function whose domain is $[-3, 4]$, then the domain of $g : g(X) = f(X) + 2$ is ....

(a) $[-3, 4]$ (b) $[-1, 6]$ (c) $[-5, 2]$ (d) $\mathbb{R}$

(35) The curve of the function $g : g(X) = |X + 3|$ is the same curve of the function $f : f(X) = |X|$ by a translation of magnitude 3 units in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overleftarrow{OX}$ (c) $\overrightarrow{Oy}$ (d) $\overleftarrow{Oy}$

(36) If $y = f(X)$ is a real function, then its image by translation 4 units to the left is $g(X) =$ ....

(a) $f(X - 4)$ (b) $f(X + 4)$ (c) $f(X) + 4$ (d) $f(X) - 4$

(37) If f is a real function whose domain is $[-2, 3]$, then the domain of $g : g(X) = f(X - 2)$ is ....

(a) $[-2, 3]$ (b) $[-4, 1]$ (c) $[0, 5]$ (d) $\mathbb{R}$

(38) If $f : f(X) = -X^2$ move 3 units to the right and 2 units down, then resulted curve is $g(X)$, then $g(4) =$ ....

(a) -3 (b) -16 (c) 16 (d) -7

(39) If the curve $f(X) = -X^3$ moves 4 units to the left and 2 units upwards to become the curve $g(X)$, then $g(-2) =$ ....

(a) -218 (b) 214 (c) 6 (d) -6

(40) The curve of the function $g : g(X) = X$ is the same as the curve of the function $f : f(X) =$ .... by reflection in the X-axis.

(a) X (b) $-X$ (c) $X + 1$ (d) $-X + 1$

(41) The product of the slopes of the two straight lines $f(X) = aX + b$ and its image by reflection in X-axis equals ....

(a) 1 (b) -1 (c) a (d) $-a^2$

(42) The curve of the function g : g (X) = 1 – | X | is the same curve of the function f : f (X) = | X | by reflection in X-axis , then a translation of magnitude one unit in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

Second Essay questions

1 **Use the curve of the function f where $f(X) = X^2$ to represent each of the functions that are defined by the following rules , from the graph find the domain and the range of the function and discuss its monotonicity and its type whether it is even , odd or otherwise and write its axis of symmetry :**

(1) $g(X) = X^2 - 3$
(2) $g(X) = X^2 + 1$
(3) $g(X) = -X^2 - 4$
(4) $g(X) = 2 - X^2$
(5) $g(X) = (X - 2)^2$
(6) $g(X) = -(X - 3)^2$
(7) $g(X) = (X + 1)^2$
(8) $g(X) = (X - 2)^2 + 1$
(9) $g(X) = (X - 1)^2 - 2$
(10) $g(X) = (X + 2)^2 - 4$
(11) $g(X) = 2 - (X + 3)^2$
(12) $g(X) = (2 - X)^2 - 1$
(13) $g(X) = -\frac{1}{2} X^2$
(14) $g(X) = 2 - \frac{1}{2}(X - 5)^2$
(15) $g(X) = X^2 + 4X + 4$
(16) $g(X) = X^2 + 4X + 1$

2 **Use the curve of the function f where $f(X) = X^3$ to represent each of the functions that are defined by the following rules , from the graph determine its domain , range , discuss its monotonicity and its type whether it is even , odd or otherwise and write its point of symmetry :**

(1) $g(X) = X^3 + 4$
(2) $g(X) = X^3 - 5$
(3) $g(X) = 4 - X^3$
(4) $g(X) = (X - 3)^3$
(5) $g(X) = (X + 2)^3$
(6) $g(X) = -(X - 1)^3$
(7) $g(X) = (2 - X)^3$
(8) $g(X) = (X - 2)^3 + 1$
(9) $g(X) = (X - 1)^3 - 2$
(10) $g(X) = (X + 1)^3 - 2$
(11) $g(X) = 2 - (X - 1)^3$
(12) $g(X) = 2X^3 - 1$

3 Use the curve of the function f where $f(x) = |x|$ to represent each of the functions that are defined by the following rules and from the graph determine its domain , range and discuss its monotonicity and its type whether it is even , odd or otherwise and write the equation of its axis of symmetry if exists :

(1) $g(x) = |x| + 1$

(2) $g(x) = |x| - 3$

(3) $g(x) = 2 - |x|$

(4) $g(x) = |x + 1|$

(5) $g(x) = |x - 3|$

(6) $g(x) = -|x + 5|$

(7) $g(x) = |x + 2| - 1$

(8) $g(x) = |x - 2| + 3$

(9) $g(x) = |2 - x| + 1$

(10) $g(x) = 4 - |x - 2|$

(11) $g(x) = 2|x|$

(12) $g(x) = 2|x - 7| + 2$

(13) $g(x) = -2|x - 1|$

(14) $g(x) = 5 - 2|x + 2|$

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4 Use the curve of the function f where $f(x) = \frac{1}{x}$ to represent each of the functions that are defined by the following rules , from the graph determine its domain , range and discuss its monotonicity and its type whether it is even , odd or otherwise and write its point of symmetry :

(1) $g(x) = \frac{1}{x} + 2$

(2) $g(x) = \frac{-1}{x} + 1$

(3) $g(x) = \frac{-1}{x + 2}$

(4) $g(x) = \frac{1}{x - 3}$

(5) $g(x) = \frac{1}{x - 2} + 3$

(6) $g(x) = \frac{1}{x + 2} + 1$

(7) $g(x) = 3 - \frac{1}{x + 2}$

(8) $g(x) = \frac{1}{4 - x} - 3$

(9) $g(x) = \frac{x + 1}{x}$

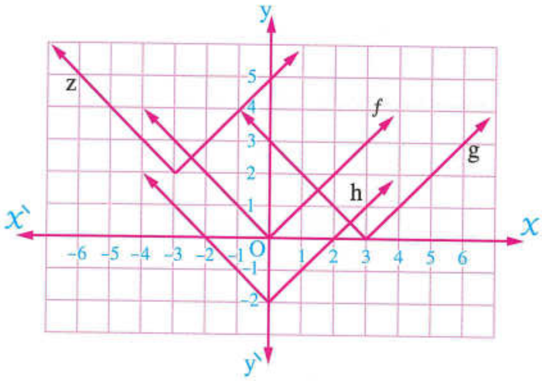
(10) $g(x) = \frac{x - 3}{x - 2}$

(11) $g(x) = \frac{2x}{x + 1}$

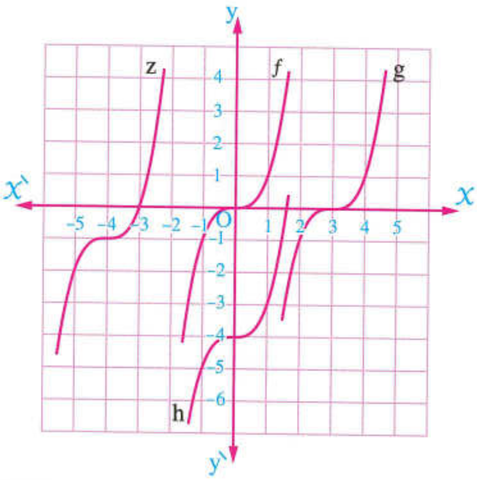
(12) $g(x) = \frac{2x - 3}{x - 2}$

Understand Apply Higher Order Thinking Skills

5 The curve of f where $f(x) = |x|$ is graphed, then translated in the directions of the coordinate axes as in the opposite figure.
Write the rule for each of the functions : g , h , z



6 The curve of the function f where $f(x) = x^3$ is graphed, then translated in the directions of the coordinate axes as in the opposite figure.
Write the rule of each of the functions : g , h , z



7 **If some geometric transformations are applied on the functions f , g , h where : $f(x) = x^2$, $g(x) = x^3$, $h(x) = \frac{1}{x}$ to get the functions represented by the following figures, complete :**

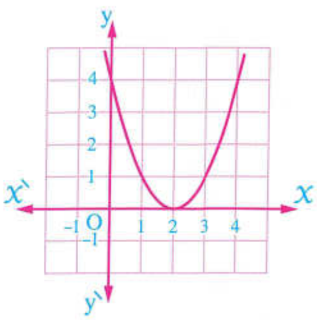


Fig. (1)

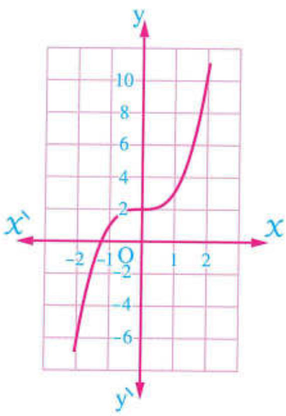


Fig. (2)

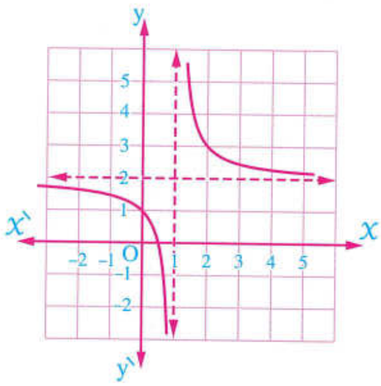


Fig. (3)

(1) The rule of the function in fig. (1) is ....

(2) The rule of the function in fig. (2) is ....

(3) The rule of the function in fig. (3) is ....

(4) The range of the function in fig. (1) is ....

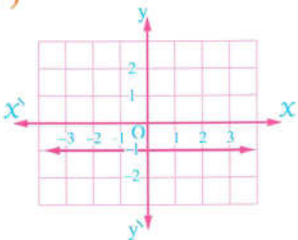
(5) The range is $\mathbb{R}$ in fig. ....

(6) The point of symmetry of the function in fig. (3) is ....

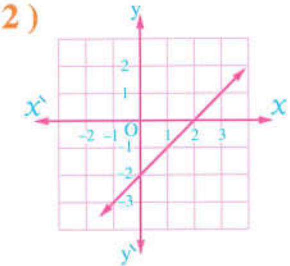
(7) The equation of symmetry line of the function in fig. (1) is ....

8 **Write the rule of the function f that is represented graphically by each of the following figures :**

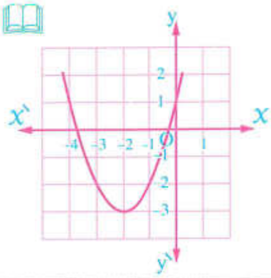
(1)



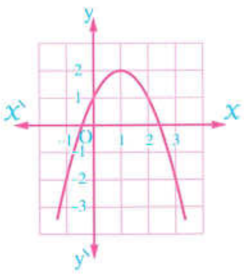
(2)



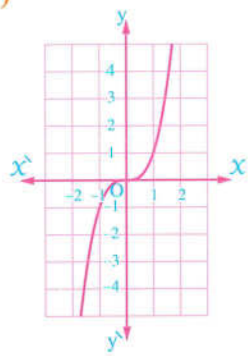
(3)



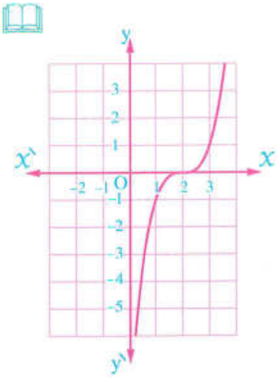
(4)



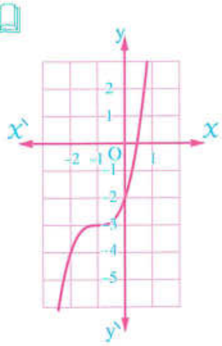
(5)



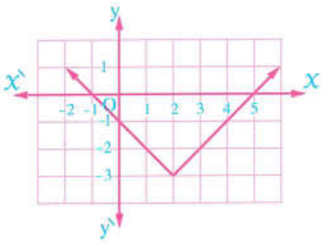
(6)



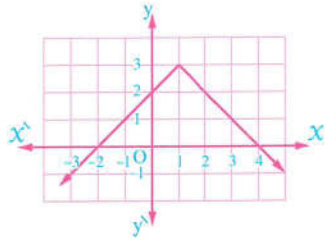
(7)



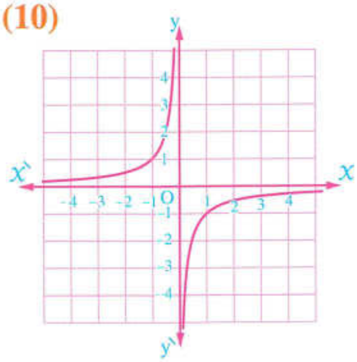
(8)



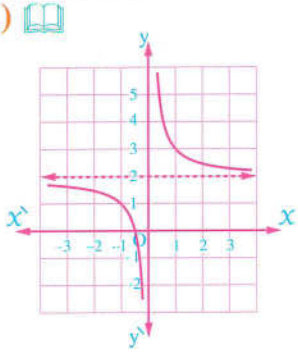
(9)



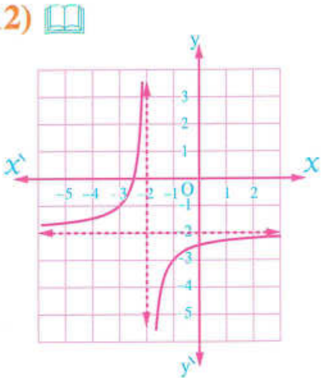
(10)



(11)



(12)



9 If f , g , k , n are real functions where $f(x) = x^2$, $g(x) = x^3$, $k(x) = |x|$, $n(x) = \frac{1}{x}$, then represent each of the functions that are defined by the following rules showing its domain and range :

(1) $f_1(x) = f(x+1)$

(2) $f_2(x) = f(x) - 1$

(3) $f_3(x) = 2 - f(x-1)$

(4) $g_1(x) = g(x-1)$

(5) $g_2(x) = g(x-1) + 2$

(6) $k_1(x) = \frac{1}{2} k(x) - 3$

(7) $n_1(x) = n(x-2)$

(8) $n_2(x) = 2 - n(x+1)$

10 Draw the curve of the function f in each of the following and determine its range and discuss its monotonicity :

(1) $f(x) = \begin{cases} x^2 + 1 & , \quad x > 0 \\ -x^2 - 1 & , \quad x < 0 \end{cases}$

(2) $f(x) = \begin{cases} x^2 + 1 & , \quad -4 \le x < 0 \\ -x^2 - 1 & , \quad 0 \le x \le 4 \end{cases}$

(3) $f(x) = \begin{cases} (x-1)^3 & , \quad x \ge 0 \\ -1 & , \quad x < 0 \end{cases}$

Third Higher skills

Choose the correct answer from those given :

(1) If f is a polynomial function and $f(x) = 0$ at $x \in \{-3 , 1 , 0\}$, then the function $g : g(x) = f(x-3)$ cuts the x-axis at $x \in$ ....

(a) $\{-3 , 1 , 0\}$ (b) $\{3 , 0 , -2\}$ (c) $\{0 , 3 , 4\}$ (d) $\{-6 , 2 , 0\}$

(2) If $f : f(x) = (x - a + 1)^2 + b - 2$ is a quadratic function whose range is $[1 , \infty[$ and the curve of f passes through $(3 , 2)$, then a = ...

(a) ± 4 (b) 3 or 5 (c) 3 or -5 (d) -3 or 5

(3) If $g(x)$ is decreasing on $]-\infty , 0[$ and is increasing on $]0 , \infty[$, then $f(x) = g(x+1)$ is increasing on ....

(a) $]-1 , \infty[$ (b) $]-\infty , -1[$ (c) $]0 , \infty[$ (d) $]-\infty , 0[$

(4) The curve $y = 3(x-5)^2 + 7$ by translation 3 units in the positive direction of x-axis and one unit in the negative direction of y-axis is ....

(a) $y = 3(x+8)^2 + 6$ (b) $y = 3(x-8)^2 - 6$

(c) $y = 3(x-8)^2 + 6$ (d) $y = 3(x+8)^2 - 6$

(5) If $f : f(x) = \begin{cases} x^3 + 2 & , \quad x \ge 0 \\ g(x) & , \quad x < 0 \end{cases}$ is symmetric about y-axis , then $g(x) =$ ....

(a) $x^3 - 2$ (b) $x^3 + 2$ (c) $-x^3 + 2$ (d) $-x^3 - 2$



Exercise 5 Solving absolute value equations

Test yourself

From the school book · Understand · Apply · Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones:

(1) The solution set of the equation : $|x-2|=3$ is ....

(a) $\{2, 3\}$ (b) $\{-1, 5\}$ (c) $[-1, 5]$ (d) $\{5, -5\}$

(2) The solution set of the equation : $|5x-1|+4=1$ in $\mathbb{R}$ is ....

(a) $\{\frac{-4}{5}\}$ (b) $\{\frac{-4}{5}, \frac{6}{5}\}$ (c) $\varnothing$ (d) $\{\frac{6}{5}\}$

(3) The solution set of the equation : $|2x-4|=|x+1|$ is ....

(a) $\{1, 5\}$ (b) $\{5, -1\}$ (c) $\{1, -5\}$ (d) $\{-5, -1\}$

(4) The solution set of the equation : $\frac{1}{|x-3|}=\frac{1}{2}$ is .... where $x \neq 3$

(a) $\{5\}$ (b) $\{1\}$ (c) $\{5, 1\}$ (d) $\varnothing$

(5) The solution set of the equation : $x^2=2-|x|$ is ....

(a) $\{1, -1\}$ (b) $\{2, -2\}$ (c) $\{1, -2\}$ (d) $\{-1, 2\}$

(6) The solution set of the equation : $\sqrt{4x^2-12x+9}=5$ is ....

(a) $\{4\}$ (b) $\{-1\}$ (c) $\{4, -1\}$ (d) $\mathbb{R}$

(7) Which of the following does not belong to the solution set of the equation :

$|x+2|+|x-1|=3$?

(a) -2 (b) zero (c) -3 (d) 1

(8) The domain of the function $f(x) = \frac{2}{|x| - 2}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-2\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{2, -2\}$

(9) The domain of the function $f : f(x) = \frac{1}{|x| + 3}$ is ....

(a) $\mathbb{R}$ (b) $\{3, -3\}$ (c) $\mathbb{R} - \{3, -3\}$ (d) $\mathbb{R} - \{3\}$

(10) The type of the function $f : f(x) = \frac{x \tan x}{|x|}$ is ....

(a) odd. (b) even.

(c) neither odd nor even. (d) one-to-one.

(11) The false statement in the following is ....

(a) $|x y| = |x| |y|$ (b) $|x| = |-x|$

(c) $|x + y| = |x| + |y|$ (d) $\sqrt{x^2} = |x|$

(12) If $a > 0$, $b < 0$, then which of the following is always negative ?

(a) $|a b|$ (b) $a |b|$ (c) $|a| b$ (d) $a + |b|$

(13) If $a - b > 0$, then which of the following could be equal to $|a - b|$?

(1) $a - b$ **(2)** $b - a$ **(3)** $|b - a|$

(a) only (3) (b) both (1), (3) (c) both (2), (3) (d) (1), (2) and (3)

(14) If $a < 0 < b$, then $|a| + |b| + |b - a| - |a - b| =$ ....

(a) $2a$ (b) $2b$ (c) $a - b$ (d) $b - a$

Second Essay questions

1 **Find algebraically in $\mathbb{R}$ the solution set of each of the following equations :**

(1) $|x| = 7$ « $\{7, -7\}$ »

(2) $|x| + 3 = 0$ « $\varnothing$ »

(3) $4|x| - 20 = 0$ « $\{5, -5\}$ »

(4) $|x - 2| = 2$ « $\{4, 0\}$ »

(5) $|x - 3| = 0$ « $\{3\}$ »

(6) $|x + 3| = 6$ « $\{3, -9\}$ »

(7) $|2x - 7| = 5$ « $\{6, 1\}$ »

(8) $|3 - 2x| = 7$ « $\{5, -2\}$ »

(9) $3 - |x + 2| = 2$ « $\{-1, -3\}$ »

(10) $2|x| = 3 - |x|$ « $\{1, -1\}$ »

(11) $|x - 3| = |x + 1|$ « $\{1\}$ »

(12) $|x + 5| = |x - 3|$ « $\{-1\}$ »

(13) $|x - 1| = |2x + 3|$ « $\{-4, -\frac{2}{3}\}$ »

(14) $|2x - 6| = |x - 3|$ « $\{3\}$ »

(15) $|2x + 1| = |x - 3|$ « $\{-4, \frac{2}{3}\}$ »

(16) $|x - 1| - 2|2 - x| = 0$ « $\{3, \frac{5}{3}\}$ »

(17) $\sqrt{x^2 - 4x + 4} = 4$ « $\{6, -2\}$ »

(18) $|x - 3|^2 - |x - 3| = 0$ « $\{3, 2, 4\}$ »

(19) $\sqrt{4x^2 - 12x + 9} = |x + 1|$ « $\{4, \frac{2}{3}\}$ »

(20) $|x^2 - 1| = |x - 1|$ « $\{1, 0, -2\}$ »

(21) $|x + 1||x - 1| = 26$ « $\{3\sqrt{3}, -3\sqrt{3}\}$ »

(22) $|x + 1|^2 - 3|x + 1| - 10 = 0$ « $\{4, -6\}$ »

(23) $(x - 5)^2 = |2x - 10|$ « $\{5, 7, 3\}$ »

(24) $|x^2 + x - 10| = 10$ « $\{0, -1, -5, 4\}$ »

2 **Find graphically in $\mathbb{R}$ the solution set of each of the following equations and verify the results algebraically :**

(1) $|x| - 4 = 0$ « $\{4, -4\}$ »

(2) $|x| + 2 = 0$ « $\varnothing$ »

(3) $|x - 4| = 3$ « $\{1, 7\}$ »

(4) $|x + 1| - 3 = 0$ « $\{2, -4\}$ »

(5) $2 - |x + 2| = 0$ « $\{0, -4\}$ »

(6) $2|x - 3| = 12$ « $\{9, -3\}$ »

(7) $|2x - 5| = 3$ « $\{1, 4\}$ »

(8) $\sqrt{x^2 - 4x + 4} = 3$ « $\{5, -1\}$ »

(9) $|x - 1| = |x + 3|$ « $\{-1\}$ »

(10) $|x - 2| = -|x + 2|$ « $\varnothing$ »

(11) $|x + 7| = |2x + 3|$ « $\{-3\frac{1}{3}, 4\}$ »

(12) $|x - 2| + |x - 1| = 0$ « $\varnothing$ »

(13) $|x - 3| = |2x + 1|$ « $\{-4, \frac{2}{3}\}$ »

3 **Show whether each of the functions defined by the following rules is even , odd or otherwise :**

(1) $f(x) = x|x|$

(2) $f(x) = x^2|x| - 1$

(3) $f(x) = x|x - 2| + 4$

(4) $f(x) = \dfrac{x^2 \cos 2x}{5 + |2x|}$

(5) $f(x) = 2|x| \tan x + 2x|\tan x|$

4 Graph the function $f : f(x) = |x - 3| + 1$, and from the graph discuss its monotonicity, then find the solution set of the equation $f(x) = 4$ « $\{0, 6\}$ »

5 Graph the function $f : f(x) = |2x + 5| - 3$, determine the range of the function and study its monotonicity.

From the graph , deduce the solution set of the equation : $|2x + 5| - 3 = 0$, then verify the solution algebraically. « $\{-1, -4\}$ »

6 Graph the function $f : f(x) = 1 - |2x|$ and from the graph deduce its range and its monotonicity.

Prove also that f is even. From the graph or by any other method , find the solution set of the equation : $1 - |2x| = -3$ « $\{-2, 2\}$ »

7 Write the domain of the function $f : f(x) = |x - 2|$, then draw the graph of f , from the graph deduce its range and its monotony and show whether the function is even or odd or otherwise , then find the solution set of the equation : $|x - 2| = 3$ graphically and verify the solution algebraically. « $\{-1 , 5\}$ »

---

8 Graph the two functions $f_1 : f_1(x) = |x - 1|$ and $f_2 : f_2(x) = |2x - 5|$, then from the graph deduce the solution set of the equation : $f_1(x) - f_2(x) = 0$ « $\{2 , 4\}$ »

---

9 If $f(x) = x^2 |x|$, tell whether the function f is even , odd or otherwise and find the solution set of the equation : $f(x) = 1$ « $\{1 , -1\}$ »

Third Higher skills

Choose the correct answer from those given :

(1) If the domain of the function $f : f(x) = \frac{x}{|x| + a}$ is $\mathbb{R} - \{2 , -2\}$, then a = ...

(a) 2 (b) -2 (c) ± 2 (d) zero

(2) If $f(x) = |x - 2| + 4$, then the solution set of the equation $f(x + 2) = 6$ in $\mathbb{R}$ is ....

(a) $\{0 , 4\}$ (b) $\{2 , -2\}$ (c) $\{2 , 4\}$ (d) $\{-2 , -4\}$

(3) If $f(x) = |x - 2| + 4$, then the solution set of the equation $f(x + 2) = 3$ in $\mathbb{R}$ is ....

(a) $\{1 , 3\}$ (b) $\mathbb{R}$ (c) $\varnothing$ (d) $\{-1 , -3\}$

(4) The solution set of the equation $|x - 3| = |3 - x|$ in $\mathbb{R}$ is ....

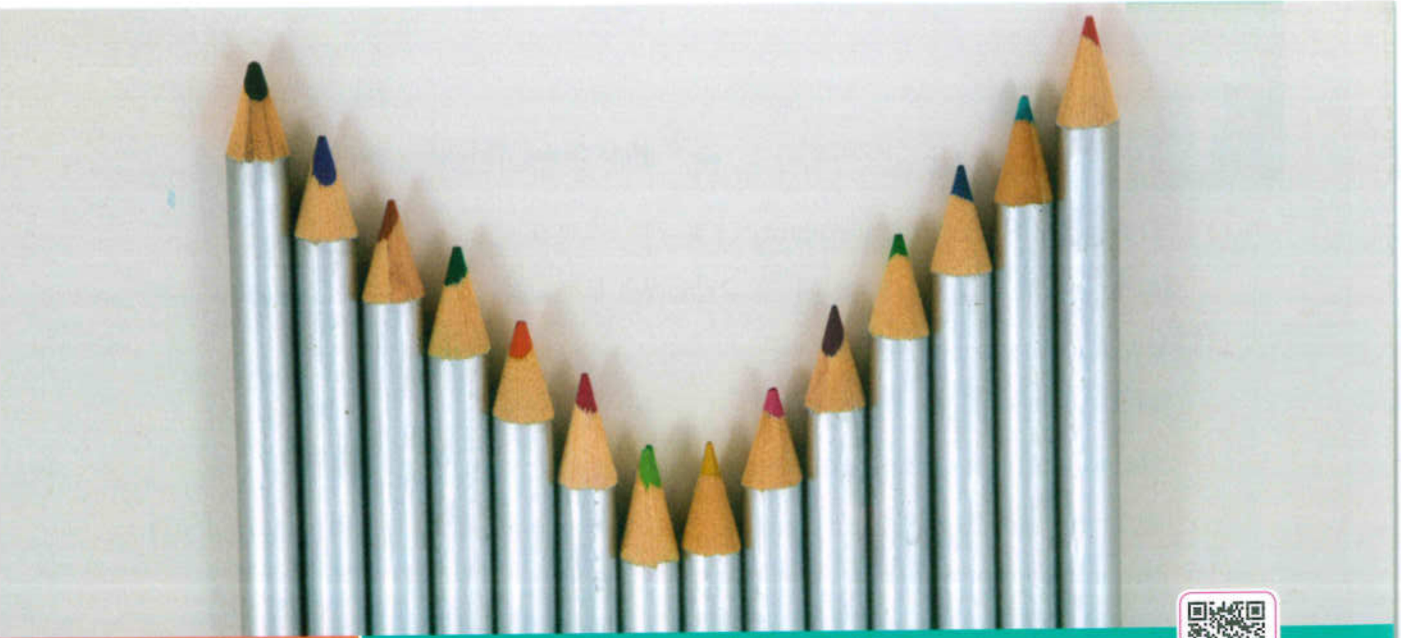
(a) $\{3\}$ (b) $\{3 , -3\}$ (c) $\mathbb{R}$ (d) $\varnothing$

(5) The solution set of the equation $|x + 1|^2 + |2x + 3| = 0$ in $\mathbb{R}$ is ....

(a) $\{-1 , \frac{-3}{2}\}$ (b) $\mathbb{R}$ (c) $\{1 , \frac{3}{2}\}$ (d) $\varnothing$

(6) The solution set of the equation $|x^2 - 4x + 3| = |x - 3|$ in $\mathbb{R}$ is ....

(a) $\{0 , 2\}$ (b) $\{2 , 3\}$ (c) $\{0 , 3\}$ (d) $\{0 , 2 , 3\}$



Exercise 6 Solving absolute value inequalities

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The solution set of the inequality : $|x| < 2$ in $\mathbb{R}$ is ....

(a) $]-\infty , 2[$ (b) $[-2 , 2]$ (c) $]-2 , 2[$ (d) $\mathbb{R} - [-2 , 2]$

(2) The solution set of the inequality : $|x| \geq 3$ in $\mathbb{R}$ is ....

(a) $[3 , \infty[$ (b) $[-3 , 3]$ (c) $\mathbb{R} -]-3 , 3[$ (d) $\mathbb{R} - [-3 , 3]$

(3) The solution set of the inequality $|x| > -1$ is ....

(a) $[0 , \infty[$ (b) $\mathbb{R}$ (c) $\varnothing$ (d) $\mathbb{R} - \{0\}$

(4) The solution set of the inequality $\frac{1}{|x|} \geq 1$ is ....

(a) $[-1 , 1]$ (b) $]-1 , 1[$ (c) $[-1 , 1] - \{0\}$ (d) $]-1 , 1[- \{0\}$

(5) The solution set of the inequality : $|3 - x| > 0$ is ....

(a) $]-3 , 3[$ (b) $\mathbb{R} - [-3 , 3]$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R}$

(6) The solution set of the inequality : $|2x + 3| \leq 1$ in $\mathbb{R}$ is ....

(a) $\mathbb{R}$ (b) $]-2 , -1[$ (c) $[-2 , -1]$ (d) $\varnothing$

(7) The solution set of the inequality : $|3 - 2x| \leq 1$ in $\mathbb{R}$ is ....

(a) $[1 , 2]$ (b) $]1 , 2[$ (c) $\mathbb{R} -]1 , 2[$ (d) $\mathbb{R} - [1 , 2]$

(8) The solution set of the inequality $\frac{1}{|x - 2|} \geq \frac{1}{2}$ is ....

(a) $]0 , 4[- \{2\}$ (b) $[0 , 4] - \{2\}$ (c) $[0 , 4]$ (d) $]0 , 4]$

(9) The solution set of the inequality $|x + 3| \leq 0$ is ....

(a) $\varnothing$ (b) $]-\infty , -3]$ (c) $]-3 , \infty[$ (d) $\{-3\}$

(10) The solution set of the inequality : $|x-1| < -2$ in $\mathbb{R}$ is ....

(a) $]-1, 3[$ (b) $\mathbb{R} - [-1, 3]$ (c) $]-2, 2[$ (d) $\varnothing$

(11) If : $|x| < a$, $a \in \mathbb{R}^+$, then $x \in$ ....

(a) $\mathbb{R} - [-a, a]$ (b) $[-a, a]$ (c) $\mathbb{R} -]-a, a[$ (d) $]-a, a[$

(12) The solution set of the inequality $|2x-5| \le 9$ is ....

(a) $]-\infty, 7[$ (b) $[-2, 7]$ (c) $\mathbb{R} -]-2, 7[$ (d) $\mathbb{R} - [-2, 7]$

(13) The solution set of the inequality $|4-6x| \ge 14$ is ....

(a) $]\frac{-5}{3}, 3[$ (b) $[\frac{-5}{3}, 3]$ (c) $\mathbb{R} -]\frac{-5}{3}, 3[$ (d) $\mathbb{R} - [\frac{-5}{3}, 3]$

(14) The solution set of the inequality : $\sqrt{x^2-2x+1} \ge 4$ is ....

(a) $[-3, 5]$ (b) $\mathbb{R} -]-3, 5[$ (c) $]-3, 5[$ (d) $\mathbb{R} - [-3, 5]$

(15) The solution set of the inequality $\sqrt{4x^2-12x+9} \le 9$ is ....

(a) $[-6, 12]$ (b) $[-3, 6]$ (c) $\mathbb{R} - [-3, 6]$ (d) $\mathbb{R} -]-3, 6[$

(16) The solution set of the inequality : $|2x-3| + |6-4x| \le 21$ is ....

(a) $[-2, 5]$ (b) $\mathbb{R} -]-2, 5[$ (c) $]-2, 5[$ (d) $\{-2, 5\}$

(17) The absolute value inequality represents that the score of a student in one test (x) is including between 70 to 90 is ....

(a) $|x| \le 90$ (b) $|x| \ge 70$ (c) $|x-80| \le 10$ (d) $|x-70| \le 90$

(18) Number of the integer solutions to the inequality : $|x-2| \le 5$ is ....

(a) zero. (b) 7 (c) 9 (d) 11

Second Essay questions

1 Find algebraically in $\mathbb{R}$ the solution set of each of the following inequalities :

(1) $|x-3| \le 5$ « $[-2, 8]$ »

(2) $|x-3| \ge 5$ « $\mathbb{R} -]-2, 8[$ »

(3) $|2x-3| < 5$ « $]-1, 4[$ »

(4) $|2x+5| > 3$ « $\mathbb{R} - [-4, -1]$ »

(5) $|3x-5| \le 10$ « $[\frac{-5}{3}, 5]$ »

(6) $|3-x| < 7$ « $]-4, 10[$ »

(7) $|2x+6| \le 4$ « $[-5, -1]$ »

(8) $|5-x| > 3$ « $\mathbb{R} - [2, 8]$ »

(9) $|2x+3| \le 7$ « $[-5, 2]$ »

(10) $|2x+3| + 2 \le 1$ « $\varnothing$ »

(11) $|2x-3| + 3 > 7$ « $\mathbb{R} - [-\frac{1}{2}, \frac{7}{2}]$ »

(12) $\sqrt{x^2-6x+9} \le 3$ « $[0, 6]$ »

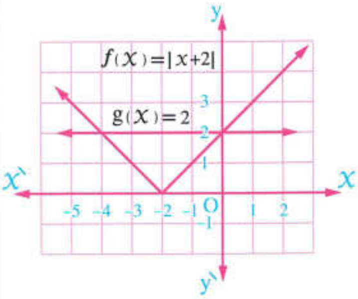
(13) $\sqrt{x^2-2x+1} \ge 4$ « $\mathbb{R} -]-3, 5[$ »

(14) $|x-2| + |2-x| < 6$ « $]-1, 5[$ »

(15) $|3x-2| + 2|6x-4| \ge 20$ « $\mathbb{R} -]-\frac{2}{3}, 2[$ »

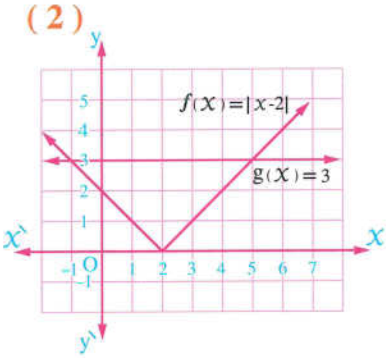
2 Using the following figures, complete :

(1)



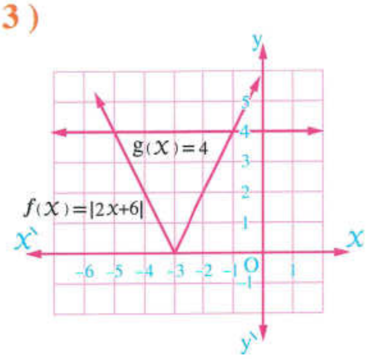
The S.S. of the inequality $f(x) < g(x)$ in $\mathbb{R}$ is ....

(2)



The S.S. of the inequality $f(x) > g(x)$ in $\mathbb{R}$ is ....

(3)



The S.S. of the inequality $f(x) \leq g(x)$ in $\mathbb{R}$ is ....

3 Find graphically in $\mathbb{R}$ the solution set of each of the following inequalities, then verify the result algebraically :

(1) $|x - 1| < 3$
(2) $|x - 2| \leq 5$
(3) $|x + 4| \leq 2$
(4) $|x + 3| \geq 2$
(5) $|2 - x| < -1$
(6) $|x + 2| > -1$
(7) $|2x - 5| \geq 2$
(8) $\sqrt{x^2 + 2x + 1} > 3$

4 Write in the form of an absolute value inequality each of the following :

(1) $-4 \leq x \leq 4$
(2) $0 < x < 6$
(3) $x \geq 2$ or $x \leq -2$

5 Write the absolute value inequality which expresses :

(1) The student's mark in an exam ranges between 60 and 100

(2) The temperature measured by a thermometer ranges between 35°C and 42°C

Third Higher skills

Choose the correct answer from those given :

(1) If $a > 0$, $b < 0$, then which of the following is always negative ?

(a) $|ab|$ (b) $a|b|$ (c) $|a|b$ (d) $a + |b|$

(2) If $x \in [-1, 4]$, then $|2x - 3| \leq$ ....

(a) 3 (b) 4 (c) 5 (d) – 5

(3) The solution set of the inequality $\sqrt{x^2 - 4x + 4} > 0$ in $\mathbb{R}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-2\}$ (c) $\mathbb{R}$ (d) $\varnothing$

(4) The smallest value of the expression $\frac{|x|+|y|}{|x+y|}$ is ....

(a) – 1 (b) zero (c) 1 (d) 2

(5) If $2^x = 61$, then $|x - 6| + |x - 5| =$ ....

(a) – 11 (b) – 1 (c) 1 (d) 11

(6) If $a^2 b > 0$, $\frac{a}{b} < 0$, then $\sqrt{a^2} + \sqrt{b^2} - (b - a) =$ ....

(a) 2 a (b) – 2 b (c) – 2 a + 2 b (d) zero

Life Applications on Unit One

From the school book

1 **Trade :**

The function $f : f(X) = \begin{cases} \frac{5}{2} X & , \ 0 \le X \le 5000 \\ 2X + 2500 & , \ 5000 < X \le 15000 \\ \frac{3}{2} X + 10000 & , \ 15000 < X \le 60000 \end{cases}$

represents the amount of money charged by a company to distribute an electrical appliance in L.E. where X represents the number of distributed appliances.

Find :

(1) $f(5000)$ (2) $f(10000)$ (3) $f(50000)$

« L.E. 12500 , L.E. 22500 , L.E. 85000 »

2 **Geometry :**

If P is the perimeter of a square of side length ℓ , write P as a function of ℓ [P (ℓ)]

, then find :

(1) P (3) (2) P$\left(\frac{15}{4}\right)$

«12 length units , 15 length units »

3 **Geometry :**

If A is the area of a circle of radius length r , write A as a function of r [A (r)]

, then find :

(1) A$\left(\frac{1}{2}\right)$ (2) A (5)

«$\frac{1}{4}\pi$ square units , 25 π square units »

4 **Trade :**

A grains merchant pays 50 L.E for each ton getting in or out of his warehouse for loading or unloading the goods. Write down the function representing the cost of loading or unloading , then represent it graphically.

5 **Urban communities :**

Rectangular pieces of land are specialized for youth housing in a new urban community. If the length of each is X metre and the area is 400 square metre.

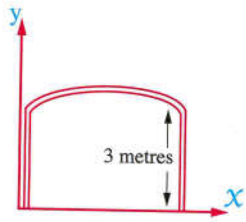
(1) Show that the length of the piece of land is inversely proportional to its width.

(2) Write down the rule of the function which shows the width of the piece of land in terms of its length. Represent it graphically.

(3) From the graph , find the width of the piece of land whose length is 25 metre , then check that algebraically. « 16 m. »

6 **Industry :**

An iron gate whose two sides are 3 metres high and its arc is in the form of a part of the curve of the function $f : f(X) = a(X-2)^2 + 4$ has been designed as shown in the opposite figure **, find :**



(1) The value of a

(2) The maximum height of the gate.

(3) The width of the gate.

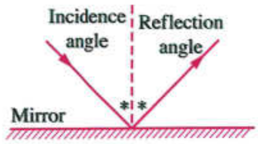
« $-\frac{1}{4}$, 4 m. , 4 m. »

7 **Roads :** Two roads; the first road is represented by the curve of the function f where $f(X) = |X - 4|$ and the second one is represented by the curve of the function g where $g(X) = 3$, if the two roads get intersected at the two points A and B , find the distance from A to B known that the length unit represents 1 km. only. « 6 km. »

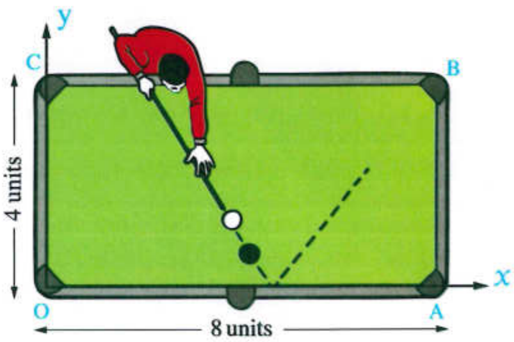
8 A meteorological station has recorded the temperature of Cairo on a day. If the temperature has been 32° in difference 7° from its normal rate on that day. What is the expected temperature recorded in Cairo on that day ? « 25° or 39° »

9 **Athletic medicine :** Bassem's weight differs from his ideal weight for 5 kg. What is his probable weight if his ideal weight is 60 kg. ? « 55 kg. or 65 kg. »

10 If a light ray falls on a reflective surface whose pathway is subjected to the modulus function , the measurement of the incidence angle equals the measurement of the reflection angle. In addition , the pathway of the billiard ball before and after colliding it against the table edge.



The opposite figure shows a billiard player shooting at the black ball. Considering $\overrightarrow{Ox}$ and $\overrightarrow{Oy}$ the perpendicular coordinates axes , and the path of the ball follows the curve of the function f where $f(x) = \frac{4}{3} | x - 5 |$



Does the black ball fall in the pocket B ?
Explain your answer mathematically.

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11 **Vacant jobs :** One of the natural gas companies allows employing a counter reader if his height ranges between 178 cm. and 192 cm. Express all possible heights for the persons applying to join this job using the absolute value inequality. « $| x - 185 | \leq 7$ »

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12 The aviation speed of planes is measured during the normal aviation on air disregarding taking off and landing time. Express the aviation speed ranging between 700 km./h. to 900 km./h. using the absolute value inequality. « $| x - 800 | \leq 100$ »

---

13 A dairy factory produces cans of weight x gram. To control the production quality , the cans pass on the production control weight line which allows the cans to pass if $| x - 1600 | \leq 15$, determine the heaviest and the lightest cans that can be sold from the production of that factory. « 1615 gm. , 1585 gm. »

UNIT 2

Exponents, logarithms and their applications

Unit Exercises

Exercise **7** Rational exponents and exponential equations.

Exercise **8** Exponential function and its applications.

Exercise **9** Logarithmic function and its graph.

Exercise **10** Some properties of logarithms.

- At the end of the unit :
  Life applications on unit two.



Exercise 7

Rational exponents and exponential equations

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) $a^m \times a^m = \dots\dots$

(a) a^{m^2} (b) a^{2m} (c) $2a^m$ (d) ma^2

(2) If $3^{x-5} = 9$, then $x = \dots\dots$

(a) -7 (b) -3 (c) 2 (d) 7

(3) If $2^{x+1} = 8$, then $x = \dots\dots$

(a) 1 (b) 2 (c) 3 (d) 4

(4) If $3^{x+5} = \frac{1}{27}$, then $x = \dots\dots$

(a) -3 (b) 8 (c) -8 (d) 3

(5) If $5^{x-3} = 4^{3-x}$, then $x = \dots\dots$

(a) $\frac{5}{4}$ (b) 3 (c) zero (d) 1

(6) The solution set of the equation : $5^{x^2-4} = 7^{x^2-4}$ is $\dots\dots$

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{2, -2\}$ (d) $\{\text{zero}\}$

(7) If $2^{x+1} = 5^{x+1}$, then : $3^{x+1} = \dots\dots$

(a) zero (b) 1 (c) -1 (d) 3

(8) $\sqrt[5]{a^3} \times \sqrt{a^3} =$ ....

(a) $\sqrt[7]{a^3}$ (b) $\sqrt[7]{a^6}$ (c) $\sqrt[7]{a^{14}}$ (d) $a^2 \sqrt[10]{a}$

(9) If $\left(\frac{2}{3}\right)^{x-2} = \frac{8}{27}$, then $x =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

(10) If $2^{|x|} = 32$, then $x =$ ....

(a) 5 (b) -5 (c) ± 5 (d) 10

(11) If $2^x = 4^y = 64$, then $x + y =$ ....

(a) 3 (b) 4 (c) 6 (d) 9

(12) If $\left(\frac{1}{2}\right)^{a^2 - a - 2} = 1$ where $a >$ zero , then $a =$ ....

(a) 1 (b) -3 (c) 2 (d) 3

(13) The solution set of the equation : $7^{x^2} = 49^{x+4}$ is ....

(a) $\{-2\}$ (b) $\{-2 , 4\}$ (c) $\{-2 , 3\}$ (d) $\{2 , -4\}$

(14) If $3^x = 2$, $2^y = 9$, then $x y =$ ....

(a) 2 (b) 3 (c) 8 (d) 18

(15) If $5^x = 2$, then $(25)^x =$ ....

(a) 10 (b) 625 (c) 4 (d) 2

(16) If $2^x = 5$, then $2^{x+2} =$ ....

(a) 15 (b) 4 (c) 10 (d) 20

(17) If $x^{\frac{3}{2}} = 64$, then $x =$ ....

(a) 512 (b) 16 (c) 4 (d) 2

(18) If $x^{\frac{2}{5}} = 4$, then $x =$ ....

(a) 4 (b) 16 (c) ± 4 (d) ± 32

(19) If $\sqrt[3]{x^2} = 9$, then $x \in$ ....

(a) $\{27\}$ (b) $\{27 , -27\}$ (c) $\{1\}$ (d) $\varnothing$

(20) If $x^{-\frac{3}{2}} = 8$, then $x =$ ....

(a) 4 (b) -4 (c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

(21) If $4 x^5 = 128$, then $x =$ ....

(a) 4 (b) ± 2 (c) 2 (d) -2

(22) $(128)^{-\frac{2}{7}} = \ldots\ldots\ldots\ldots$

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 4

(23) $(64)^{-\frac{1}{6}} = \ldots\ldots\ldots\ldots$

(a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(24) $\sqrt[3]{x^{-3}} = \ldots\ldots\ldots\ldots$

(a) x^{-1} (b) $-x$ (c) $|x^{-1}|$ (d) $-|x|$

(25) If $2 \times 4^{x-3} = 16$, then $x = \ldots\ldots\ldots\ldots$

(a) 5 (b) 7 (c) $4\frac{1}{2}$ (d) $-1\frac{1}{2}$

(26) $\dfrac{6^{-\frac{1}{5}} \times 6^{\frac{3}{5}}}{\sqrt[5]{36}} = \ldots\ldots\ldots\ldots$

(a) 1 (b) 6 (c) $\frac{1}{6}$ (d) $\sqrt{6}$

(27) If x , y $\in \mathbb{R}$, then $\sqrt{x^2 y^6} = \ldots\ldots\ldots\ldots$

(a) $x y^2$ (b) $|x y^3|$ (c) $\frac{1}{2} x^2 y^6$ (d) $\pm x y^3$

(28) $\sqrt[4]{x^4 y^8} = \ldots\ldots\ldots\ldots$

(a) $x y^2$ (b) $|x| y^2$ (c) $\pm x y^2$ (d) $x |y^2|$

(29) If $2^{x-1} = 44$, then $2^{x-2} = \ldots\ldots\ldots\ldots$

(a) 18 (b) 22 (c) 10 (d) 16

(30) If $x^{\frac{5}{3}} = 2\, y^{\frac{4}{3}} = 32$, then $x + y = \ldots\ldots\ldots\ldots$

(a) 16 (b) zero. (c) 16 , – 16 (d) zero , 16

(31) The solution set of the equation : $3^{x+1} + 3^x = 12$ in $\mathbb{R}$ is $\ldots\ldots\ldots\ldots$

(a) $\{0\}$ (b) $\{3\}$ (c) $\{1\}$ (d) $\{1 , 0\}$

(32) The solution set of the equation : $3^x + 3^{3-x} = 12$ is $\ldots\ldots\ldots\ldots$

(a) $\{1 , 2\}$ (b) $\{0 , 3\}$ (c) $\{3 , 4\}$ (d) $\{-1 , -2\}$

(33) The solution set of the equation : $\sqrt[3]{x^2} - 3\sqrt[3]{x} + 2 = 0$ is $\ldots\ldots\ldots\ldots$

(a) $\{1 , 8\}$ (b) $\{9 , 3\}$ (c) $\{8\}$ (d) $\{1\}$

(34) The solution set of the equation : $9^x - 30 \times 3^{x-1} + 9 = 0$ is $\ldots\ldots\ldots\ldots$

(a) $\{0 , 1\}$ (b) $\{1 , 2\}$ (c) $\{0 , 2\}$ (d) $\{0 , 3\}$

(35) The number of real roots of the equation : $x^n = a$ where n is an odd number is $\ldots\ldots\ldots\ldots$

(a) 1 (b) 2 (c) 3 (d) n

(36) The number of real roots of the equation : $x^6 = a$ where $a > 0$, is ....

(a) 1 (b) 2 (c) 3 (d) 6

(37) The number of roots of the equation : $x^3 = 4$ is ....

(a) 1 (b) 2 (c) 3 (d) 4

(38) The number of real roots of the equation : $x^4 = -16$ is ....

(a) zero (b) 1 (c) 2 (d) 4

(39) The set of the real roots of the equation : $(x - 2)^4 = 16$ equals ....

(a) $\{0\}$ (b) $\{4\}$ (c) $\{8\}$ (d) $\{0 , 4\}$

(40) The solution set of the equation : $(x - 3)^{\frac{5}{3}} = 32$ in $\mathbb{R}$ is ....

(a) $\{2\}$ (b) $\{11\}$ (c) $\{11 , -5\}$ (d) $\{-11 , 11\}$

(41) Which of the following is not equal to $\sqrt[5]{x^4}$?

(a) $\left(\sqrt[5]{x}\right)^4$ (b) $\sqrt[4]{x^5}$ (c) $x^{\frac{4}{5}}$ (d) $\left(x^{\frac{1}{5}}\right)^4$

(42) If $a < 0 < b < c$, then which of the following does not belong to $\mathbb{R}$?

(a) $\sqrt[3]{ab}$ (b) $\sqrt[4]{bc}$ (c) $\sqrt[5]{ab + c}$ (d) $\sqrt[6]{ac}$

(43) If $3^{x-2} = \sqrt[4]{27}$, then $x =$ ....

(a) $\frac{11}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 6

(44) If $2^x = 20$, $n < x < n + 1$, n is an integer , then n = ...

(a) 4 (b) 5 (c) 6 (d) 7

(45) If $2^x = a$, $3^x = b$, $5^x = c$, then $(90)^x =$ ....

(a) abc (b) a^2bc (c) ab^2c (d) a + 2 b + c

Second Essay questions

1 Write down each of the following in an exponential form :

(1) $\sqrt[3]{x}$

(2) $\sqrt[4]{a^3}$

(3) $2\sqrt[3]{n}$

(4) $3\sqrt[5]{y^4}$

(5) $\sqrt{a^3 b^6}$

(6) $\sqrt[4]{a^2 b^3}$

(7) $x\sqrt[3]{x}$

(8) $\dfrac{\sqrt[3]{x}}{\sqrt[5]{x^3}}$

(9) $\dfrac{a^3}{\sqrt{a}}$

2 Write down each of the following in a root form :

(1) $a^{\frac{1}{2}}$ (2) $b^{\frac{2}{3}}$ (3) $5\,a^{\frac{4}{5}}$

(4) $8\,b^{\frac{4}{9}}$ (5) $(3\,x)^{-\frac{2}{3}}$ (6) $x^{\frac{1}{2}} \times x^{\frac{1}{3}}$

3 If $x^n = a$, find the values of x in $\mathbb{R}$ (if found) in each of the following cases :

(1) $n = 5$, $a = 0$ (2) $n = 4$, $a = 81$

(3) $n = 2$, $a = -4$ (4) $n = 3$, $a = -8$

4 Find the value of each of the following in the simplest form :

(1) $(16)^{\frac{3}{2}}$ (2) $\left(\frac{16}{625}\right)^{-\frac{3}{4}}$

(3) $\sqrt[3]{(-8)^2}$ (4) $\left(\sqrt[3]{10^2}\right)^{-\frac{3}{2}}$

(5) $\sqrt[4]{\left(2-\sqrt{3}\right)^4}$ (6) $\sqrt[6]{\left(1-\sqrt{7}\right)^6}$

(7) $\sqrt[5]{\left(2-\sqrt{5}\right)^5}$ (8) $\sqrt[5]{243} + \sqrt[9]{512}$

(9) $(27)^{\frac{2}{3}} - (64)^{\frac{5}{6}}$ (10) $(16)^{\frac{3}{2}} \div (8)^{\frac{2}{3}}$

(11) $\sqrt{16\,x^2}$ (12) $\sqrt[5]{-32\,x^5}$

(13) $-\sqrt[3]{8\,a^6\,b^9}$ (14) $\pm\sqrt{64\,(a^2+3)^6}$

(15) $\sqrt[4]{81\,a^{12}}$ (16) $\sqrt[7]{128\,(a+b)^7}$

5 Find in the simplest form the value of each of the following :

(1) $\left(a^{-\frac{2}{3}}\right)^{-3}$ (2) $\sqrt[3]{x} \times x^{\frac{1}{2}}$

(3) $x^{\frac{1}{2}} \div x^{\frac{1}{3}}$ (4) $\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)$

(5) $\left(a^{\frac{1}{3}} - b^{\frac{1}{3}}\right)\left(a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}\right)$ (6) $\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2$

(7) $\left(x^{\frac{3}{2}} - x^{-\frac{3}{2}}\right)^2$ (8) $\frac{\sqrt{a}}{a\,\sqrt[3]{a}}$

6 Simplify to the simplest form :

(1) $\frac{3^4 \times 2^6}{6^3}$ « 24 »

(2) $\frac{6^2 \times 9^2 \times 8}{(12)^2 \times 3^5}$ « $\frac{2}{3}$ »

(3) $\frac{4^{2n+1} \times 2^{1-n}}{8^{n+2}}$ « $\frac{1}{8}$ »

(4) $\frac{6^{4n} \times (30)^{-2n} \times 2^{2n}}{(18)^{2n} \times (15)^{-2n}}$ « 2^{2n} »

(5) $\frac{(27)^{-3} \times (12)^2}{16 \times (81)^{-2}}$ « 3 »

(6) $\frac{9^{4n+1} \times 4^{2-2n}}{3^{9n+1} \times 48^{1-n}}$ « 1 »

(7) $\frac{16^{x-\frac{1}{4}} \times 9^{x+\frac{1}{2}}}{8^{x-1} \times 18^{x+2}}$ « $\frac{1}{27}$ »

(8) $\frac{25}{27} \times \left(\frac{1}{25}\right)^{\frac{1}{2}} \times (81)^{\frac{3}{4}}$ « 5 »

(9) $(125)^{\frac{2}{3}} \times (81)^{\frac{1}{4}} \times (15)^{-1}$ « 5 »

(10) $\frac{8^{\frac{3}{8}} \times 4^{-\frac{3}{16}}}{2^{-\frac{5}{4}}}$ « 4 »

7 Prove that :

(1) $\frac{2^x \times 9^{x+1}}{3 \times 18^x} = 3$

(2) $\frac{(343)^{2x-\frac{1}{3}} \times (4)^{3x+1}}{(196)^{3x} \times 4} = \frac{1}{7}$

8 Find the error :

(1) $\sqrt[4]{x^4} = x$

(2) If $x^{\frac{2}{3}} = 4$, then $x = 8$

9 Find in $\mathbb{R}$ the solution set of each of the following equations :

(1) $x^2 = 36$

(2) $x^2 = -49$

(3) $x^3 = 125$

(4) $x^5 = -32$

(5) $x^7 = -128$

(6) $x^4 = 1296$

(7) $x^{-4} = \frac{1}{16}$

(8) $x^{\frac{7}{2}} = 128$

(9) $x^{\frac{5}{2}} = \frac{1}{32}$

(10) $x^{-\frac{5}{3}} = \sqrt[3]{32}$

(11) $3x^{-\frac{3}{4}} = \frac{3}{8}$

(12) $\sqrt{x^{-5}} = 243$

(13) $\sqrt[3]{x^2} = \frac{1}{25}$

(14) $(x+1)^{\frac{3}{4}} = 8$

(15) $(x-5)^{\frac{5}{2}} = 32$

(16) $\sqrt[3]{x^4} = 81$

(17) $\sqrt[3]{(x-1)^5} = 32$

(18) $(2x+3)^{\frac{4}{3}} = 81$

(19) $\left(\sqrt{x}+2\right)^{\frac{1}{2}} = 3$

(20) $x^{\frac{4}{5}} - 5x^{\frac{2}{5}} + 4 = 0$

10 **Find in $\mathbb{R}$ the solution set of each of the following equations :**

(1) $3^{x+4} = 9$ « $\{-2\}$ »

(2) $2^{x-5} = \frac{1}{32}$ « $\{0\}$ »

(3) $7^{x-2} = 1$ « $\{2\}$ »

(4) $3^{x+5} = 1$ « $\{-5\}$ »

(5) $4^{1-x} = \frac{1}{4}$ « $\{2\}$ »

(6) $5^{x+3} = 4^{x+3}$ « $\{-3\}$ »

(7) $5^{x+2} = x^{x+2}$ « $\{5, -2\}$ »

(8) $2 \times 3^{x-2} = 54$ « $\{5\}$ »

(9) $2^{3x-6} = 5^{x-2}$ « $\{2\}$ »

(10) $2^{x^2-9} = 1$ « $\{3, -3\}$ »

(11) $2 \times 4^{x+3} = \frac{1}{32}$ « $\{-6\}$ »

(12) $\left(\frac{3}{5}\right)^{2x-1} = \frac{27}{125}$ « $\{2\}$ »

(13) $\left(\frac{3}{2}\right)^{x-2} = \frac{8}{27}$ « $\{-1\}$ »

(14) $2^{x} \times 5^{-x} = \frac{4}{25}$ « $\{2\}$ »

(15) $\left(\sqrt{7}\right)^{|x+2|} = 49$ « $\{2, -6\}$ »

(16) $5^{x^2} = 25^{x+4}$ « $\{-2, 4\}$ »

11 **Find in $\mathbb{R}$ the solution set of each of the following equations :**

(1) $3^{x} + 3^{1+x} = 36$ « $\{2\}$ »

(2) $5^{x+1} + 5^{x-1} = 26$ « $\{1\}$ »

(3) $3^{x+3} - 3^{x+2} = 162$ « $\{2\}$ »

(4) $2^{3x+1} - 2^{3x-2} = 56$ « $\{\frac{5}{3}\}$ »

(5) $9^{x} - 3 \times 3^{x} = 0$ « $\{1\}$ »

(6) $2^{x} + 2^{5-x} = 12$ « $\{2, 3\}$ »

(7) $5^{x} + \frac{125}{5^{x}} = 30$ « $\{1, 2\}$ »

(8) $4^{x} + 2^{x+1} = 8$ « $\{1\}$ »

Third Higher skills

Choose the correct answer from the given ones :

(1) If $\sqrt{2} \times \sqrt[3]{3} = \sqrt[6]{x}$, then $x =$ ....

(a) 27 (b) 48 (c) 72 (d) 108

(2) The number $(5^{x+1} + 5^{x})$ is divisible by .... for all natural values of x

(a) 7 (b) 6 (c) 13 (d) 17

(3) If $3^{a} = 4^{b}$, then $9^{\frac{a}{b}} + 16^{\frac{b}{a}} =$ ....

(a) 7 (b) 12 (c) 20 (d) 25

(4) If $2^{a} = 3$, $3^{b} = 7$, $7^{c} = 11$, then $2^{abc} =$ ....

(a) 11 (b) 27 (c) 21 (d) 231

(5) If $x \in \mathbb{R}^*$, n is an even integer , which of the following is true ?

(a) $x^n > 0$ (b) $x^n < 0$ (c) $x^n \leq 0$ (d) $x^n = 0$

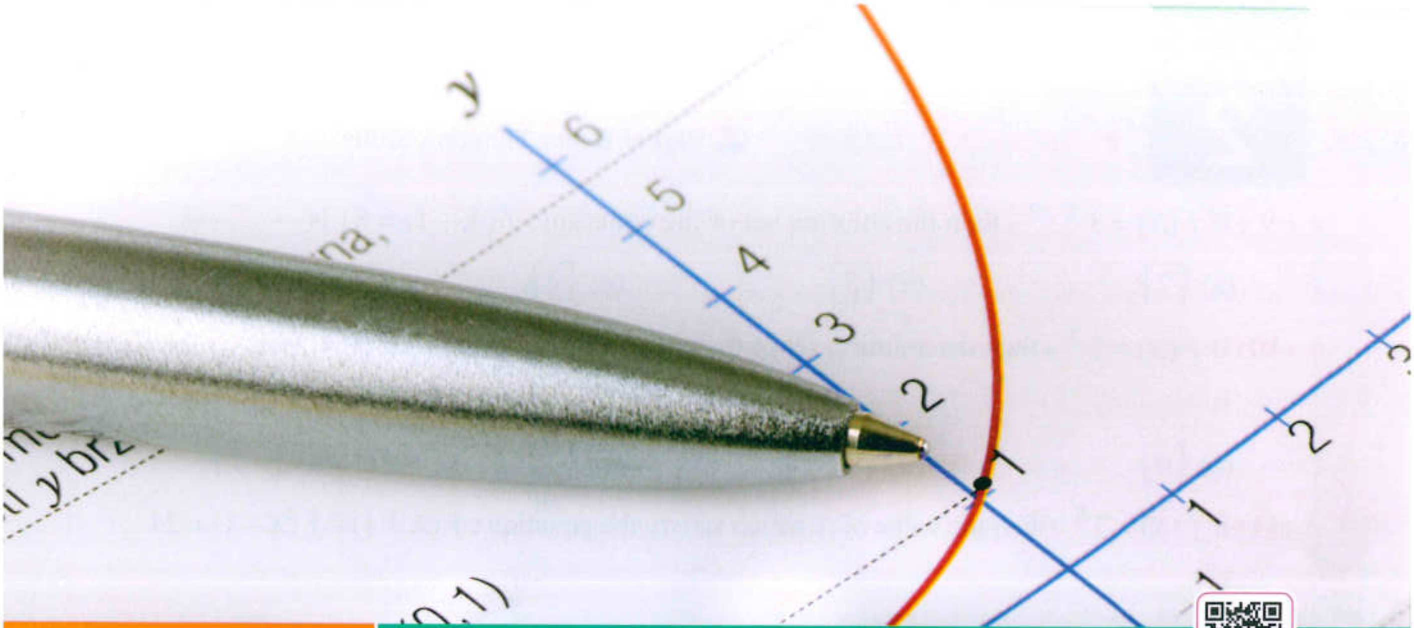
(6) The equation $x^{\frac{2}{3}} = a$ has a solution in $\mathbb{R}$ if ....

(a) $a \in \mathbb{R}$ (b) $a \in \mathbb{R}^+$ (c) $a \in \mathbb{R}^-$ (d) $a \in \mathbb{R}^+ \cup \{0\}$

Activity **Use the calculator to simplify the following operations (Round to two decimals) :**

(1) $75\,(1.21)^{\frac{19}{2}}$

(2) $\dfrac{\sqrt[5]{2^{-1}} \times \sqrt[3]{7^{-2}}}{\sqrt{4^{-3}}}$



Exercise 8 — Exponential function and its applications

Test yourself

From the school book | ● Understand | ● Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If $f : f(x) = a^{x}$ is an exponential function , then $a \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^{+}$ (c) $\mathbb{R}^{-}$ (d) $\mathbb{R}^{+} - \{1\}$

(2) If $f : f(x) = 3^{x+2}$, then $f(-2) =$ ....

(a) 3 (b) zero (c) -1 (d) 1

(3) If $f(x) = 4^{x-1}$, then $f(x+1) =$ ....

(a) 4^{x} (b) 4^{x+1} (c) 4^{x+2} (d) 2^{x}

(4) If $f(x) = 2^{x}$, then $f(-x) =$ ....

(a) -2^{x} (b) $\left(\frac{1}{2}\right)^{x}$ (c) 2^{x+1} (d) $\left(\frac{1}{2}\right)^{-x}$

(5) If $f(x) = (5)^{-x}$, then $\frac{f(x-1)}{f(x+1)} =$ ....

(a) 5 (b) $\frac{1}{5}$ (c) 25 (d) $\frac{1}{25}$

(6) If $f(x-1) = 2^{x+1}$, then $f(x) =$ ....

(a) 2^{x} (b) 2^{x-1} (c) 2^{x+2} (d) 2^{x-2}

(7) If $f(x) = a^{x}$, then $f(x+1) \times f(x-1) = f($..............$)$

(a) $2x + 1$ (b) a^{2x} (c) $2x$ (d) 2

(8) If $f(x+1) = 2^{x}$ and $f(a) = 8$, then $a =$ ....

(a) 3 (b) 2 (c) 4 (d) 5

(9) If $f(X) = 3^{X-2}$, then the solution set of the equation : $f(X-1) = 81$ is ....

(a) $\{7\}$ (b) $\{5\}$ (c) $\{4\}$ (d) $\{3\}$

(10) If $f(X) = 2^{X}$, then the solution set of the equation $f(2X) - f(X+1) = \text{zero}$ is ....

(a) $\{0\}$ (b) $\{0, 1\}$ (c) $\{1\}$ (d) $\{-1\}$

(11) If $f(X) = 3^{X}$, then the value of X which satisfy the equation : $f(X+1) - f(X-1) = 24$ is ....

(a) 2 (b) 3 (c) 8 (d) zero.

(12) If $f(X) = 3^{X}$, then the value of X which satisfy the relation :
$f(2X) - 24 f(X-1) - f(2) = 0$ is ....

(a) $2, \frac{1}{3}$ (b) 2 , zero (c) 2 (d) $2, -1$

(13) The exponential function of base a is increasing if ....

(a) $a > 0$ (b) $a > 1$ (c) $0 < a < 1$ (d) $a = 1$

(14) The function $f : f(X) = a^{X}$ is decreasing on its domain if ....

(a) $a = 1$ (b) $a > 1$ (c) $0 < a < 1$ (d) $a = -1$

(15) The range of the function $f : f(X) = \left(\frac{1}{2}\right)^{X}$ is ....

(a) $]-\infty, \infty[$ (b) $]-\infty, 0[$ (c) $]0, \infty[$ (d) $]1, \infty[$

(16) If $f(X) = 2^{-X}$, then $f(X)$ is decreasing when $X \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^{+}$ (c) $\mathbb{R}^{-}$ (d) $\varnothing$

(17) Which of the following functions is increasing on its domain ?

(a) $f(X) = \left(\frac{1}{2}\right)^{X}$ (b) $f(X) = 3^{-X}$ (c) $f(X) = \left(\frac{2}{3}\right)^{X}$ (d) $f(X) = 5^{X}$

(18) If $n(X) = \left(-\frac{1}{2}\right)^{X+1}$, then it represents ....

(a) an exponential function with base $\left(-\frac{1}{2}\right)$

(b) an exponential function with exponent $(X+1)$

(c) not an exponential function because the base < 0

(d) both (a) and (b)

(19) If $f(X) = 2^{X+1}$ and the point $\left(a, \frac{1}{2}\right) \in$ to the curve of the function f , then a = ...

(a) $\frac{1}{2}$ (b) -1 (c) 2 (d) -2

(20) If $f(X) = a^X$, then ....

(a) $f(X + y) = f(X) + f(y)$ (b) $f(X - y) = f(X) - f(y)$

(c) $f(X + y) = f(X) \cdot f(y)$ (d) $f(X y) = f(X) \cdot f(y)$

(21) If the curve of the function $f : f(X) = 2^X$ is shifted one unit to the left , then the new function is $g : g(X) =$ ....

(a) 2^{X+1} (b) 2^{X-1} (c) -2^X (d) -2^{1-X}

(22) 📖 If the curve of the function $f : f(X) = a^X$ passes through the point (1 , 3) , then a = ...

(a) zero (b) 1 (c) – 1 (d) 3

(23) 📖 The curve of the function $f : f(X) = 3^X$ is the image of the curve of the function $g : g(X) = \left(\frac{1}{3}\right)^X$ by reflection in ....

(a) $X = 0$ (b) $y = 0$ (c) $y = X$ (d) $y = 3X$

(24) 📖 If the curve of the function $f_1 : f_1(X) = 3^X$ intersects the curve of the function $f_2 : f_2(X) = 4 - X$ at the point (k , 3) , then the solution set of the equation $3^X = 4 - X$ is ....

(a) {1} (b) {2} (c) {3} (d) {4}

(25) 📖 The curves of the two functions $f : f(X) = 2^X$ and $g : g(X) = 3^X$ are intersecting at $X =$ ....

(a) – 1 (b) zero (c) 1 (d) 2

(26) The curve of the function $f : f(X) = 5^X$ intersects the y-axis at the point ....

(a) (1 , 0) (b) (0 , 1) (c) (5 , 1) (d) (1 , 5)

(27) The curve of the function $f : f(X) = 2^{X+2}$ intersects the y-axis at the point ....

(a) (0 , 1) (b) (0 , 2) (c) (0 , 4) (d) (0 , 8)

(28) The straight line y = 9 cuts the curve of the function $f : f(X) = 3^X$ at the point ....

(a) (0 , 1) (b) (2 , 0) (c) (2 , 9) (d) (1 , 9)

(29) If the point (a , b) where $a \neq 0$ lies on the curve of the function $y = 2^X$, which of the following points lies on the curve of the function $y = \left(\frac{1}{2}\right)^X$?

(a) (a , b) (b) (– a , b) (c) (a , – b) (d) $\left(a , \frac{1}{2} b\right)$

(30) If the point (a , b) lies on the curve of the function $y = 2^{X}$, then which of the following points lies on the curve of the function $y = 2^{X+3}$

(a) (a , b) (b) (a + 3 , b) (c) (a , b + 3) (d) (a , 8 b)

(31) Which of the functions that are defined by the following rules represents an exponential growth function ?

(a) $f(X) = 2^{-X}$ (b) $f(X) = \left(\frac{1}{3}\right)^{X}$ (c) $f(X) = 3^{X}$ (d) $f(X) = \left(\frac{2}{3}\right)^{X}$

(32) Which of the functions that are defined by the following rules represents an exponential decay function ?

(a) $f(X) = 2^{X}$ (b) $f(X) = \left(\frac{1}{3}\right)^{-X}$ (c) $f(X) = 3^{X}$ (d) $f(X) = \left(\frac{2}{3}\right)^{X}$

(33) Which of the following functions represents an increasing exponential function on its domain $\mathbb{R}$?

(a) $y = 3\,(1.05)^{X}$ (b) $y = 3\left(\frac{1}{1.05}\right)^{X}$ (c) $y = 3 + (0.5)^{X}$ (d) $y = (0.05)^{X}$

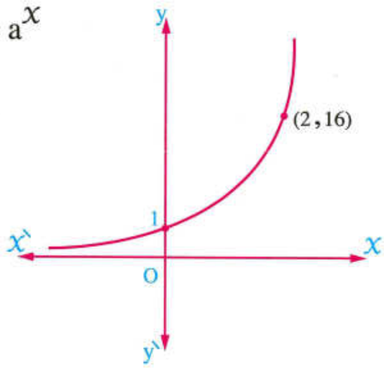
(34) The opposite figure represents the curve of the function $y = a^{X}$, then a = ...

(a) 2

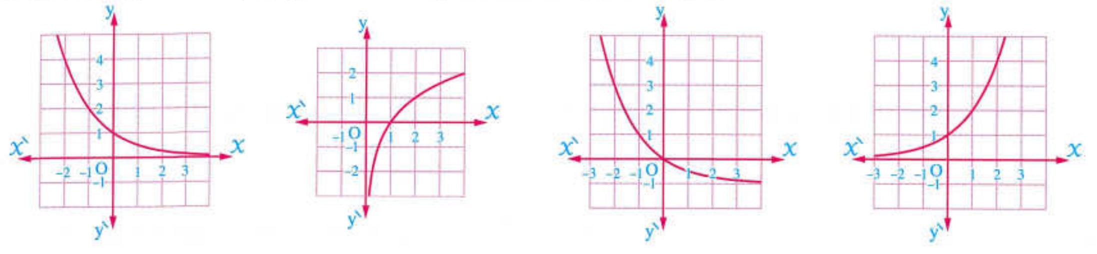
(b) 3

(c) 4

(d) 9



(35) The function f where $f(X) = 2^{X}$ is represented by the figure ....



(a) (b) (c) (d)

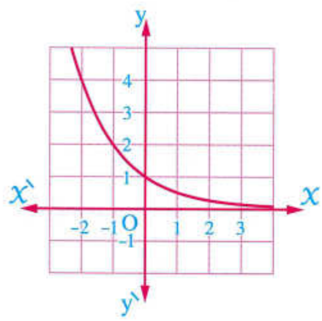
(36) The opposite figure shows the function f where ....

(a) $f(X) = 2^{X+1}$

(b) $f(X) = 2^{-X}$

(c) $f(X) = 3^{-X}$

(d) $f(X) = 2^{X}$



(37) An amount of 5000 pounds is deposited in a bank gives a yearly compound interest 5% for 7 years ≃ …………… pounds.

(a) 6750 (b) 7035.5 (c) 5350 (d) 8500

(38) Galal bought a car for 200000 pounds , if the car price depreciated by 0.4 % yearly. Which of the following functions express the car price after n years ?

(a) $y = 200000 \times (0.4)^n$ (b) $y = 200000\ (0.996)^n$

(c) $y = 200000 \times (1.4)^n$ (d) $y = 200000\ (0.2)^n$

Second Essay questions

1 **Show which of the functions defined by the following rules is an exponential function , then determine the base and the power of each :**

(1) $f(X) = 2X^3$

(2) $f(X) = \frac{2}{3}(5)^X$

(3) $f(X) = \frac{1}{X-1}$

(4) $f(X) = 3X^2 - 1$

(5) $f(X) = \left(\frac{2}{3}\right)^{X-1}$

(6) $f(X) = (-7)^X$

---

2 If $f(X) = 5^X$, **then find the value of :** $\frac{f(X+4) - f(X+3)}{f(X+5) - f(X+4)}$ « $\frac{1}{5}$ »

---

3 If $f(X) = 3^X$, **prove that :** $f(a) \times f(b) = f(a+b)$

---

4 If $f(X) = 5^{X+1}$, **prove that :** $\frac{f(X) \times f(X-1)}{f(X-2) \times f(X+1)} = 1$

---

5 If $f(X) = 2^X$, **then prove that :** $\frac{f(X+1)}{f(X-1)} + \frac{f(X-1)}{f(X+1)} = \frac{17}{4}$

---

6 If $f(X) = 2^X$, **find the solution set for each of the following equations :**

(1) $f(X) = 8$ « {3} »

(2) $f(X+1) = \frac{1}{32}$ « {−6} »

---

7 If $f(X) = 3^{X+1}$, **find the solution set for each of the following equations :**

(1) $f(X) = 27$ « {2} »

(2) $f(X-1) = \frac{1}{9}$ « {−2} »

---

8 If $f(X) = 7^{X-2}$, **find the solution set for each of the following equations :**

(1) $f(X) = 343$ « {5} »

(2) $f(2X) = \frac{1}{49}$ « {0} »

● Understand ● Apply ● Higher Order Thinking Skills

9 If $f(x) = 7^{x+1}$, **then find the value of x that satisfies :**

$f(2x-1) + f(x-2) = 50$ « 1 »

10 If $f(x) = 3^{x}$, **then find the value of x satisfying :** $f(x+1) + f(x-1) = 90$ « 3 »

11 If $f(x) = 4^{x}$, **then find the value of x satisfying :** $f(x+1) + f(x-1) = 68$ « 2 »

12 If $f_1(x) = 3^{x}$ and $f_2(x) = 9^{x}$, **then find the value of x that satisfies :**

$f_1(2x-1) + f_2(x+1) = 756$ « 2 »

13 If $f(x) = 7^{x}$, **then find the value of x satisfying :**

$f(2x-1) + f(2x+1) = \frac{50}{49}$ « $\frac{-1}{2}$ »

14 If $f(x) = 3^{x-1}$, **then find the value of x satisfying :** $f(x+2) + f(4-x) = 30$ « 0 or 2 »

15 If $f(x) = 2^{x}$, **then find in $\mathbb{R}$ the S.S. of the equation :** $f(2x) - 6f(x) + f(3) = 0$ « {1 , 2} »

16 If $f(x) = 3^{x}$, **then prove that :** $\frac{f(2x+2) + f(2x-1)}{5f(2x) - 7f(2x-1)} = \frac{7}{2}$

17 If $f(x) = 3^{3x-1}$, **then prove that :** $\frac{f(x+1) \times f(x+2)}{f(x+3)} = f(x)$

18 **Represent graphically each of the functions defined by the following rules , then find the domain and the range of each , also determine which is increasing and which is decreasing :**

(1) $f(x) = 3^{x}$

(2) $f(x) = 2^{x}$

(3) $f(x) = \left(\frac{1}{2}\right)^{x}$

(4) $f(x) = 2^{-x+1}$

(5) $f(x) = 2^{x-1}$

19 **Find graphically in $\mathbb{R}$ the solution set of each of the following equations :**

(1) $3^{x} = 3$

(2) $2^{x+1} = 5$

(3) $3^{x} = 4 - x$

(4) $3^{x+1} = -x$

(5) $2^{x} = \frac{1}{2}x + 1$

(6) $2^{x} = 2x$

20 **If $f : \mathbb{R} \longrightarrow \mathbb{R}^{+}$ where $f(x) = 3^{x-1}$, graph the function where $x \in [-2 , 3]$, from the graph find :**

(1) $f\left(\frac{3}{2}\right)$

(2) The value of x when : $3^{x-1} = 7\frac{1}{2}$ « 1.7 , 2.8 »

Applications on the exponential growth and decay

21 **Saving :**

Find the sum of L.E. 8000 deposited in a bank giving a yearly compound profit of 5% for 7 years. « L.E. 11256.8 »

22 **In-habitation :**

If the population of a country at the end of the year 2000 is 43.3 million and the rate of population increasing is 1.5% yearly.

(1) Find a form represents the population of this country after n years from the year 2000

(2) Use this form to find the expected population of this country at the year 2020

« 58.3 million people »

23 **Sport :**

The number of spectators of a football team decreases at the rate of 4 % each match as a result of recurrent loss in a championship. If the number of spectators in the first match was 36400 , write the exponential function which represents the number of spectators (y) in the match (t) , then estimate the number of fans in the tenth match. « 24200 fans »

24 If the maximal production of a gold mine is 1850 kg. per year and this production starts to decrease yearly in ratio 9% :

(1) Write an exponential function representing the gold production of this mine after n year.

(2) Estimate the production of this mine after 8 years to the nearest kg. « 870 kg. »

25 A man deposited L.E. 2000 in a bank given yearly complex profit 7% , find the sum of money after 10 years in each of the following cases :

(1) Yearly interest. (2) 6 month's interest. (3) Monthly interest.

« L.E. 3934.3 , L.E. 3979.58 , L.E. 4019.32 »

26 If the marketing price of a car decreases according to the relation $x = 160000\ (0.95)^n$ such that x is the price of the car in L.E. and n is the time in years from the moment of buying it. Find :

(1) The car price when it was brand new.

(2) The car price after 5 years of its buying date. « L.E. 160000 , L.E. 123804.95 »

27 **Fish wealth :**

If the number of Salmons in a lake is increasing according to the function of the exponential growth $f : f\ (n) = 200\ (1.03)^n$, where n is the number of weeks , find the number of Salmons in this lake after 8 weeks. « 253 Salmons »

28 **Population :**

The number of population in a city of A.R.E. reached 4.6 million people with an average increase 4 % annually.

(1) Write the exponential growth function after t years.

(2) Estimate the number of population after 5 years. « 5.6 million people »

---

29 The price of an article is increasing yearly in the ratio 8%. If the original price of this article is L.E. 4000 :

(1) Write a mathematical form gives the price after n year.

(2) Evaluate the price of the article after 4 years to the nearest pound. « L.E. 5442 »

---

30 **Investment :**

If a man has invested L.E. 1000000 in a project in a way that this amount of money grows according to an exponential function with yearly increase of 6% :

(1) Find a formula showing the growth of this money after n year.

(2) Estimate this money after 10 years. « L.E. 1790847.697 »

---

31 **The price of an article is increasing yearly in the ratio 10% If its original price is L.E. 2000 :**

(1) Write a mathematical form giving the price after n years.

(2) After how many years the price will be L.E. 2420 ? « After two years »

Third Higher skills

Choose the correct answer from the given ones :

(1) The function $f : f(X) = (2a)^X$ is decreasing when a ∈ ....

(a) $]0 , 1[$ (b) $]1 , \infty[$ (c) $]0 , 2[$ (d) $]0 , \frac{1}{2}[$

(2) If the function $f : f(X) = \left(\frac{a}{3}\right)^X$ is an increasing exponential function , then ....

(a) $a > 0$ (b) $a > 1$ (c) $a > 3$ (d) $a < 3$

(3) Which of the following curves intersects X-axis ?

(a) $f(X) = \left(\frac{1}{3}\right)^X$ (b) $f(X) = 2^X + 3$ (c) $f(X) = 3^X - 1$ (d) $f(X) = 3^{X-1}$

(4) If $f(X) = \frac{9^X}{9^X + 3}$, then $f(X) + f(1 - X) =$ ....

(a) $\frac{2}{9^X + 3}$ (b) $\frac{9^X + 3}{2}$ (c) $\frac{1}{3}$ (d) 1



Exercise 9 Logarithmic function and its graph

Test yourself

From the school book · Understand · Apply · Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The form $\log_a X = y$ is equivalent to ....

(a) $\log_a y = X$ (b) $a^y = X$ (c) $a^X = y$ (d) $y = a X$

(2) $\text{Log}_{\frac{5}{2}} \frac{16}{625} =$ ....

(a) – 2 (b) – 4 (c) 3 (d) 5

(3) If $\log 0.01 = 3 X + 1$, then $X =$ ....

(a) – 3 (b) – 1 (c) 2 (d) 7

(4) If $\log_3 X = 2$, then $X =$ ....

(a) 3 (b) 5 (c) 8 (d) 9

(5) If $\log_{\frac{1}{4}} X = -1$, then $X =$ ....

(a) – 4 (b) – 1 (c) 1 (d) 4

(6) If $\log_2 X = \log_3 9$, then $X =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

(7) If $\log_5 X = 2$, then $\log (40 X) =$ ....

(a) 3 (b) 25 (c) 100 (d) 1000

(8) If $\text{Log}_5 x = 3$, then $\log_5 \frac{x}{5} =$ ....

(a) 2 (b) 3 (c) 25 (d) 125

(9) If $\log (x + 11) = 2$, then $x =$ ....

(a) – 9 (b) 22 (c) 89 (d) 91

(10) If $\log_9 \sqrt{x + 7} = \frac{1}{2}$, then $x =$ ....

(a) 2 (b) 4 (c) 6 (d) 8

(11) The S.S. of the equation $\log_x 81 = 4$ in $\mathbb{R}$ is ....

(a) $\{-3\}$ (b) $\{3\}$ (c) $\{3 , -3\}$ (d) $\{9\}$

(12) The solution set of the equation : $\log_x 3 = -2$ in $\mathbb{R}$ is ....

(a) $\left\{\frac{1}{9}\right\}$ (b) $\{9\}$ (c) $\left\{\sqrt{3}\right\}$ (d) $\left\{\frac{1}{\sqrt{3}}\right\}$

(13) If $\text{Log}_x 25 = 2$, then $x^3 + x^2 - x =$ ....

(a) 95 (b) 105 (c) 145 (d) 155

(14) If $\log_3 (2x + 3) = 2$, then $x =$ ....

(a) 3 (b) 2 (c) 9 (d) 4

(15) The S.S. of the equation $\log_{(x+3)} 125 = 3$ in $\mathbb{R}$ is ....

(a) $\{5\}$ (b) $\{3\}$ (c) $\varnothing$ (d) $\{2\}$

(16) The solution set of the equation $\log (x - 1) =$ zero is ....

(a) $\left\{\frac{1}{10}\right\}$ (b) $\{1\}$ (c) $\{2\}$ (d) $\{-1\}$

(17) The S.S. of the equation $\log_x (3x - 2) = 2$ in $\mathbb{R}$ is ....

(a) $\{1 , 2\}$ (b) $\{1\}$ (c) $\{2\}$ (d) $\varnothing$

(18) S.S. of the equation $\log_x (x + 6) = 2$ in $\mathbb{R}$ is ....

(a) $\{3 , -2\}$ (b) $\{3\}$ (c) $\{3 , 1\}$ (d) $\{6 , 1\}$

(19) If the solution set of the equation : $\log_x 64x = 4$ in $\mathbb{R}$ is ....

(a) $\{2\}$ (b) $\{4\}$ (c) $\{6\}$ (d) $\{0 , 4\}$

(20) If $\log_{|x+2|} 64 = 3$, then $x \in$ ....

(a) $\{6 , -2\}$ (b) $\{2 , -6\}$ (c) $\{0 , -8\}$ (d) $\{4 , -8\}$

(21) The value of $\log_6 33 \approx$ .... (by using calculator)

(a) 1.95 (b) 0.0512 (c) 2.297 (d) 0.74

(22) The value of x where $\log x = 0.35$ is .... (to the nearest thousandths)

(a) 3.534 (b) 2.839 (c) 2.239 (d) ± 2.239

(23) The curve of the function $f : f(\mathcal{X}) = \log_2(\mathcal{X} + 1)$ intersects the $\mathcal{X}$-axis at the point ....

(a) (0 , 0) (b) (1 , 0) (c) (2 , 0) (d) (1 , 1)

(24) The curve of the function $f : f(\mathcal{X}) = \log_2(3 - \mathcal{X})$ intersects the $\mathcal{X}$-axis at the point ....

(a) (1 , 0) (b) (2 , 0) (c) (0 , 1) (d) (3 , 0)

(25) The curve of the function $f : f(\mathcal{X}) = \log_2 \mathcal{X}$ is passing through the point (8 , ...)

(a) 2 (b) 3 (c) $\log_2 3$ (d) 256

(26) The function $f : f(\mathcal{X}) = \log_a \mathcal{X}$ is decreasing for every a ∈ ....

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $]0 , 1[$ (d) $]1 , \infty[$

(27) If the function $f : f(\mathcal{X}) = \log_{\frac{1}{2}} \mathcal{X}$, then $f\left(\frac{1}{4}\right) + f(8) =$ ....

(a) – 3 (b) – 1 (c) 2 (d) 5

(28) If $\log_{\frac{1}{2}} f(\mathcal{X}) = \mathcal{X}$, then $8 f(2) + f(-3) + f(0) =$ ....

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) 11 (d) 22

(29) If the curve of the function $y = \log_4(1 - a\mathcal{X})$ passes through the point $\left(\frac{1}{4} , -\frac{1}{2}\right)$, then a = ...

(a) 2 (b) 3 (c) 4 (d) 8

(30) If the curve of the function f where $f(\mathcal{X}) = \log_a \mathcal{X}$ passes through the point (8 , 3) , then $f(4) =$ ....

(a) 1 (b) 2 (c) $\frac{1}{4}$ (d) –2

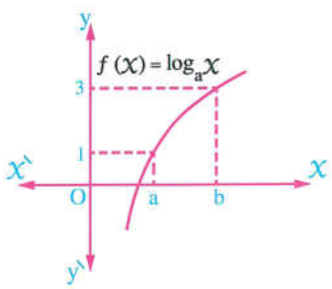
(31) The domain of the function f where $f(\mathcal{X}) = \log_{(1-\mathcal{X})} 3$ is ....

(a) $]-\infty , 0[\cup]0 , 1[$ (b) $]-\infty , 1[$ (c) $]1 , \infty[$ (d) $]-1 , 1[$

(32) The domain of the function $f : f(\mathcal{X}) = \log_{(1-\mathcal{X})} \mathcal{X}$ is ....

(a) $\mathcal{X} > 0$ (b) $\mathcal{X} < 1$ (c) $0 < \mathcal{X} < 1$ (d) $0 \le \mathcal{X} \le 1$

(33) The opposite figure shows the curve of the function $f : f(\mathcal{X}) = \log_a \mathcal{X}$, then b = ...



(a) a^2 (b) a + 3

(c) a^3 (d) 3^a

Second Essay questions

1 **Express each of the following in the equivalent exponential form :**

(1) $\log_2 128 = 7$ (2) $\log_{49} 7 = \frac{1}{2}$ (3) $\log_{\frac{2}{5}} \frac{4}{25} = 2$

(4) $\log_3 \frac{1}{81} = -4$ (5) $\log 0.001 = -3$ (6) $\log_2 4\sqrt{2} = \frac{5}{2}$

2 **Express each of the following in the equivalent logarithmic form :**

(1) $125 = 5^3$ (2) $81 = 9^2$ (3) $5^0 = 1$

(4) $\left(\sqrt{2}\right)^4 = 4$ (5) $5^{-3} = \frac{1}{125}$ (6) $c = 2^n$

3 **Find the value of each of the following :**

(1) $\log_2 16$ « 4 »

(2) 📖 $\log_7 7$ « 1 »

(3) 📖 $\log_5 1$ « 0 »

(4) 📖 $\log_3 9$ « 2 »

(5) $\log 0.00001$ « –5 »

(6) $\log_4 2\sqrt{2}$ « $\frac{3}{4}$ »

(7) $\log_{\frac{1}{2}} 128$ « –7 »

(8) 📖 $\log_9 \frac{1}{27}$ « $-\frac{3}{2}$ »

(9) $\log_{0.2} 125$ « –3 »

(10) $\log_{\sqrt{2}} 8\sqrt{2}$ « 7 »

4 **Solve each of the following equations in $\mathbb{R}$:**

(1) $\log_2 X = 7$ « 128 »

(2) $\log_{\frac{1}{3}} X = -1$ « 3 »

(3) $\log_{\sqrt{3}} X = 4$ « 9 »

(4) $\log_3 X^2 = 4$ « ± 9 »

(5) $\log_{81} X = \frac{3}{4}$ « 27 »

(6) $\log_{81} 3X = \frac{1}{4}$ « 1 »

(7) $\log_{0.2} X = -2$ « 25 »

(8) 📖 $\log_5 \frac{1}{X} = -2$ « 25 »

(9) $\log_{0.5} 2^X = -4$ « 4 »

(10) $\log_3 (2X - 5) = 0$ « 3 »

(11) 📖 $\log_5 (X + 2) = 3$ « 123 »

(12) 📖 $\log_2 (X + 5) = 3$ « 3 »

(13) 📖 $\log_3 (X - 1) = 2$ « 10 »

(14) 📖 $\log_5 (3X - 1) = 1$ « 2 »

(15) $\log_6 \sqrt{X + 4} = \frac{1}{2}$ « 2 »

(16) $\log_3 |X| = 1$ « ± 3 »

(17) $\log_5 |2X + 1| = 1$ « 2 or – 3 »

(18) $\log_2 X(X + 6) = 4$ « 2 or – 8 »

(19) $\log_2 (X - 1)^2 = 2$ « 3 or – 1 »

(20) $\log_3 (X^2 - 2X) = 1$ « 3 or – 1 »

(21) $(\log_3 X)^2 - 9\log_3 X + 20 = 0$ « 81 or 243 »

(22) $|\log_{10} X - 2| = 2$ « 10^4 or 1 »

5 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

(1) $\log_x 9 = 2$ «{3}»

(2) 📖 $\log_x 9 = \frac{2}{3}$ «{27}»

(3) $\log_x 0.001 = \frac{-3}{4}$ «{10000}»

(4) $\log_{-x} 81 = 4$ «{-3}»

(5) 📖 $\log_{x-1} 9 = 2$ «{4}»

(6) $\log_{x-1} 27 = 3$ «{4}»

(7) $\log_{x-1} (7 - x) = 2$ «{3}»

(8) 📖 $\log_{x+1} 8 = \frac{3}{4}$ «{15}»

(9) $\log_x 5x = 2$ «{5}»

(10) 📖 $\log_x (x + 2) = 2$ «{2}»

(11) $\log_x (2x + 8) = 2$ «{4}»

(12) $\log_x \left(\sqrt{x-2} + 2\right) = 1$ «{2 , 3}»

6 **Find the value of x in each of the following :**

(1) $\log_3 81 = x$ « 4 »

(2) $\log_3 \frac{1}{27} = x$ « − 3 »

(3) 📖 $\log_5 625 = x - 1$ « 5 »

(4) 📖 $\log_3 27 = x + 2$ « 1 »

(5) $\log_{27} 1 = x^3$ « 0 »

(6) $\log_{\sqrt{8}} 64 = x$ « 4 »

(7) $\log_4 8\sqrt{2} = x$ «$\frac{7}{4}$»

(8) $\log_{\sqrt{5}} 625\sqrt{5} = x^2$ « ± 3 »

7 📖 **Using the calculator , find the value of each of the following approximating to the nearest 4 decimals :**

(1) $\log 15$ (2) $\log_2 27$ (3) $4 \log 7 - 5 \log 13$

8 **Using the calculator , find the value of x in each of the following approximating to the nearest 4 decimals :**

(1) $\log x = 0.2345$ (2) $\log x = 1.412$ (3) $\log x = -0.3$

9 **Determine the domain of each of the functions that are defined by the following rules :**

(1) $f(x) = \log_3 (2x + 1)$

(2) $f(x) = 2 \log x$

(3) 📖 $f(x) = \log_3 (x - 2)$

(4) $f(x) = \log_x x$

(5) $f(x) = \log_{x-2} x$

(6) $f(x) = \log_{2-x} x$

10 **Represent graphically each of the functions defined by the following rules , from the graph find its range and investigate its monotony :**

(1) 📖 $k(x) = \log_2 x$

(2) 📖 $h(x) = \log_3 x$

(3) 📖 $f(x) = \log_{\frac{1}{2}} x$

(4) $\ell(x) = \log_{\frac{1}{2}} (x + 1)$

(5) 📖 $r(x) = \log_{\frac{1}{2}} (x - 1)$

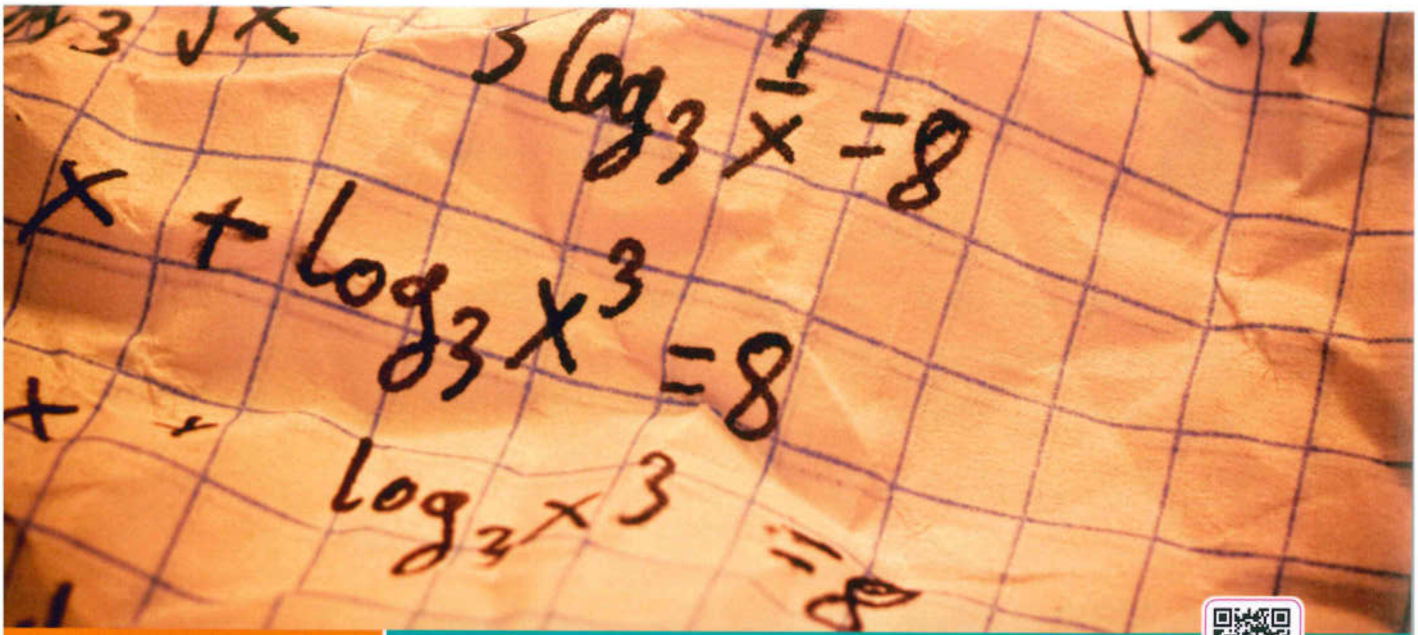
(6) 📖 $t(x) = 1 - \log_2 x$

11 If the curve of the function $f : f(X) = \log_a X$ passes through the point (4 , 2) , find the value of a , then graph the function f taking $X \in [\frac{1}{8}, 8]$, from the graph deduce the range and monotonicity , then find the approximated value to the number $\log_2 1.5$

12 **If the curve of the function $f : f(X) = \log_a X$ is passing through the point (81 , 4) , find the value of a , then graph the function f taking $X \in [\frac{1}{9}, 9]$, from the graph :**

(1) Deduce domain and range and monotonicity and the point of intersection with X-axis.

(2) Find approximated value to the number $\log_3 5$



Exercise 10 Some properties of logarithms

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) $\log_2 5 \times \log_5 2 =$ ....

(a) 1 (b) 10 (c) $\log_2 10$ (d) $\log_5 10$

(2) $1 + \log 2 =$ ....

(a) $\log 5$ (b) $\log 2$ (c) $\log 20$ (d) $-\log 5$

(3) The value of the expression : $2 \log 25 + \log \frac{8}{15} + 2 \log 3 - \log 30 =$ ....

(a) 6 (b) 2 (c) 3 (d) -1

(4) Which of the following statements is true ?

(a) $\log 2 - \log \sqrt{2} = \log \sqrt{2}$ (b) $\log_1 1 =$ zero

(c) $\log \frac{7}{5} = \frac{\log 7}{\log 5}$ (d) $\log 7 \div \log 2 = \log 5$

(5) $\log_{0.09} (0.3)^{-2} =$ ....

(a) -1 (b) -2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

(6) If $\log_2 x = 3$, then $\log_8 x =$ ....

(a) 8 (b) 4 (c) 2 (d) 1

(7) If $\log x - \log 2 = \log 4$, then $x =$ ....

(a) 4 (b) 6 (c) 8 (d) 16

(8) If $\log x + \log 5 = 2$, then $x =$ ....

(a) 3 (b) 8 (c) 17 (d) 20

(9) $2 \log_5 3 + 3 \log_5 2 =$ ....

(a) $\log_5 6$ (b) $6 \log_5 6$ (c) $\log_5 72$ (d) $\log_5 36$

(10) $\frac{1}{\log_2 14} + \frac{1}{\log_7 14} =$ ....

(a) 1 (b) 2 (c) 7 (d) 14

(11) $2 \log_a x + \log_a y - \log_a (x y) =$ ....

(a) $\log x$ (b) $\log_a x$ (c) $\log_a x y$ (d) $\log_a x^2$

(12) The simplest form of the expression : $\log_b a^2 \times \log_c b^3 \times \log_a c =$ ....

(a) 2 (b) 3 (c) 6 (d) 1

(13) $\log \left(\frac{a^2}{bc}\right) =$ .... where a , b , c $\in \mathbb{R}^+$

(a) $2 \log a + \log b + \log c$ (b) $2 \log a - \log b + \log c$

(c) $2 \log a - \log b - \log c$ (d) $2 (\log a - \log b - \log c)$

(14) If $a \in \mathbb{R}^+ - \{1\}$, x and $y \in \mathbb{R}^+$, $\log_a y \neq 0$, then $\frac{\log_a x}{\log_a y} =$ ....

(a) $\log_a \frac{x}{y}$ (b) $\log_a (x - y)$ (c) $\log_a x - \log_a y$ (d) $\log_y x$

((15) $\log_{ab} \frac{1}{a} + \log_{ab} \frac{1}{b} =$ ....

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) -1 (d) 1

(16) The expression $\frac{3 \log 2}{\log 4 + \log 3}$ is equivalent to the expression ....

(a) $\log_3 2$ (b) $\log_7 2$ (c) $\log_{12} 8$ (d) $\log_7 8$

(17) If $3^x = 5$, then $x =$ ....

(a) 3 (b) $\log_3 5$ (c) $\log_5 3$ (d) $\frac{5}{3}$

(18) The solution set of the equation : $\log_2 (2^{x-4}) = 5 - x$ is ....

(a) $\{4\}$ (b) $\{4.5\}$ (c) $\{5\}$ (d) $\{5.4\}$

(19) The solution set of the equation : $\log_{\sqrt{2}} x + \log_{\sqrt{2}} (x + 1) = 2$ is ....

(a) $\{1 , 2\}$ (b) $\{-2\}$ (c) $\{1 , -2\}$ (d) $\{1\}$

(20) The solution set of the equation : $2 \log 2 - \log x = \log (x + 3) - \log 7$ is ....

(a) $\{7\}$ (b) $\{4\}$ (c) $\{7 , 4\}$ (d) $\varnothing$

(21) If $x = 5 + 2\sqrt{6}$, then $\log\left(x + \frac{1}{x}\right) =$ ....

(a) 1 (b) $5 - 2\sqrt{6}$ (c) 10 (d) $5 + 2\sqrt{6}$

(22) If $\log_3 x = \log_9 25$, then $x =$ ....

(a) 5 (b) 3 (c) 9 (d) $\frac{25}{3}$

(23) The solution set of the equation : $\log_2 x + \log_4 x = 3$ is ....

(a) $\{2\}$ (b) $\{4\}$ (c) $\{2 , 4\}$ (d) $\{0\}$

(24) If $\log_2 x + \log_2 x^2 = 6$, then $x =$ ....

(a) 2 (b) 4 (c) 6 (d) 216

(25) The solution set of the equation : $\log x^2 - (\log x)^2 = 0$ is ....

(a) $\{1\}$ (b) $\{1 , 10\}$ (c) $\{1 , 100\}$ (d) $\{100\}$

(26) The solution set of the equation : $(\log_3 x)^2 - \log_3 x^3 + 2 = 0$ is ....

(a) $\{3\}$ (b) $\{3 , 9\}$ (c) $\{9\}$ (d) $\{1 , 2\}$

(27) The solution set of the equation : $\log_3 x - 2\log_x 3 = 1$ in $\mathbb{R}$ is ....

(a) $\{8 , 3\}$ (b) $\left\{8 , \frac{1}{3}\right\}$ (c) $\left\{9 , \frac{1}{3}\right\}$ (d) $\{8\}$

(28) If $\log 23 = a$, then $\log 2300 =$ ....

(a) $a + 2$ (b) $a - 2$ (c) $100\,a$ (d) a^2

(29) If $\log 3 = x$, $\log 4 = y$, then $\log 12 =$ ....

(a) $x + y$ (b) $x\,y$ (c) $x - y$ (d) $\log x + \log y$

(30) ABC is a right-angled triangle at A in which AB = ($\log_4 3$) cm. , AC = ($\log_3 64$) cm.
, then its area = ... cm^2

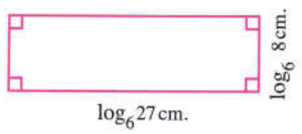
(a) 1.5 (b) 3 (c) $\log 16$ (d) $\log_3 16$

(31) If L and M are the two roots of the equation : $3x^2 - 16x + 12 = 0$
, then $\log_2 L + \log_2 M =$ ....

(a) 2 (b) 4 (c) 12 (d) 16

(32) In the opposite figure :

The perimeter of the figure = ... cm.



(a) $2\log_6 35$ (b) $\log_6 70$

(c) 3 (d) 6

UNIT 2

● Understand ○ Apply Higher Order Thinking Skills

Second Essay questions

1 Without using the calculator , find the value of each of the following :

(1) $\log 2 + \log 5$ « 1 »

(2) $\log_3 2 + \log_3 \frac{1}{2}$ « 0 »

(3) $\log_2 4 + \log_2 16$ « 6 »

(4) $\log_5 15 - \log_5 3$ « 1 »

(5) $\log_3 5 \times \log_5 3$ « 1 »

(6) $3 \log_3 5 + \log_3 \frac{243}{125}$ « 5 »

(7) $\log_3 81 \times \log_9 3$ « 2 »

(8) $\log_6 3 + \log_6 2$ « 1 »

(9) $\log_6 54 - \log_6 9$ « 1 »

(10) $\log_2 12 + \log_2 \frac{2}{3}$ « 3 »

(11) $\log 48 + \log 125 - \log 6$ « 3 »

(12) $\log_2 15 + \log_2 14 - \log_2 105$ « 1 »

(13) $\log 54 - 3 \log 3 - \log 2$ « 0 »

(14) $\log_2 \frac{3}{25} + 5 \log_2 5 + \log_2 27 - \log_2 \frac{125}{12} - \log_2 243$ « 2 »

(15) $\log 0.009 - \log \frac{27}{16} + \log 15 \frac{5}{8} - \log \frac{1}{12}$ « 0 »

(16) $\log_{100} \frac{7}{170} - \log_{100} \frac{35}{18} + \log_{100} \frac{17}{36}$ « −1 »

(17) $\log_2 16 + \log_3 \sqrt{3} + \log 0.1$ « $3\frac{1}{2}$ »

(18) $\frac{1 - \log 2}{\log 125}$ « $\frac{1}{3}$ »

(19) $\frac{\log 49 + 3 \log 7}{\log 7}$ « 5 »

(20) $1 + \log 3 - \log 2 - \log 15$ « 0 »

(21) $\log 25 + \frac{\log 8 \times \log 16}{\log 64}$ « 2 »

(22) $\log_{xy} x + \log_{xy} y$ « 1 »

(23) $\log_{abc} a + \log_{abc} b + \log_{abc} c$ « 1 »

(24) $\frac{1}{2} \log_3 a + \frac{1}{2} \log_3 b + 2 \log_3 c - \log_3 \sqrt{ab} - \log_3 3c^2$ « − 1 »

(25) $\frac{1}{\log_2 12} + \frac{1}{\log_8 12} + \frac{1}{\log_9 12}$ « 2 »

2 **Without using the calculator , prove each of the following :**

(1) $\log_4 16 + \log_4 64 = 5$

(2) $\log_3 243 - \log_3 9 = 3$

(3) $\log_5 125 + \log 10 + \log_3 (25 + 2) = 7$

(4) $\log_2 \frac{4}{11} - \log_2 \frac{7}{130} + \log_2 \frac{77}{65} = \log_5 125$

(5) $\log (4^2 - 2^2) - (\log 4^2 - \log 2^2) = \log 3$

(6) $(1 - \log 5)(2 - \log 25) = 2 (\log 2)^2$

(7) $\frac{\log_2 243 - \log_3 32}{\log_2 27 - \log_3 8} = \frac{5}{3}$

(8) $\frac{(\log 5)^2 - \log 5^2}{\log 5 - \log 100} = 1 - \log 2$

---

3 **Using the calculator , find the value of x in each of the following , approximating to two decimals :**

(1) $5^{x} = 13$ « 1.59 »

(2) $2^{x+1} = 8$ « 2 »

(3) $3^{x+2} = 6$ « – 0.37 »

(4) 📖 $5^{x-1} = 2$ « 1.43 »

(5) 📖 $4 \times 7^{x-2} = 1$ « 1.29 »

(6) $\left(\frac{2}{5}\right)^{x} = 0.042$ « 3.46 »

(7) $7^{x+1} = 3^{x-2}$ « – 4.89 »

(8) $3^{2x-3} = 11^{1-x}$ « 1.24 »

(9) 📖 $2^{x-3} = 3^{x+1}$ « – 7.84 »

(10) $x^{1.6} = 94.5$ « ± 17.17 »

(11) $7^{x+1} + 7^{x-1} = 300$ « 1.92 »

(12) $25^{x} - 27 \times 5^{x} + 50 = 0$ « 2 or 0.43 »

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4 **If $\log_2 7 \simeq 2.807$, then find without using the calculator :**

(1) $\log_2 14$ (2) $\log_2 56$ (3) $\log_2 \frac{7}{4}$

---

5 **If $\log 2 \simeq 0.301$, $\log 3 \simeq 0.4771$, then find without using the calculator :**

(1) $\log 6$ (2) $\log 9$ (3) $\log 12$

---

6 **Find in $\mathbb{R}$ the solution set of each of the following equations :**

(1) 📖 $\log x = \log 3 + \log 10$ « {30} »

(2) 📖 $\log_5 x - \log_5 2 = 2$ « {50} »

(3) $\log_3 (x + 6) = 2 \log_3 x$ « {3} »

(4) $\log_2 x + \log_2 (x+2) = 3$ « $\{2\}$ »

(5) $\log x + \log (x-3) = 1$ « $\{5\}$ »

(6) $\log (x+3) - \log 3 = \log x$ « $\{\frac{3}{2}\}$ »

(7) $\log_2 (x-1) - \log_2 (x-2) = 2$ « $\{\frac{7}{3}\}$ »

(8) $\log_3 x + \log_3 x^2 = 3$ « $\{3\}$ »

(9) $\log (x+1) + \log (x-1) = \log (x+5)$ « $\{3\}$ »

(10) $\log (x+8) - \log (x-1) = 1$ « $\{2\}$ »

(11) $\log_5 x^2 + \log_5 2 = \log_5 18$ « $\{3 , -3\}$ »

(12) $\log_3 (7x^2 - 4) = 2\log_3 x + \frac{1}{2}\log_3 9$ « $\{1\}$ »

(13) $4\log_{11} 3 - \log_{11} 18 - 3\log_{11} x + \log_{11} \frac{3}{4} = 0$ « $\{\frac{3}{2}\}$ »

(14) $\log (x+2) + \log (x-2) = 1 - \log 2$ « $\{3\}$ »

(15) $\log_2 x = \log_4 9$ « $\{3\}$ »

(16) $\log_3 x = \log_x 3$ « $\{3 , \frac{1}{3}\}$ »

(17) $\log x = \frac{(\log 3)^2 - \log 27}{\log 0.003}$ « $\{3\}$ »

(18) $(\log x)^2 - \log x^2 = 3$ « $\{0.1 , 1000\}$ »

---

7 **Find in $\mathbb{R}$ the solution set of each of the following equations :**

(1) $\frac{1}{\log_2 x} + \frac{1}{\log_3 x} = 2$ « $\{\sqrt{6}\}$ »

(2) $\log x - \frac{3}{\log x} = 2$ « $\{0.1 , 1000\}$ »

(3) $\log x - \frac{1}{\log x} = \frac{3}{2}$ « $\{\frac{1}{\sqrt{10}} , 100\}$ »

(4) $\log 7 \times \log 729 = \log 49 \times \log x^3$ « $\{3\}$ »

(5) $\log_2 x + \log_x 2 = 2$ « $\{2\}$ »

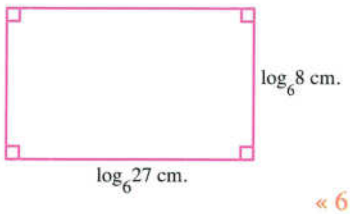
(6) $(\log x)^3 = \log x^9$ « $\{1 , 1000 , 0.001\}$ »

(7) $\log_2 (x^2 + 6x + 9) - \log_2 (x-1) = \log_5 625$ « $\{5\}$ »

(8) $\log_2 x + \log_4 x = \frac{-3}{2}$ « $\{\frac{1}{2}\}$ »

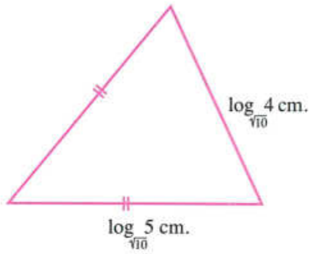
8 **Find the perimeter of each of the following figures :**

(1)



« 6 cm. »

(2)



« 4 cm. »

---

9 **Prove that :** $\log_b a \times \log_c b \times \log_d c \times \log_a d = 1$ **, then calculate the value of :**

$\log_2 3 \times \log_3 5 \times \log_5 16$ « 4 »

Third Higher skills

Choose the correct answer from the given ones :

(1) Which of the following statements is true ?

(a) $\log 3 + \log 3 = \log 6$ (b) $1 - \log 5 = 2$

(c) $\log 2 \times \log 2 = \log 4$ (d) $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

(2) If $\frac{\log \mathcal{X}}{\log 5} = \frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$, then $\mathcal{X} + y =$ ....

(a) 25 (b) 8 (c) 17 (d) 33

(3) $\frac{1}{1 + \log_b a + \log_b c} + \frac{1}{1 + \log_c a + \log_c b} + \frac{1}{1 + \log_a b + \log_a c} =$ ....

(a) $\log_a bc$ (b) $\log_b ac$ (c) $\log_c ab$ (d) 1

(4) If $\frac{1}{\log_2 \mathcal{X}} + \frac{1}{\log_4 \mathcal{X}} + \frac{1}{\log_8 \mathcal{X}} + \frac{1}{\log_{16} \mathcal{X}} = 5$, then $\mathcal{X} =$ ....

(a) 1 (b) 2 (c) 4 (d) 8

Life Applications on Unit Two

📖 From the school book

1 📖 **Economy :**

If it known that the profit (r) of a bank on a sum of money (a) after (n) year is given by the relation : $r = \left(\frac{c}{a}\right)^{\frac{1}{n}} - 1$ where c is the total money after n year. If Gamal has deposited L.E. 10000 and after 3 years the sum of money becomes L.E. 12597 , find the yearly percentage of the profit. « 8 % »

2 📖 **Trade :**

Mohammed has started a project to grow rabbits. If the number of rabbits at the beginning of the project was 75 rabbits and the number of rabbits in their reproduction has followed the relation : $z = 75\,(4.22)^{\frac{n}{6}}$ where n is the number of months.
Find the number of rabbits expected over 5 months. « 249 rabbits »

3 📖 **Volumes :**

If the edge length of a cube ℓ is determined by the relation : $\ell = \sqrt[3]{V}$ where V is the volume of the cube in cubic units. Find the edge length of a cube whose volume is 1331 cm^3

« 11 cm »

4 **Geometry :**

If the radius length of a sphere r is given by the relation $r = \left(\frac{3\,V}{4\,\pi}\right)^{\frac{1}{3}}$ where V is the volume of the sphere , find the increase in the radius length when the volume changes from $\frac{32}{3}\pi$ to $36\,\pi$ cube unit. « 1 length unit »

5 📖 The number of marine organisms decreases according to the function of the exponential decay $y = 8192\left(\frac{1}{2}\right)^{n-1}$ where n is the number of weeks from now.

(1) Find the number of the organisms after 4 weeks from now. « 1024 organisms »

(2) After how many weeks does the number of these organisms get 256 ? « 6 weeks »

6 📖 **Biology :**

A microorganism reproduces by binary fission where the number of these organisms is replicated each hour because each cell is divided into two cells. If the number of cells at the beginning was 20000 cells :

(1) Find the number of cells after 5 hours. « 640000 cells »

(2) After how many hours does the number of cells get 2560000 ? « 7 hours »

7 If the relation between retention of materials of a student in the first secondary form and the number of months (t) starting from the end of study of the class is :
$f(t) = 70 - 4 \log_2 (t + 1)$, find the score of the student :

(1) At the end of the study of the class (t = 0) « 70 marks »

(2) After 7 months from the end of the study of the class. « 58 marks »

8 **A country use a taxes system such that the taxpayer pays yearly the decided taxes according to the following function :**

$$f : f(X) = \begin{cases} 10\,\%\, X & , \quad \text{where } X \leq 5000 \\ 10\,\%\, X + 100 \log (X - 4999) & \text{where } X > 5000 \end{cases}$$

Where X is the yearly net profit , find :

(1) The decided taxes on a taxpayer whose yearly net profit is 3600 pounds.

(2) The decided taxes on a taxpayer whose yearly net profit 8000 pounds.

« 360 pounds , 1147.7266 pounds »

9 **Population :**

If the population of a city increases by yearly rate 7 % :

(1) Find a formula of the population of the city after 1 year.

(2) After how many years the population is doubled assuming that it rises at the same rate ? « 10 years »

10 **If the population of a city starting from 2010 is given by $N = 10^5 (1.3)^{t-2010}$, where N is the number of population , t is the year.**

(1) Find the population of this city in 2015

(2) In which year the population of this city is 1.4 million people ? « 371293 people , 2020 »

11 **Industry :**

If the efficiency of a machine decreases yearly according to the relation $k = k_o (0.9)^n$ where k is the machine efficiency , k_o is the primary efficiency of the machine and n is the number of years the machine works.
If you know that the machine stops working if its efficiency is 40 % of its primary efficiency , how many years does the machine work before it stops working ? « 9 years »

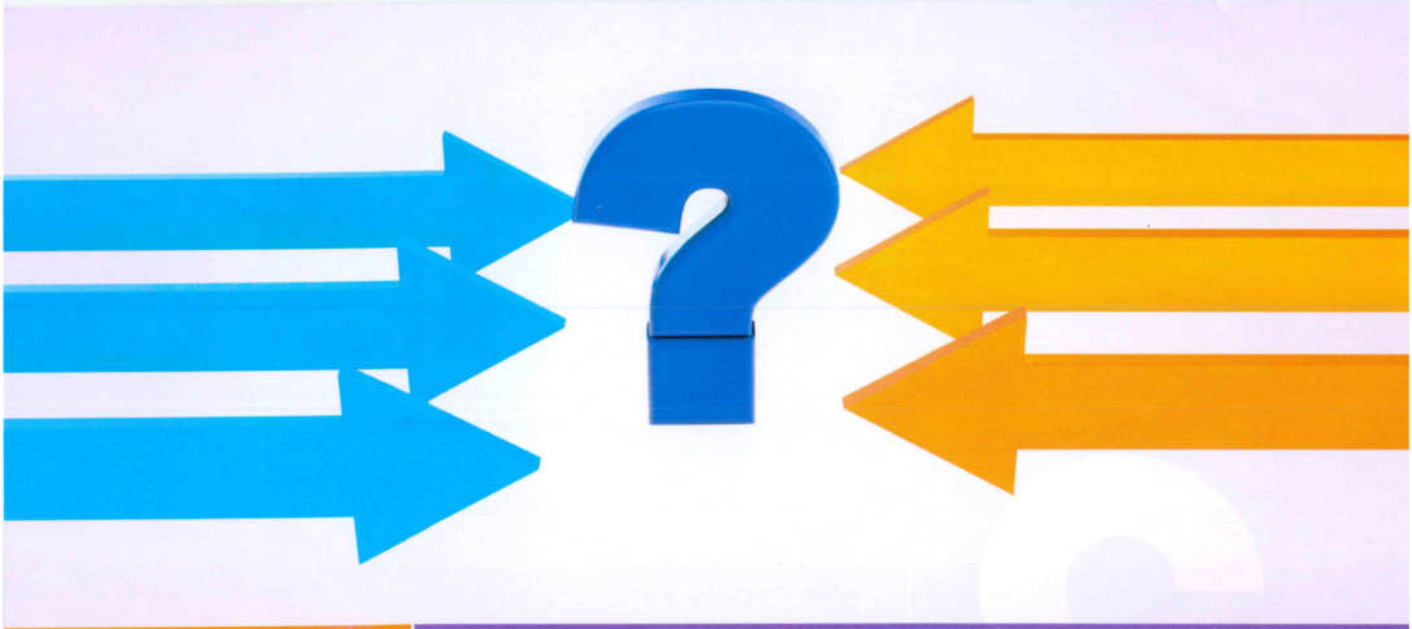
Second

Calculus and Trigonometry



3 Limits.

Unit Exercises



Exercise 11

Introduction to limits of functions
"Evaluation of the limit numerically and graphically"

From the school book | ● Understand | ○ Apply | Higher Order Thinking Skills

First Multiple choice questions

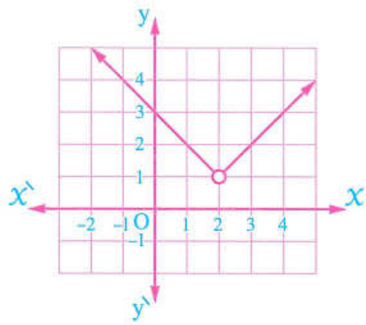
Choose the correct answer from those given :

(1) All the following are unspecified quantities except ....

(a) zero ÷ zero (b) $\infty - \infty$ (c) $\infty + \infty$ (d) $\infty \div \infty$

(2) In the opposite figure :

$\lim\limits_{x \to 2} f(x) =$ ....



(a) 1

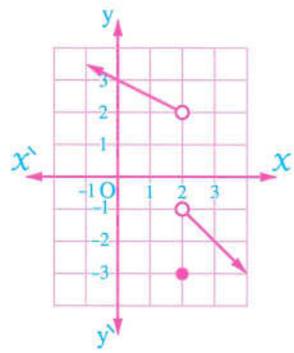
(b) – 1

(c) does not exist.

(d) 2

(3) In the opposite figure :

$\lim\limits_{x \to 2} f(x) =$ ....



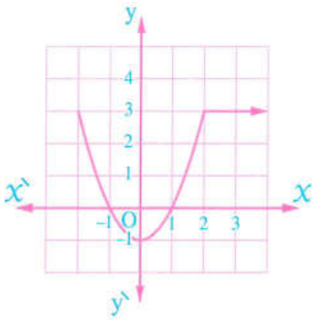
(a) – 3

(b) 2

(c) – 1

(d) does not exist.

(4) In the opposite figure :



$\lim_{x \to 2} f(x) =$ ....

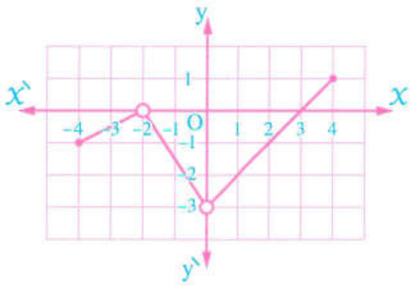
(a) zero

(b) 2

(c) 3

(d) does not exist.

(5) From the opposite figure :



First : $\lim_{x \to -2} f(x) =$ ....

(a) zero (b) – 3

(c) – 2 (d) does not exist.

Second : $\lim_{x \to 0} f(x) =$ ....

(a) zero (b) – 2

(c) – 3 (d) does not exist.

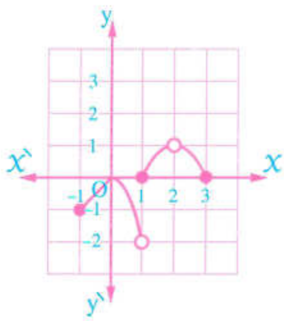
Third : $\lim_{x \to -4} f(x) =$ ....

(a) zero. (b) – 4 (c) – 1 (d) does not exist.

Fourth : $\lim_{x \to 4} f(x) =$ ....

(a) zero. (b) 4 (c) 1 (d) does not exist.

(6) Using the opposite figure :



First : $\lim_{x \to 1} f(x) =$ ....

(a) zero (b) – 2

(c) 1 (d) does not exist.

Second : $\lim_{x \to -1} f(x) =$ ....

(a) zero (b) – 1

(c) – 2 (d) does not exist.

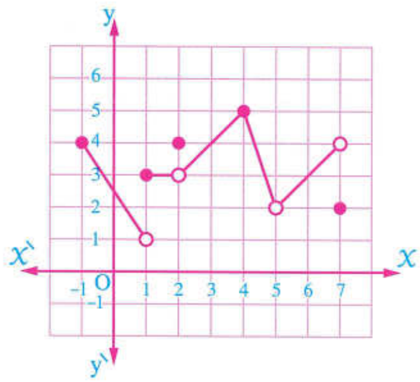
Third : $\lim_{x \to 2} f(x) =$ ....

(a) zero. (b) 1 (c) 2 (d) does not exist.

Fourth : $\lim_{x \to 0} f(x) =$ ....

(a) zero. (b) – 1 (c) – 2 (d) does not exist.

(7) Using the opposite figure :



First : $\lim\limits_{x \to -1} f(x) =$ ....

(a) zero. (b) – 1

(c) 4 (d) does not exist.

Second : $f(2) =$ ....

(a) zero. (b) 3

(c) 4 (d) undefined.

Third : $f(5) =$ ....

(a) zero. (b) 2 (c) 5 (d) undefined.

Fourth : $\lim\limits_{x \to 5} f(x) =$ ....

(a) zero. (b) 2 (c) 3 (d) does not exist.

Fifth : $\lim\limits_{x \to 7} f(x) =$ ....

(a) zero. (b) 2 (c) 4 (d) does not exist.

Second Essay questions

1 Complete the following table and deduce $\lim\limits_{x \to 2} f(x)$ where $f(x) = 5x + 4$:

x	1.9	1.99	1.999	⟶	2	⟵	2.001	2.01	2.1
$f(x)$	....	....	....	⟶	?	⟵	....	....	....

2 Complete the following table and deduce $\lim\limits_{x \to 2} \frac{x-2}{x^2-4}$:

x	1.9	1.99	1.999	⟶	2	⟵	2.001	2.01	2.1
$f(x)$	....	....	....	⟶	?	⟵	....	....	....

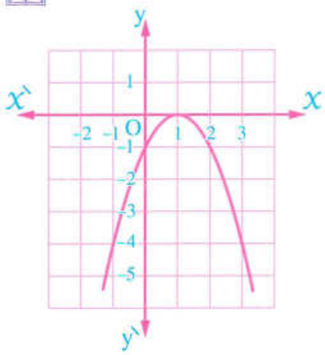
3 Find each of the following limits graphically and numerically :

(1) $\lim\limits_{x \to 4} (2x - 5)$

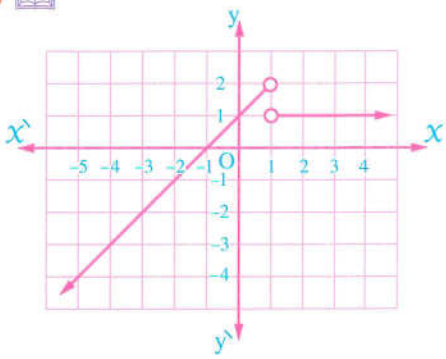
(2) $\lim\limits_{x \to 3} \frac{x^2 - 9}{x - 3}$

4 In each of the following figures , find : $\lim\limits_{x \to 1} f(x)$

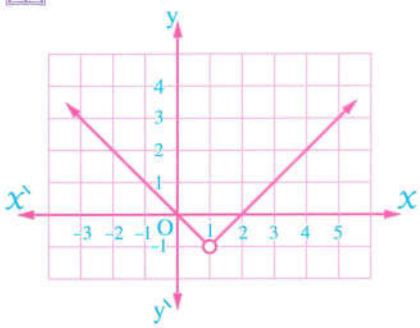
(1)



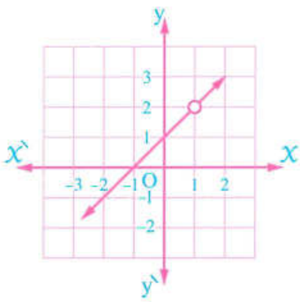
(2)



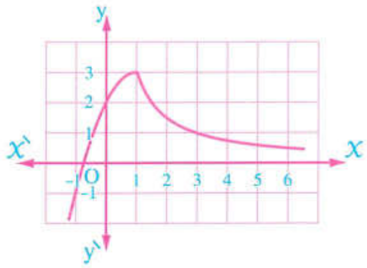
(3)



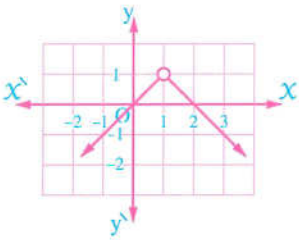
(4)



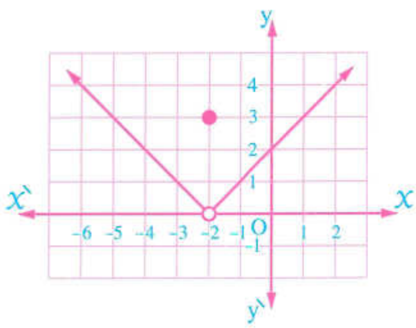
(5)



(6)



5 From the opposite figure , find :



(1) $\lim\limits_{x \to -2} f(x)$

(2) $f(-2)$

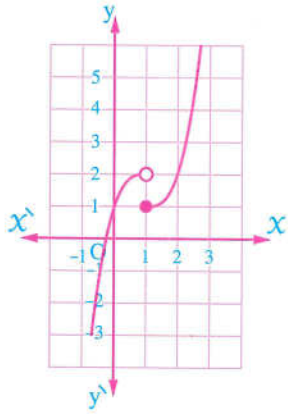
(3) $\lim\limits_{x \to 0} f(x)$

(4) $f(0)$

6 Study the opposite figure , then find :

(1) $f(1)$

(2) $\lim_{x \to 1} f(x)$



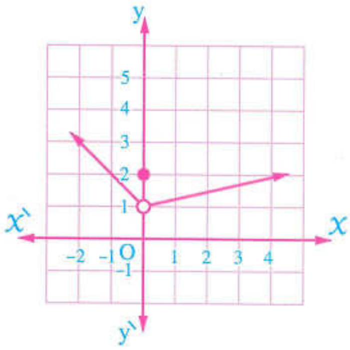
7 Study the opposite figure , then find :

(1) $f(0)$

(2) $\lim_{x \to 0} f(x)$

(3) $f(2)$

(4) $\lim_{x \to 2} f(x)$

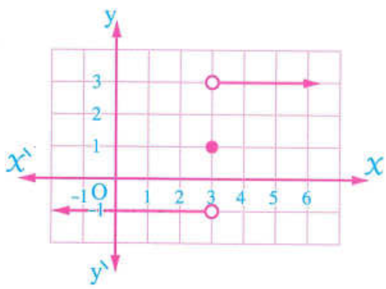


8 From the opposite figure :

Find each of the following if possible :

(1) $f(3)$

(2) $\lim_{x \to 3} f(x)$

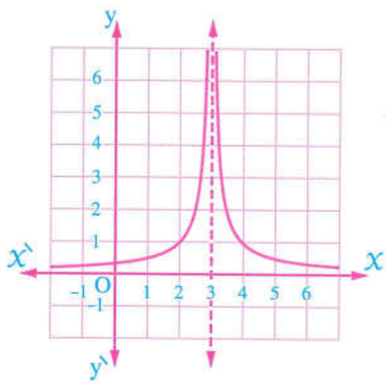


9 From the opposite figure :

Find (if possible) each of the following :

(1) $f(3)$

(2) $\lim_{x \to 3} f(x)$

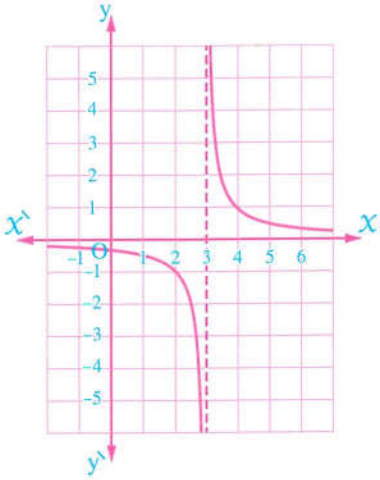


10 **From the opposite figure :**

Find (if possible) each of the following :

(1) $f(3)$

(2) $\lim_{X \to 3} f(X)$



11 **From the opposite figure :**

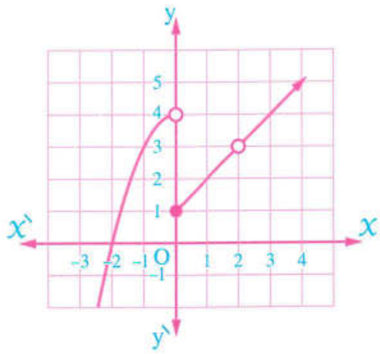
Find :

(1) $f(0)$

(2) $\lim_{X \to 0} f(X)$

(3) $f(2)$

(4) $\lim_{X \to 2} f(X)$



12 If the function f where $f(X) = \begin{cases} X & \text{when } X < 2 \\ X + 2 & \text{when } X \geq 2 \end{cases}$

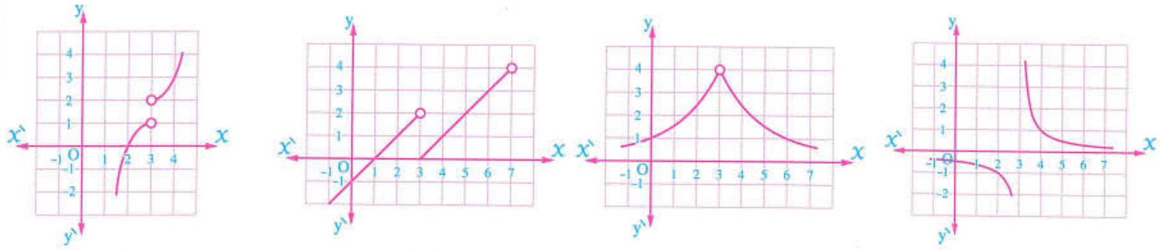
Graph the curve of this function

, then investigate graphically the presence of $\lim_{X \to 2} f(X)$

Third Higher skills

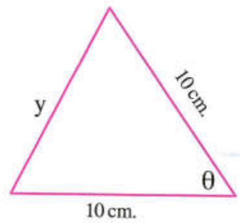
Choose the correct answer from those given :

(1) Which of the functions represented by the following figures does have a limit at $x = 3$?



(a) (b) (c) (d)

(2) In the opposite figure :



When $\theta \longrightarrow \frac{\pi}{2}$

, then : $y \longrightarrow$ .... cm.

(a) 0 (b) 5 (c) 10 (d) $10\sqrt{2}$

(3) If the curve of the polynomial function f intersects the x-axis at $x = 3$, then ....

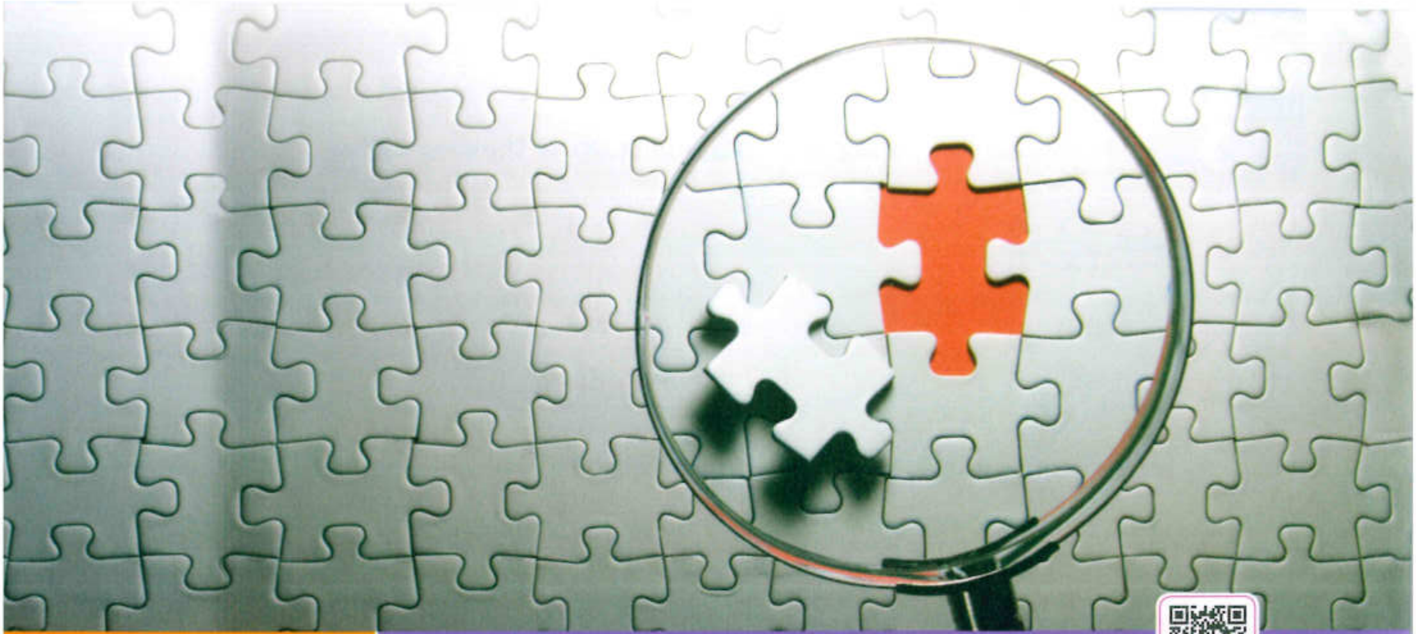
(a) $\lim\limits_{x \to 3} f(x) = 0$ (b) $\lim\limits_{x \to 0} f(x) = 3$

(c) $\lim\limits_{x \to 0} f(x) = 0$ (d) $\lim\limits_{x \to 3} f(x) = 3$

(4) If the curve of the polynomial function f intersects the y-axis at y = 3, then ....

(a) $\lim\limits_{x \to 3} f(x) = \text{zero}$ (b) $\lim\limits_{x \to 3} f(x) = 3$

(c) $\lim\limits_{x \to 0} f(x) = \text{zero}$ (d) $\lim\limits_{x \to 0} f(x) = 3$



Exercise 12 Finding the limit of a function algebraically

Test yourself

From the school book | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) $\lim_{x \to \frac{1}{2}} (10) =$ ....

(a) 5 (b) 20 (c) 10 (d) $10\frac{1}{2}$

(2) $\lim_{x \to 4} \left(3x - \sqrt{x}\right) =$ ....

(a) 8 (b) 10 (c) 14 (d) 16

(3) $\lim_{x \to -1} 3x^2 =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

(4) $\lim_{x \to -2} \frac{1}{|x|} =$ ....

(a) 1 (b) – 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(5) $\lim_{x \to -2} \frac{3x^2 - 12}{x + 2} =$ ....

(a) 18 (b) – 3 (c) 12 (d) – 12

(6) $\lim_{x \to 2} \sqrt{\frac{3 + 2x}{4x - 1}} =$ ....

(a) – 3 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(7) $\lim\limits_{x \to 3} \frac{2x-6}{7x-21} = \text{..............}$

(a) $\frac{2}{3}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) 3

(8) $\lim\limits_{x \to 0} \frac{x^2-x}{x} = \text{..............}$

(a) zero. (b) – 1 (c) does not exist. (d) 1

(9) $\lim\limits_{x \to 3} \frac{x^2-7x+12}{x-3} = \text{..............}$

(a) 1 (b) – 1 (c) 7 (d) – 2

(10) $\lim\limits_{x \to -1} \frac{x^2+x}{x^3+1} = \text{..............}$

(a) zero. (b) $-\frac{1}{3}$ (c) – 1 (d) has no existence.

(11) $\lim\limits_{x \to 3} \frac{x^2-x-6}{x^2+x-12} = \text{..............}$

(a) $\frac{5}{7}$ (b) $\frac{1}{7}$ (c) – 1 (d) – 5

(12) $\lim\limits_{x \to -4} \frac{3x^2-48}{x+4} = \text{..............}$

(a) 32 (b) – 32 (c) 24 (d) – 24

(13) $\lim\limits_{x \to 2} \frac{x^2+x-6}{x^2-4} = \text{..............}$

(a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) $\frac{2}{5}$ (d) $\frac{-2}{5}$

(14) $\lim\limits_{x \to \sqrt{5}} \frac{x^4-x^2-20}{x-\sqrt{5}} = \text{..............}$

(a) 9 (b) $2\sqrt{5}$ (c) $9\sqrt{5}$ (d) $18\sqrt{5}$

(15) $\lim\limits_{x \to 4} \frac{(x-3)^2-1}{x-4} = \text{..............}$

(a) zero (b) 2 (c) 3 (d) 4

(16) 📖 $\lim\limits_{x \to 0} \frac{\sqrt{x+1}-1}{x} = \text{..............}$

(a) zero (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) has no existence.

(17) $\lim\limits_{x \to 3} \frac{\sqrt{x+1}-2}{x-3} = \text{..............}$

(a) 4 (b) $\frac{1}{4}$ (c) 6 (d) $\frac{1}{6}$

(18) $\lim\limits_{x \to 9} \dfrac{\sqrt{2}-\sqrt{x-7}}{x-9} = \text{..............}$

(a) $2\sqrt{2}$ (b) $\frac{\sqrt{2}}{4}$ (c) $\frac{-\sqrt{2}}{4}$ (d) $-2\sqrt{2}$

(19) $\lim\limits_{x \to 2} \dfrac{(x-3)^2-1}{\sqrt{x+2}-2} = \text{..............}$

(a) -6 (b) -8 (c) -2 (d) does not exist.

(20) $\lim\limits_{x \to 1} \dfrac{\sqrt{2x-1}-1}{\sqrt{3x+1}-2} = \text{..............}$

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{4}{3}$

(21) $\lim\limits_{x \to 1} \left(\dfrac{x^3}{x-1} - \dfrac{1}{x-1}\right) = \text{..............}$

(a) zero. (b) -3 (c) 3 (d) does not exist.

(22) $\lim\limits_{x \to 2} \dfrac{x^3-7x+6}{3x^2-8x+4} = \text{..............}$

(a) $\frac{5}{4}$ (b) $\frac{3}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{3}$

(23) $\lim\limits_{x \to 0} \dfrac{7+2x}{\cos x} = \text{..............}$

(a) 7 (b) 8 (c) 9 (d) 1

(24) 📖 $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\tan x}{x} = \text{..............}$

(a) 0 (b) 1 (c) $\frac{4}{\pi}$ (d) does not exist.

(25) 📖 $\lim\limits_{x \to \pi} \dfrac{\cos 2x}{x} = \text{..............}$

(a) 2 (b) 1 (c) $\frac{1}{\pi}$ (d) zero

(26) 📖 $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin x}{x} = \text{..............}$

(a) 1 (b) $\frac{\pi}{2}$ (c) $\frac{2}{\pi}$ (d) has no existence.

(27) 📖 If $\lim\limits_{x \to 2} \dfrac{a}{x+1} = 4$, then a = ...

(a) 3 (b) 4 (c) $\frac{2}{3}$ (d) 12

(28) $\lim\limits_{x \to 2} \dfrac{x-3}{x-2} = \text{..............}$

(a) -1 (b) $\frac{-3}{2}$ (c) $\frac{3}{2}$ (d) does not exist.

(29) If $\lim\limits_{x \to m} \frac{2x^2 - x - 3}{4x^2 - 9} = \frac{5}{12}$, then m = ...

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

(30) If $\lim\limits_{x \to 1} \frac{x^2 + 3x - 4}{x + a} = 5$, then a = ...

(a) – 1 (b) 1 (c) 0 (d) 4

(31) If $\lim\limits_{x \to 2} \frac{x^2 - 4a}{x - 2}$ exists , then a = ...

(a) – 1 (b) 1 (c) 2 (d) 4

Second Essay questions

1 Find each of the following :

(1) $\lim\limits_{x \to 5} \frac{x^2 - 25}{x - 5}$ « 10 »

(2) $\lim\limits_{x \to 3} \frac{x^2 - 8x + 15}{x - 3}$ « – 2 »

(3) $\lim\limits_{x \to 4} \frac{x^2 - 7x + 12}{x - 4}$ « 1 »

(4) $\lim\limits_{x \to 0} \frac{x^2}{3x^3 - 2x^2}$ « $-\frac{1}{2}$ »

(5) $\lim\limits_{x \to 2} \frac{5x - 10}{4x - 8}$ « $\frac{5}{4}$ »

(6) $\lim\limits_{x \to 4} \frac{4x^2 - 64}{x - 4}$ « 32 »

(7) $\lim\limits_{x \to 4} \frac{2x - 8}{x^2 - x - 12}$ « $\frac{2}{7}$ »

(8) $\lim\limits_{x \to -1} \frac{x^2 - 1}{x^2 + x}$ « 2 »

(9) $\lim\limits_{x \to 5} \frac{x^3 - 25x}{x - 5}$ « 50 »

(10) $\lim\limits_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 9}$ « $\frac{1}{3}$ »

(11) $\lim\limits_{x \to 2} \frac{x^2 - 2x}{x^2 - x - 2}$ « $\frac{2}{3}$ »

(12) $\lim\limits_{x \to -1} \frac{x^2 - 3x - 4}{x^2 - x - 2}$ « $\frac{5}{3}$ »

(13) $\lim\limits_{x \to \frac{1}{2}} \frac{2x^2 - 5x + 2}{2x - 1}$ « $\frac{-3}{2}$ »

(14) $\lim\limits_{x \to \frac{3}{2}} \frac{2x^2 - x - 3}{4x^2 - 9}$ « $\frac{5}{12}$ »

(15) $\lim\limits_{x \to -3} \frac{2x^2 + 5x - 3}{x^2 + x - 6}$ « $\frac{7}{5}$ »

(16) $\lim\limits_{x \to 9} \frac{9 - x}{x^2 - 81}$ « $-\frac{1}{18}$ »

2 Find each of the following :

(1) $\lim\limits_{x \to 0} \frac{(x + 2)^2 - 4}{x^2 + x}$ « 4 »

(2) $\lim\limits_{x \to 0} \frac{(2x - 1)^2 - 1}{5x}$ « $\frac{-4}{5}$ »

(3) $\lim\limits_{x \to 2} \frac{(x - 3)^2 - 1}{2x^2 - 3x - 2}$ « $\frac{-2}{5}$ »

(4) $\lim\limits_{x \to -2} \frac{(x + 5)^2 - 9}{x^2 - 4}$ « $\frac{-3}{2}$ »

(5) $\lim\limits_{x \to 2} \frac{x^4 + x^2 - 20}{x - 2}$ « 36 »

(6) $\lim\limits_{x \to 2} \frac{(x^2 - 4)^2}{x - 2}$ « zero »

(7) $\lim\limits_{x \to -2} \frac{x + 2}{x^4 - 16}$ « $-\frac{1}{32}$ »

(8) 📖 $\lim\limits_{x \to 1} \frac{x^{\frac{7}{2}} - x^{\frac{1}{2}}}{x^2 - x}$ « 3 »

(9) 📖 $\lim\limits_{x \to 1} \frac{x^3 - x^2 + 2x - 2}{x - 1}$ « 3 »

(10) 📖 $\lim\limits_{x \to -1} \frac{2x^3 - x^2 - 2x + 1}{x^3 + 1}$ « 2 »

(11) $\lim\limits_{x \to 3} \left(\frac{5}{x} + \frac{x^2 - 3x}{x - 3}\right)$ « $\frac{14}{3}$ »

(12) $\lim\limits_{x \to -1} \left(\frac{x^2}{x^2 - 1} - \frac{3x + 4}{x^2 - 1}\right)$ « $\frac{5}{2}$ »

(13) $\lim\limits_{x \to 1} \left(\frac{1}{x - 1} - \frac{3}{x^3 - 1}\right)$ « 1 »

3 Find each of the following :

(1) $\lim\limits_{x \to 4} \frac{x^3 - 15x - 4}{x - 4}$ « 33 »

(2) $\lim\limits_{x \to 1} \frac{x^3 - 2x + 1}{x^2 + x - 2}$ « $\frac{1}{3}$ »

(3) $\lim\limits_{x \to 4} \frac{x^4 - 21x^2 + 20x}{x - 4}$ « 108 »

(4) 📖 $\lim\limits_{x \to 2} \frac{x^3 - x^2 - 5x + 6}{x - 2}$ « 3 »

(5) 📖 $\lim\limits_{x \to -3} \frac{x^3 - 10x - 3}{x^2 + 2x - 3}$ « $-\frac{17}{4}$ »

(6) $\lim\limits_{x \to -2} \frac{2x^3 + 3x^2 + 4}{x^3 + 8}$ « 1 »

(7) $\lim\limits_{x \to -2} \frac{x^2 + 4x + 4}{x^3 + x^2 - 8x - 12}$ « $-\frac{1}{5}$ »

4 Find each of the following :

(1) $\lim\limits_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$ « $\frac{1}{6}$ »

(2) 📖 $\lim\limits_{x \to 5} \frac{\sqrt{x - 1} - 2}{x - 5}$ « $\frac{1}{4}$ »

(3) 📖 $\lim\limits_{x \to -1} \frac{x + 1}{\sqrt{x + 5} - 2}$ « 4 »

(4) $\lim\limits_{x \to 6} \frac{x - 6}{\sqrt{x - 2} - 2}$ « 4 »

(5) 📖 $\lim\limits_{x \to 1} \frac{\sqrt{x + 3} - 2}{x - 1}$ « $\frac{1}{4}$ »

(6) 📖 $\lim\limits_{x \to 3} \frac{\sqrt{4x - 3} - 3}{x - 3}$ « $\frac{2}{3}$ »

(7) $\lim\limits_{x \to 0} \frac{\sqrt{2x + 9} - 3}{x^2 + x}$ « $\frac{1}{3}$ »

(8) 📖 $\lim\limits_{x \to 5} \frac{x^2 - 5x}{\sqrt{x + 4} - 3}$ « 30 »

(9) $\lim\limits_{x \to 1} \frac{\sqrt{x} - 1}{x^2 + 2x - 3}$ « $\frac{1}{8}$ »

(10) $\lim\limits_{x \to 3} \frac{x^2 - x - 6}{\sqrt{5x - 6} - 3}$ « 6 »

(11) $\lim\limits_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{2x}$ « $\frac{1}{2}$ »

(12) $\lim\limits_{x \to 3} \frac{\sqrt{x + 1} - 2}{\sqrt{x - 2} - 1}$ « $\frac{1}{2}$ »

5 If $\lim\limits_{x \to 2} \frac{f(x) - 5}{x - 2} = 1$, where f is a polynomial function , then find : $\lim\limits_{x \to 2} f(x)$ « 5 »

6 If $\lim\limits_{x \to -1} \frac{x^2 - (a - 1)x - a}{x + 1} = 4$, then find a « –5 »

Third Higher skills

Choose the correct answer from those given :

(1) If f is a function satisfying that : $x(f(x) + 1) = f(x) + x^2$

, then $\lim\limits_{x \to 1} f(x) =$ ....

(a) 1 (b) 2 (c) 3 (d) zero

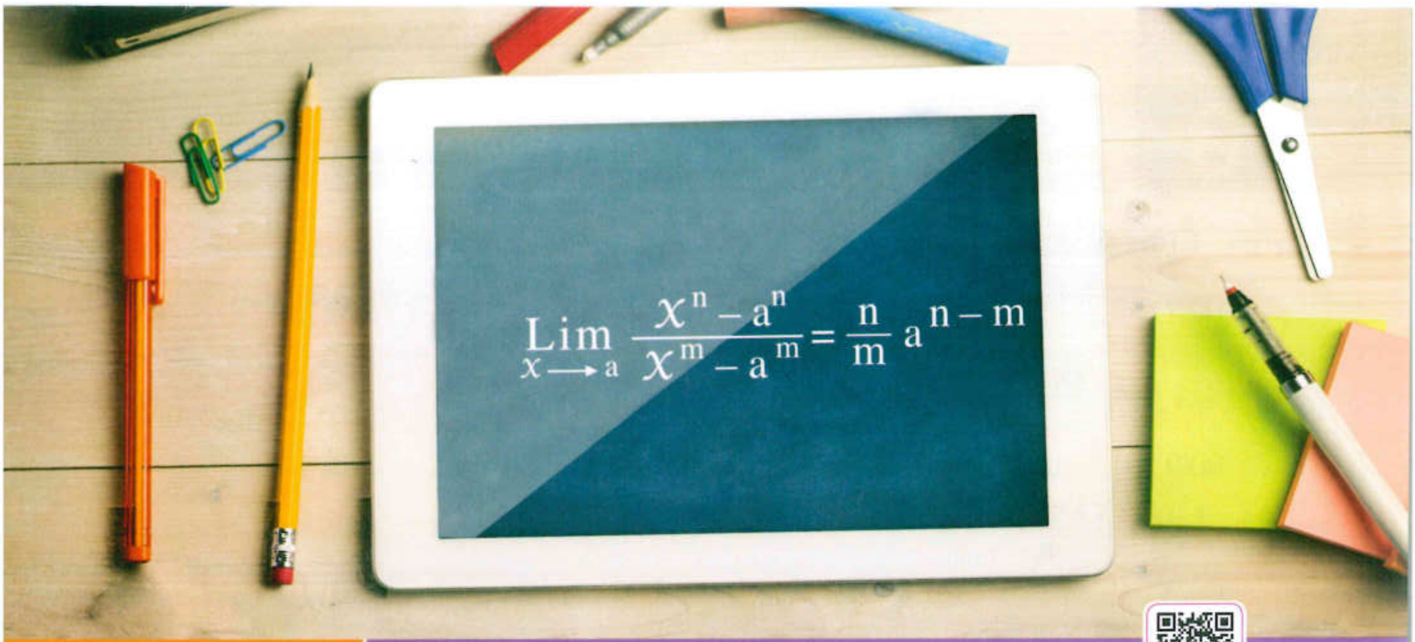
(2) If $\lim\limits_{x \to 1} \frac{x^2 + ax + b}{x - 1} = 5$, then a – b = ...

(a) – 1 (b) – 4 (c) 3 (d) 7

(3) If $\lim\limits_{x \to m} \left(2 f(x) - 5 g(x)\right) = 10$, $\lim\limits_{x \to m} \left(f(x) - g(x)\right) = 6$

, then $\lim\limits_{x \to m} \frac{f(x)}{g(x)} =$ ....

(a) $\frac{40}{7}$ (b) $\frac{2}{7}$ (c) 10 (d) 20



Exercise 13

Theorem (4) " The law "

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) $\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m} = \dots\dots\dots\dots$

(a) $\frac{m}{n}$ (b) $\frac{m}{n} (a)^{m-n}$ (c) $\frac{n}{m} (a)^{m-n}$ (d) $\frac{n}{m} (a)^{n-m}$

(2) $\lim_{y \to 2} \frac{y^5 - 32}{y - 2} = \dots\dots\dots\dots$

(a) $31 y^4$ (b) 32×2^4 (c) 64 (d) 5×2^4

(3) $\lim_{x \to 1} \frac{x^{10} - 1}{x - 1} = \dots\dots\dots\dots$

(a) zero. (b) 1 (c) 9 (d) 10

(4) $\lim_{x \to 2} \frac{x^{-1} - 2^{-1}}{x^{-4} - 2^{-4}} = \dots\dots\dots\dots$

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{32}$ (d) 8

(5) $\lim_{x \to -1} \frac{x^5 + 1}{x + 1} = \dots\dots\dots\dots$

(a) 5 (b) 4 (c) -5 (d) -4

(6) 📖 $\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8} = \dots\dots\dots\dots$

(a) 4 (b) $\frac{5}{3}$ (c) zero (d) $6\frac{2}{3}$

(7) $\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{32x^5 - 1} = \dots\dots\dots\dots$

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{2}{9}$ (d) $\frac{1}{8}$

(8) $\lim_{x \to -2} \frac{x^7 + 128}{x^4 - 16} =$ ....

(a) 9 (b) – 9 (c) – 14 (d) 14

(9) $\lim_{x \to 0} \frac{(x+1)^9 - 1}{x} =$ ....

(a) 9 (b) 1 (c) zero (d) 10

(10) $\lim_{h \to 0} \frac{(x+h)^7 - x^7}{h} =$ ....

(a) x^7 (b) $7x^6$ (c) zero. (d) 1

(11) $\lim_{x \to 0} \frac{(x+2)^5 - 32}{x} =$ ....

(a) 25 (b) 64 (c) 80 (d) 100

(12) $\lim_{x \to 0} \frac{\sqrt[3]{x+1} - 1}{x} =$ ....

(a) 1 (b) $\frac{1}{3}$ (c) zero (d) $\frac{-2}{3}$

(13) $\lim_{x \to 1} \frac{x^6 - 64}{x - 2} =$ ....

(a) $6 (2)^5$ (b) 128 (c) $64 (2)^5$ (d) 63

(14) $\lim_{x \to 1} \frac{x^{\frac{13}{2}} - x^{\frac{1}{2}}}{x^{\frac{7}{2}} - x^{\frac{1}{2}}} =$ ....

(a) $\frac{13}{7}$ (b) 1 (c) 2 (d) x

(15) $\lim_{h \to 0} \frac{(2 - 3h)^7 - 128}{4h} =$ ....

(a) 336 (b) – 336 (c) 448 (d) – 448

(16) $\lim_{h \to 0} \frac{(3h - 1)^5 + 1}{5h} =$ ....

(a) – 3 (b) $\frac{3}{5}$ (c) 3 (d) 5

(17) $\lim_{x \to 5} \frac{(x-3)^7 - 128}{x - 5} =$ ....

(a) 7 (b) 28 (c) 64 (d) 448

(18) $\lim_{x \to 2} \frac{(x+1)^4 - 81}{x - 2} =$ ....

(a) 18 (b) 81 (c) – 108 (d) 108

(19) $\lim_{x \to 1} \frac{1 - \sqrt[n]{x}}{1 - \sqrt[m]{x}} =$ ....

(a) 1 (b) $\frac{n}{m}$ (c) – 1 (d) $\frac{m}{n}$

(20) $\lim\limits_{x \to 2} \dfrac{x^5 - 32}{x^2 + 3x - 10} = \dots\dots$

(a) 80 (b) $\frac{80}{7}$ (c) $\frac{7}{80}$ (d) $\frac{1}{80}$

(21) $\lim\limits_{x \to 1} \dfrac{\sqrt[5]{x} + 2\sqrt{x} - 3}{x - 1} = \dots\dots$

(a) $\frac{6}{5}$ (b) 2 (c) 5 (d) 3

(22) If $f(x) = x^5$, $g(x) = x^2 - 4$, then $\lim\limits_{x \to 2} \dfrac{f(x) - 32}{g(x)} = \dots\dots$

(a) -20 (b) 20 (c) ± 20 (d) 32

(23) If $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x^m - a^m} = \dfrac{n}{m}$, then a = ……

(a) 1 (b) n (c) m (d) $\frac{m}{n}$

(24) If $\lim\limits_{x \to 2} \dfrac{(x)^n - (2)^n}{x - 2} = 32$, then n = ……

(a) 3 (b) 4 (c) 9 (d) 12

(25) If $\lim\limits_{x \to k} \dfrac{x^5 - k^5}{x - k} = 80$, then k = ……

(a) 2 (b) -2 (c) ± 2 (d) 16

(26) If $\lim\limits_{x \to a} \dfrac{x^8 - a^8}{x^6 - a^6} = 48$, then a = ……

(a) 4 (b) 6 (c) ± 4 (d) ± 6

Second Essay questions

1 Find each of the following :

(1) $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x - 2}$ « 12 »

(2) $\lim\limits_{x \to -5} \dfrac{x^4 - 625}{x + 5}$ « – 500 »

(3) $\lim\limits_{x \to a} \dfrac{x^5 - a^5}{x - a}$ « $5a^4$ »

(4) $\lim\limits_{x \to 2} \dfrac{x^5 - 32}{x^2 - 4}$ « 20 »

(5) $\lim\limits_{x \to 2} \dfrac{x^7 - 128}{x^3 - 8}$ « $\frac{112}{3}$ »

(6) $\lim\limits_{x \to \frac{1}{2}} \dfrac{x^3 - \frac{1}{8}}{x^2 - \frac{1}{4}}$ « $\frac{3}{4}$ »

(7) $\lim\limits_{x \to 2} \dfrac{x^6 - 64}{x^7 - 128}$ « $\frac{3}{7}$ »

(8) $\lim\limits_{x \to -3} \dfrac{x^5 + 243}{x + 3}$ « 405 »

(9) $\lim\limits_{x \to -3} \dfrac{x^4 - 81}{x + 3}$ « – 108 »

(10) $\lim\limits_{x \to -3} \dfrac{x^4 - 81}{x^5 + 243}$ « $-\frac{4}{15}$ »

(11) $\lim\limits_{x \to -2} \dfrac{x^5 + 32}{x^3 + 8}$ « $\frac{20}{3}$ »

(12) $\lim\limits_{x \to 4} \dfrac{2x^3 - 128}{x^2 - 16}$ « 12 »

(13) $\lim\limits_{x \to -2} \frac{x^6 - 64}{3x + 6}$ « − 64 »

(14) $\lim\limits_{x \to -1} \frac{x^{10} + x}{x^7 - x}$ « $\frac{-3}{2}$ »

(15) $\lim\limits_{x \to 1} \frac{1 - x^9}{x^7 - 1}$ « $\frac{-9}{7}$ »

(16) $\lim\limits_{2x \to 1} \frac{128x^7 - 1}{32x^5 - 1}$ « $\frac{7}{5}$ »

(17) $\lim\limits_{x \to -\frac{1}{2}} \frac{32x^5 + 1}{64x^6 - 1}$ « $\frac{-5}{6}$ »

(18) $\lim\limits_{x \to \frac{-2}{3}} \frac{243x^5 + 32}{27x^3 + 8}$ « $\frac{20}{3}$ »

2 Find each of the following :

(1) $\lim\limits_{x \to 2} \frac{x^{-7} - (2)^{-7}}{x - 2}$ « $\frac{-7}{256}$ »

(2) $\lim\limits_{x \to -1} \frac{x^{-4} - 1}{x^{-18} - 1}$ « $\frac{2}{9}$ »

(3) $\lim\limits_{x \to 2} \frac{x^{-5} - \frac{1}{32}}{x^{-7} - \frac{1}{128}}$ « $\frac{20}{7}$ »

(4) $\lim\limits_{x \to 4} \frac{\frac{1}{x^2} - \frac{1}{16}}{x - 4}$ « $\frac{-1}{32}$ »

(5) $\lim\limits_{x \to 2} \frac{x^{-8} - (16)^{-2}}{x - 2}$ « $-\frac{1}{64}$ »

(6) $\lim\limits_{x \to 1} \frac{\sqrt[7]{x} - 1}{x - 1}$ « $\frac{1}{7}$ »

(7) $\lim\limits_{x \to 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{x - 2}$ « $\frac{1}{3\sqrt[3]{4}}$ »

(8) $\lim\limits_{x \to 1} \frac{x^{\frac{21}{2}} - x^{\frac{1}{2}}}{x^{\frac{14}{3}} - x^{\frac{2}{3}}}$ « $\frac{5}{2}$ »

(9) $\lim\limits_{x \to 1} \frac{x^{17} - 1}{3x^2 + 2x - 5}$ « $\frac{17}{8}$ »

3 Find each of the following :

(1) $\lim\limits_{x \to 0} \frac{(1 + x)^{10} - 1}{(1 + x)^7 - 1}$ « $\frac{10}{7}$ »

(2) $\lim\limits_{x \to 6} \frac{(x - 5)^7 - 1}{x - 6}$ « 7 »

(3) $\lim\limits_{x \to -2} \frac{(x + 3)^5 - 1}{x + 2}$ « 5 »

(4) $\lim\limits_{x \to 0} \frac{(x + 2)^5 - 32}{x}$ « 80 »

(5) 📖 $\lim\limits_{h \to 0} \frac{(3 + h)^4 - 81}{6h}$ « 18 »

(6) $\lim\limits_{h \to 0} \frac{(1 + 4h)^8 - 1}{h}$ « 32 »

(7) 📖 $\lim\limits_{x \to 1} \frac{(x + 2)^4 - 81}{x - 1}$ « 108 »

(8) 📖 $\lim\limits_{x \to 1} \frac{(x - 2)^4 - 1}{x - 1}$ « − 4 »

(9) $\lim\limits_{x \to 0} \frac{(1 - 2x)^5 - 1}{5x}$ « − 2 »

(10) $\lim\limits_{h \to 0} \frac{(x + 3h)^5 - x^5}{h}$ « $15x^4$ »

(11) $\lim\limits_{x \to 0} \frac{\sqrt[3]{1 + 3x} - 1}{2x}$ « $\frac{1}{2}$ »

(12) 📖 $\lim\limits_{x \to 2} \frac{(x - 4)^5 + 32}{x - 2}$ « 80 »

(13) $\lim_{x \to 5} \frac{\sqrt[3]{x+3} - 2}{x - 5}$ « $\frac{1}{12}$ »

(14) $\lim_{x+1 \to 0} \frac{(3x+2)^9 + 1}{x + 1}$ « 27 »

(15) $\lim_{x \to 1} \frac{x^{19} + x^8 - 2}{x - 1}$ « 27 »

(16) $\lim_{x \to -1} \frac{x^7 + x^9 + 2}{x + 1}$ « 16 »

4 **Find each of the following :**

(1) $\lim_{x \to 2} \left(\frac{x^3 - 8}{x^5 - 32} + \frac{x^4 - 16}{x^7 - 128} \right)$ « $\frac{31}{140}$ »

(2) $\lim_{x \to -1} \left(\frac{x^2 - 2x - 3}{x + 1} + \frac{x^3 + 1}{x^5 + 1} \right)$ « $-\frac{17}{5}$ »

(3) $\lim_{x \to 3} \left(\frac{x^5 - 243}{x^2 - 4} \times \frac{x - 2}{x - 3} \right)$ « 81 »

(4) $\lim_{x \to -3} \left(\frac{x^4 - 81}{x^3 + 27} \right)^3$ « − 64 »

5 **Find the value of a if :** $\lim_{x \to a} \frac{x^{12} - a^{12}}{x^{10} - a^{10}} = 30$ « ± 5 »

6 **Find the value of k if :** $\lim_{x \to -1} \frac{x^{15} + 1}{x + 1} = \lim_{x \to k} \frac{x^5 - k^5}{x^3 - k^3}$ « ± 3 »

7 If $\lim_{x \to 2} \frac{x^n - 64}{x - 2} = \ell$ **, find the value of each of :** n and ℓ « 6 , 192 »



Exercise 14 The limit of the function at infinity

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) $\lim\limits_{x \to \infty} \left(\frac{3}{x^2} - 2\right) =$ ....

(a) 3 (b) 2 (c) – 3 (d) – 2

(2) $\lim\limits_{x \to \infty} \frac{3x}{4x+5} =$ ....

(a) ∞ (b) $\frac{3}{4}$ (c) $\frac{1}{5}$ (d) zero.

(3) $\lim\limits_{x \to \infty} \frac{2x^2+1}{x^2+1} =$ ....

(a) zero. (b) does not exist. (c) ∞ (d) 2

(4) $\lim\limits_{x \to \infty} \frac{x^2+5}{6} =$ ....

(a) zero. (b) $\frac{5}{6}$ (c) 1 (d) ∞

(5) $\lim\limits_{x \to \infty} \frac{\sqrt{x}-3}{x} =$ ....

(a) zero. (b) 1 (c) – 2 (d) ∞

(6) $\lim\limits_{x \to \infty} (3x^{-5} + 4x^{-2} + 5) =$ ....

(a) 12 (b) ∞ (c) 5 (d) zero.

(7) $\lim\limits_{x \to \infty} \frac{3x^{-3} + 4x^{-2} - 2}{7x^{-3} - x^{-2} + 6} =$ ....

(a) ∞ (b) zero. (c) $\frac{3}{7}$ (d) $\frac{-1}{3}$

(8) $\lim_{x \to \infty} \frac{x^7 - 2x^3}{2x^4 - 3x^2 - 1} = \ldots\ldots\ldots\ldots$

(a) zero. (b) 3 (c) ∞ (d) $\frac{1}{2}$

(9) $\lim_{x \to \infty} \frac{3x^2}{x(2x - 1)} = \ldots\ldots\ldots\ldots$

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) zero. (d) 3

(10) $\lim_{x \to \infty} \sqrt{\frac{1 - x}{1 - 4x}} = \ldots\ldots\ldots\ldots$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

(11) $\lim_{x \to \infty} \frac{x^3 + 5}{x(2x^2 + 3)} = \ldots\ldots\ldots\ldots$

(a) $\frac{5}{8}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{5}{3}$

(12) $\lim_{x \to \infty} \frac{5 + x^{-2}}{3x^{-2} + 1} = \ldots\ldots\ldots\ldots$

(a) $\frac{5}{4}$ (b) $\frac{5}{3}$ (c) 5 (d) $\frac{1}{3}$

(13) $\lim_{x \to \infty} \frac{2x}{\sqrt{9x^2 + 1}} = \ldots\ldots\ldots\ldots$

(a) $\frac{2}{9}$ (b) zero (c) $\frac{2}{3}$ (d) ∞

(14) $\lim_{x \to \infty} \frac{1}{x}\sqrt{8 + 9x^2} = \ldots\ldots\ldots\ldots$

(a) $2\sqrt{2}$ (b) 3 (c) $-2\sqrt{2}$ (d) -3

(15) $\lim_{x \to \infty} \frac{\sqrt[3]{64x^3 + 7x - 2}}{3x + 2} = \ldots\ldots\ldots\ldots$

(a) 4 (b) 3 (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

(16) 📖 $\lim_{x \to \infty} \frac{\sqrt{x^2}}{x} = \ldots\ldots\ldots\ldots$

(a) zero. (b) 1 (c) 2 (d) -1

(17) $\lim_{x \to \infty} \frac{\sqrt[3]{8x^3 + 1}}{|x|} = \ldots\ldots\ldots\ldots$

(a) 2 (b) 8 (c) zero (d) 4

(18) $\lim_{x \to \infty} \frac{\sqrt{9x^2 + 3}}{\sqrt[3]{1 - 8x^3}} = \ldots\ldots\ldots\ldots$

(a) $\frac{-2}{3}$ (b) $\frac{-9}{8}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

(19) $\lim\limits_{x \to \infty} \dfrac{4 - 3x^3}{\sqrt{x^6 + 9}} = \ldots\ldots$

(a) -3 (b) 1 (c) zero (d) ∞

(20) $\lim\limits_{x \to 4} \left(\dfrac{3x^2 + 2x + 1}{x^2 - 3x + 2}\right)^4 = \ldots\ldots$

(a) 3 (b) 9 (c) 27 (d) 81

(21) $\lim\limits_{x \to \infty} \dfrac{(12)^{\frac{1}{x}}}{x + 7} = \ldots\ldots$

(a) 1 (b) zero. (c) $\frac{12}{7}$ (d) ∞

(22) $\lim\limits_{x \to \infty} \dfrac{k^{\frac{1}{x}}}{3} = \ldots\ldots$ where k is a positive constant.

(a) $\frac{k}{3}$ (b) $\frac{1}{3}$ (c) 3 (d) 3 k

(23) If $\lim\limits_{x \to \infty} \dfrac{a x + 6}{2x - 7} = 4$, then a = …… where $a \in \mathbb{R}$

(a) 2 (b) 4 (c) 6 (d) 8

(24) If $\lim\limits_{x \to \infty} \dfrac{a^2 x + 7}{2x - 5} = 8$, then a = …… where $a \in \mathbb{R}$

(a) 2 (b) zero (c) ± 4 (d) ± 8

(25) If $\lim\limits_{x \to \infty} \dfrac{a x^2 - 5x}{2x + 3x^2} = 3$, then a = ……

(a) 3 (b) 6 (c) 9 (d) 12

(26) If $m \in \mathbb{R}$ and $\lim\limits_{x \to \infty} \dfrac{(m + 2)x^3 - x + 4}{2m x^3 - 2x + 5} = 2$, then m = ……

(a) zero. (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

(27) $\lim\limits_{x \to \infty} (x^3 + 7x^2 + 8) = \ldots\ldots$

(a) ∞ (b) zero. (c) $-\infty$ (d) 1

(28) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + x} - x\right) = \ldots\ldots$

(a) $\frac{1}{2}$ (b) zero. (c) $\sqrt{2}$ (d) does not exist.

(29) If : a > b , then $\lim\limits_{x \to \infty} \dfrac{x^a}{x^b} = \ldots\ldots$

(a) zero. (b) ∞ (c) 1 (d) a – b

(30) If : a < b < zero , then $\lim\limits_{x \to \infty} \dfrac{x^a}{x^b} = \ldots\ldots$

(a) ∞ (b) $-\infty$ (c) zero. (d) a – b

Second Essay questions

1 Find each of the following :

(1) $\lim\limits_{x \to \infty} \frac{2x-5}{3x+8}$ « $\frac{2}{3}$ »

(2) $\lim\limits_{x \to \infty} \frac{2x-5}{3x^2+8}$ « zero »

(3) $\lim\limits_{x \to \infty} \frac{2x^2-5}{3x+8}$ « ∞ »

2 Find each of the following :

(1) $\lim\limits_{x \to \infty} \frac{5x-4}{3x-2}$ « $\frac{5}{3}$ »

(2) $\lim\limits_{x \to \infty} \frac{4-2x}{3x-1}$ « $\frac{-2}{3}$ »

(3) 📖 $\lim\limits_{x \to \infty} \frac{2x^2+5x+1}{3x^2-7}$ « $\frac{2}{3}$ »

(4) 📖 $\lim\limits_{x \to \infty} \frac{5-6x-3x^2}{2x^2+x+4}$ « $-\frac{3}{2}$ »

(5) 📖 $\lim\limits_{x \to \infty} \frac{x^3-2}{|x|^3+1}$ « 1 »

(6) $\lim\limits_{x \to \infty} \frac{6x-4x^3}{2-7x^3}$ « $\frac{4}{7}$ »

(7) $\lim\limits_{x \to \infty} \frac{2x^4+2x^2-1}{5-x^2-2x^4}$ « -1 »

(8) $\lim\limits_{x \to \infty} \frac{4x^5+5}{8x^5+x^4-2}$ « $\frac{1}{2}$ »

(9) $\lim\limits_{x \to \infty} \frac{7x^2+1}{4x^3-8x+1}$ « zero »

(10) $\lim\limits_{x \to \infty} \frac{2x^5+3x-2}{3x^4+5x-1}$ « ∞ »

(11) $\lim\limits_{x \to \infty} \frac{5x^7+2x-1}{6x^4+13}$ « ∞ »

(12) $\lim\limits_{x \to \infty} \frac{5-7x^8+3x^{14}}{7-6x^{14}+2x^6}$ « $-\frac{1}{2}$ »

(13) $\lim\limits_{x \to \infty} \left(\frac{7}{x^2}+\frac{2}{x}-3\right)$ « -3 »

(14) $\lim\limits_{x \to \infty} \frac{5x^{-3}+4x^{-2}-3}{7x^{-3}-2x^{-2}+8}$ « $\frac{-3}{8}$ »

(15) $\lim\limits_{x \to \infty} \frac{5x^3-4x^2+2}{7-x+|2x|^3}$ « $\frac{5}{8}$ »

(16) 📖 $\lim\limits_{x \to \infty} (x^3+5x^2+1)$ « ∞ »

(17) $\lim\limits_{x \to \infty} (x^2-x+5)$ « ∞ »

3 Find each of the following :

(1) $\lim\limits_{x \to \infty} \frac{(3x+9)(2x+7)}{(x+1)(5x+4)}$ « $\frac{6}{5}$ »

(2) $\lim\limits_{x \to \infty} \frac{3x^2-4x+5}{(x+2)^2}$ « 3 »

(3) $\lim\limits_{x \to \infty} \frac{(2x+3)^2}{5-3x-x^2}$ « -4 »

(4) $\lim\limits_{x \to \infty} \frac{6x^2-5x}{(3-x)(2+x)}$ « -6 »

(5) $\lim\limits_{x \to \infty} \frac{(x+1)(5x-3)}{x^2+3}$ «5»

(6) $\lim\limits_{x \to \infty} \frac{8x^3-x+1}{(x+1)(2x^2-3)}$ «4»

(7) $\lim\limits_{x \to \infty} \frac{x^3-4x+5}{(2x-1)^3}$ «$\frac{1}{8}$»

(8) $\lim\limits_{x \to \infty} \frac{(2x+3)(4x^2-5)}{(3x^2-8)(5x-3)}$ «$\frac{8}{15}$»

(9) $\lim\limits_{x \to \infty} \frac{(2x+3)(5x-1)(x-2)}{x(x+1)(3x-1)}$ «$\frac{10}{3}$»

(10) $\lim\limits_{x \to \infty} \frac{\left(7+\sqrt{x}\right)\left(3+\sqrt{x}\right)}{4x-3}$ «$\frac{1}{4}$»

4 Find each of the following :

(1) $\lim\limits_{x \to \infty} \frac{x+2}{\sqrt{9x^2+25}}$ «$\frac{1}{3}$»

(2) $\lim\limits_{x \to \infty} \frac{\sqrt{x^2+81}}{2x-1}$ «$\frac{1}{2}$»

(3) $\lim\limits_{x \to \infty} \frac{\sqrt{4x^2+1}}{x-2}$ «2»

(4) $\lim\limits_{x \to \infty} \frac{4-3x^2}{\sqrt{x^4+5}}$ «−3»

(5) $\lim\limits_{x \to \infty} \frac{1}{x}\sqrt{3+4x^2}$ «2»

(6) $\lim\limits_{x \to \infty} \frac{2x+1}{\sqrt{4x^2+3x-4}}$ «1»

(7) $\lim\limits_{x \to \infty} \frac{2x-3}{\sqrt[3]{125x^3+5}}$ «$\frac{2}{5}$»

(8) $\lim\limits_{x \to \infty} \frac{\sqrt[3]{8x^3+5x-2}}{3x+2}$ «$\frac{2}{3}$»

(9) $\lim\limits_{x \to \infty} \frac{4-3x^3}{\sqrt{x^6+9}}$ «−3»

(10) $\lim\limits_{x \to \infty} \frac{\sqrt{9x^2-3x+8}}{\sqrt[3]{3x^2+125x^3+2}}$ «$\frac{3}{5}$»

(11) $\lim\limits_{x \to \infty} \frac{\sqrt{x^2+1}}{\sqrt[4]{x^4+2}}$ «1»

(12) $\lim\limits_{x \to \infty} \frac{\sqrt{4x2+7}+3x}{2x+9}$ «$\frac{5}{2}$»

5 Find each of the following :

(1) $\lim\limits_{x \to \infty} \left(\frac{2}{x}+\frac{x^2-x}{x^2-1}\right)$ «1»

(2) $\lim\limits_{x \to \infty} \left(7+\frac{2x^2}{(x+3)^2}\right)$ «9»

(3) $\lim\limits_{x \to \infty} \left(\frac{x}{2x+1}+\frac{3x^2}{(x-3)^2}\right)$ «$\frac{7}{2}$»

(4) $\lim\limits_{x \to \infty} \left(\frac{2}{3}-\frac{3x}{2x+7}\right)$ «$-\frac{5}{6}$»

(5) $\lim\limits_{x \to \infty} \left(\frac{3}{x}+\frac{2x^5+1}{x^2(x^3+2)}\right)$ «2»

(6) $\lim\limits_{x \to \infty} \left(\frac{2x^3}{2x^2+1}-x\right)$ «zero»

(7) $\lim\limits_{x \to \infty} \left(\frac{x^2-1}{x+2}-\frac{x^2+1}{x-2}\right)$ «−4»

(8) $\lim\limits_{x \to \infty} \left(\sqrt{x^2-2}-\sqrt{x^2+x}\right)$ «$-\frac{1}{2}$»

(9) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + x - 1} - \sqrt{x^2 - x + 1}\right)$ «1»

(10) $\lim\limits_{x \to \infty} \frac{1}{x}\left(\sqrt{4x^2 + 1} - \sqrt{x^2 + 1}\right)$ «1»

(11) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 5x} - x\right)$ «$\frac{5}{2}$» | (12) $\lim\limits_{x \to \infty} x\left(\sqrt{4x^2 + 1} - 2x\right)$ «$\frac{1}{4}$»

---

6 **Find the value of each of a and n if :** $\lim\limits_{x \to \infty} \frac{4a x^n - 4x + 5}{3 - 9x + 8x^2} = 3$ «6 , 2»

---

7 **Find the value of a if :** $\lim\limits_{x \to \infty} \frac{\sqrt[3]{a x^3 + 3}}{\sqrt{4x^2 + 7}} = -1$ «−8»

4 Trigonometry

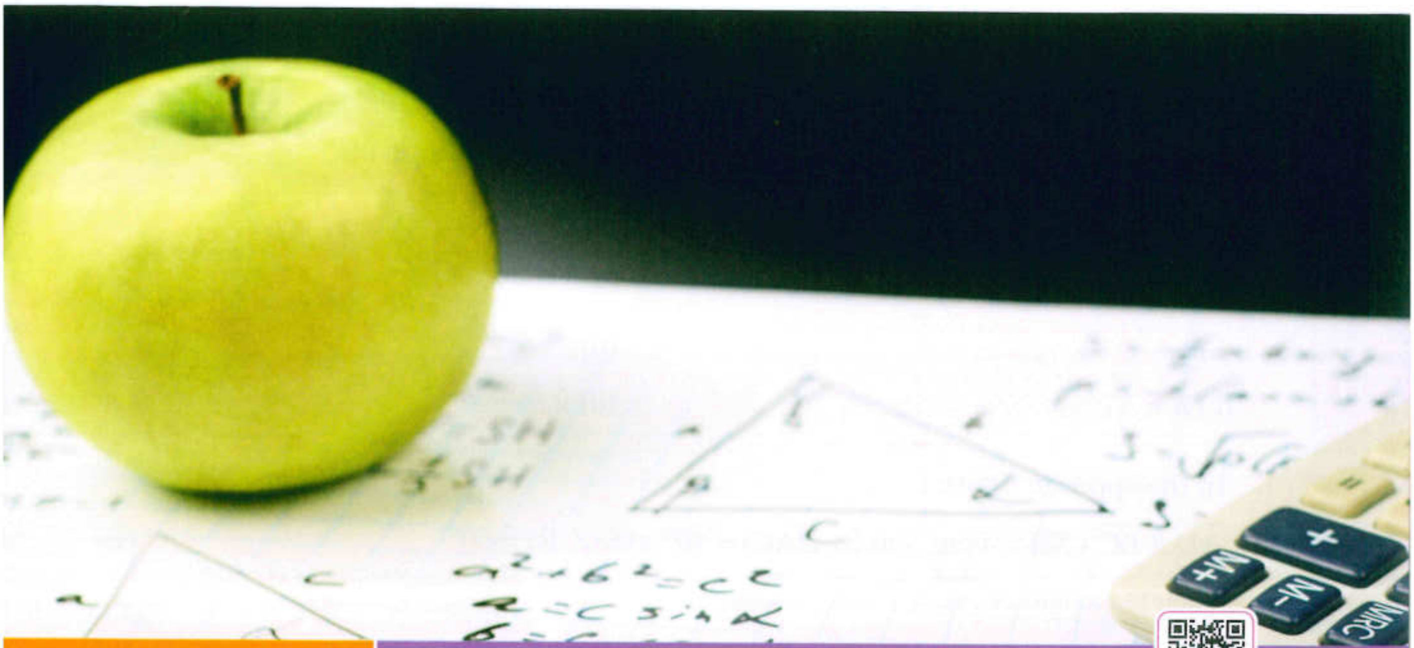
Unit Exercises

Exercise **15** The sine rule.

Exercise **16** The cosine rule.

Exercise **17** Solution of the triangle.

- At the end of the unit :
  Life applications on unit four.



Exercise 15 The sine rule

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) In any triangle XYZ , XY : YZ = ...

(a) sin X : sin Y (b) sin Y : sin Z (c) sin Z : sin X (d) sin Z : sin Y

(2) In Δ ABC , if m ($\angle$ A) = 30° , C = $15\sqrt{3}$ cm. , m ($\angle$ C) = 60° , then a = ... cm.

(a) 30 (b) 45 (c) 15 (d) 60

(3) DEF is a triangle in which m ($\angle$ D) = 80° and m ($\angle$ E) = 60° , if f = 12 cm.,
then d = ... cm.

(a) $\frac{12 \sin 80°}{\sin 40°}$ (b) $\frac{12 \sin 80°}{\sin 60°}$ (c) $\frac{12 \sin 40°}{\sin 80°}$ (d) $\frac{12 \cos 80°}{\cos 40°}$

(4) In Δ ABC , if a = 4 cm. , b = 7 cm. , m ($\angle$ C) = 120° , then the area of the triangle = ... cm^2.

(a) $7\sqrt{3}$ (b) $14\sqrt{3}$ (c) 7 (d) 14

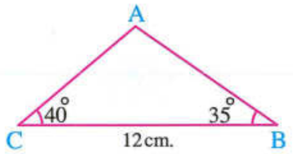
(5) XYZ is an equilateral triangle , the length of its side is $10\sqrt{3}$ cm. , then the length of the diameter of its circumcircle is .... cm.

(a) 5 (b) 10 (c) 15 (d) 20

(6) In Δ XYZ , $\frac{x}{\sin X} = 6$, then the length of the diameter of its circumcircle is .... length units.

(a) 6 (b) 12 (c) 3 (d) 9

(7) In the opposite figure :

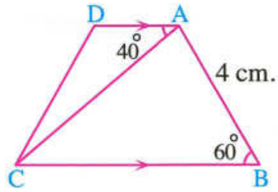


The length of $\overline{AB} \approx$ .... cm.

(a) 6 (b) 7

(c) 8 (d) 9

(8) In the opposite figure :



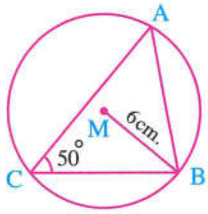
$\overline{AD} \parallel \overline{BC}$, AB = 4 cm. , m (∠ DAC) = 40° , m (∠ B) = 60°

, then the length of $\overline{AC} \simeq$ .... cm.

(a) 5 (b) 3

(c) 2 (d) 4

(9) In the opposite figure :



M is the centre of the circle

, BM = 6 cm. , then AB = ... cm.

(a) 6 sin 50° (b) 12 sin 50°

(c) 6 cos 50° (d) 12 cos 50°

(10) A circle with diameter of length 20 cm. , passes through the vertices of Δ ABC which is an acute-angled triangle in which BC = 10 cm. , then m (∠ A) = ...°

(a) 30 (b) 60 (c) 45 (d) 150

(11) In triangle ABC , m (∠ A) = 45° , the length of the radius of its circumcircle = 6 cm. , then a = ... cm.

(a) 13 (b) $6\sqrt{2}$ (c) 12 (d) $\sqrt{2}$

(12) If the length of a side in any triangle = 12 cm. and the measure of the opposite angle to this side = 55° , then the circumference of the circle that passes through the vertices of this triangle ≃ .... cm.

(a) 36 (b) 42 (c) 46 (d) 52

(13) If the perimeter of triangle ABC equals 15 cm. , m (∠ A) = 53° , m (∠ B) = 47° , then the length of $\overline{AB} \simeq$ .... cm.

(a) 6 (b) 7 (c) 5 (d) 8

(14) In triangle ABC , a = 27 cm. , m (∠ B) = 82° , m (∠ C) = 56° , then its surface area ≃ .... cm².

(a) 540 (b) 447 (c) 350 (d) 400

(15) In triangle ABC , m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 4 , AB = 12 cm. , then the length of $\overline{AC} \simeq$ .... cm.

(a) 10 (b) 11 (c) 16 (d) 18

(16) In triangle ABC , which of the following statements is true ?

(a) sin A + cos B = a + b (b) a sin B = b sin A

(c) a = b sin c (d) $\frac{a}{\sin A} = \frac{\sin B}{b}$

(17) In Δ XYZ , 2 r sin X = ... "where r is the radius length of its circumcircle"

(a) z (b) y (c) x (d) area of Δ XYZ

(18) 📖 If r is the length of the radius of the circumcircle of the triangle XYZ , then $\frac{y}{2 \sin Y}$ = ...

(a) r (b) 2 r (c) $\frac{1}{2}$ r (d) 4 r

(19) 📖 In any triangle LMN , $\frac{\ell}{\sin L}$ = ...

(a) $\frac{m}{\sin N}$ (b) $\frac{n}{\sin M}$ (c) $\frac{m + n}{\sin N + \sin M}$ (d) 3 r

(20) 📖 In Δ ABC , if $\frac{\sin A}{a} = \frac{\sin B}{b}$, then $\frac{2 \sin A - \sin B}{\cdots\cdots} = \frac{\sin A}{a}$

(a) a + b (b) 2 a + b (c) a − 2 b (d) 2 a − b

(21) In acute-angled triangle ABC , $2a = \frac{b}{\sin B}$, then m (∠ A) = ...

(a) 30° (b) 45° (c) 60° (d) 75°

(22) In Δ ABC , sin A = 2 sin C , BC = 6 cm. , then AB = ... cm.

(a) 2 (b) 3 (c) 4 (d) 6

(23) If the radius length of circumcircle of Δ ABC equals 3 cm. and sin A + sin B + sin C = 2, then the perimeter of triangle ABC = ... cm.

(a) 6 (b) 9 (c) 12 (d) 24

(24) 📖 In any triangle ABC , $\frac{\sin (A + B)}{\sin A + \sin B}$ = ...

(a) 1 (b) $\frac{c}{a + b}$ (c) $\frac{a}{b + c}$ (d) $\frac{b}{a + c}$

(25) In Δ ABC , $\frac{a}{a + b} = \frac{\sin A}{\cdots\cdots}$

(a) sin B (b) sin C (c) sin A + sin B (d) sin A + sin C

(26) In Δ XYZ , if 3 sin X = 4 sin Y = 2 sin Z , then $x : y : z =$ ....

(a) 2 : 3 : 4 (b) 6 : 4 : 3 (c) 3 : 4 : 6 (d) 4 : 3 : 6

(27) ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then a : b : c = ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

(28) In Δ ABC : If $\frac{\sin A}{4} = \frac{\sin B}{9} = \frac{\sin C}{7}$, then the greatest angle in measure is ....

(a) $\angle$ A (b) $\angle$ B (c) $\angle$ C (d) right

(29) In triangle ABC , m ($\angle$ A) : m ($\angle$ B) : m ($\angle$ C) = 3 : 5 : 4

, then $c^2 : a^2 =$ ....

(a) $\sqrt{6} : 2$ (b) 2 : 3 (c) 4 : 3 (d) 3 : 2

(30) In Δ ABC , $\frac{a}{b} \times \frac{\sin B}{\sin A} =$ ....

(a) $\frac{c}{\sin C}$ (b) $\frac{\sin C}{c}$ (c) 4 r (d) 1

(31) In Δ ABC , if the radius of its circumcircle = 4 cm.

, then $\frac{a + b + c}{\sin A + \sin B + \sin C} =$ ....

(a) 4 (b) 2 (c) 8 (d) 16

(32) If the radius of the circumcircle of Δ ABC equals r , then the perimeter of the triangle = ... (sin A + sin B + sin C)

(a) r (b) 2 r (c) $4 r^2$ (d) $8 r^3$

(33) In Δ ABC , a – b = 4 cm. , sin A = $\frac{3}{2}$ sin B , then a = ... cm.

(a) 4 (b) 6 (c) 8 (d) 12

(34) If the perimeter of Δ ABC is 24 cm. and sin A + sin B = 3 sin C , then C = ... cm.

(a) 4 (b) 6 (c) 8 (d) 9

(35) ABC is a triangle , sin B + sin C = 4 sin A and b + c = 2 a + 10 cm.

, then a = ... cm.

(a) 2 (b) 3 (c) 4 (d) 5

(36) In Δ ABC , AB = 8 cm. , BC = 12 cm. , m ($\angle$ A) – m ($\angle$ C) = 90°

, then tan C = ...

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(37) If r is the radius length of the circumcircle of Δ ABC and a = r , then m ($\angle$ A) = ...

(a) 30° only. (b) 30° or 120° (c) 150° only. (d) 30° or 150°

(38) If the area of the triangle ABC is Δ and r is the radius length of the circumcircle of the triangle ABC , then : $\frac{4 r \Delta}{abc}$ = ...

(a) 1 (b) 2 (c) 4 (d) 8

(39) In Δ ABC , $\frac{2 b}{\sin B}$ = ... r (where r is the radius of its circumcircle)

(a) 1 (b) 2 (c) 4 (d) 8

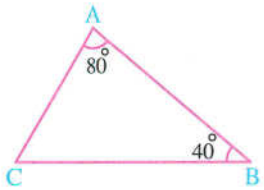
(40) If the triangle ABC is an isosceles right-angled triangle and r is the radius length of the circumcircle of the triangle ABC , then the area of Δ ABC = ... (in terms of r)

(a) $\frac{1}{2} r^2$ (b) $2 r^2$ (c) r^2 (d) $4 r^2$

(41) **In the opposite figure :**

If the perimeter of Δ ABC = 20 cm. ,

then the diameter length of its circumcircle ≈ .... cm.



(a) 2 (b) 4

(c) 6 (d) 8

Second Essay questions

1 XYZ is a triangle in which : m ($\angle$ X) = 80° , m ($\angle$ Y) = 60° and z = 10 cm. , find each of x and y to the nearest cm. « 15 cm. , 13 cm. »

2 ABC is a triangle in which : c = 19 cm. , m ($\angle$ A) = 112° and m ($\angle$ B) = 33°
Find to the nearest hundredth each of b and the length of the radius of the circumcircle of the triangle. « 18.04 cm. , 16.56 cm. »

3 XYZ is a triangle , if y = 68.4 cm. , m ($\angle$ Y) = 100° and m ($\angle$ Z) = 40°
, find : (1) x

(2) The radius length of the circumcircle of the triangle XYZ

(3) The area of the triangle XYZ « 44.64 cm. , 34.73 cm. , 981.34 cm^2 »

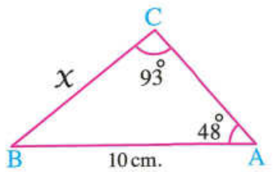
4 ABC is a triangle in which : b = 10 cm. , m ($\angle$ A) = 40° and m ($\angle$ C) = 80°
Find the length of the greatest side of Δ ABC « 11 cm. »

5 ABC is a triangle in which : c = 4.5 cm. , m (∠ A) = 100° and m (∠ B) = 15°

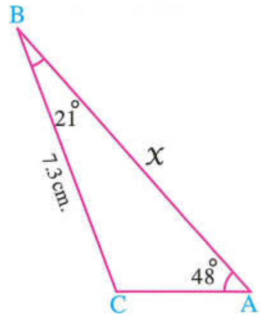
Find the length of the smallest side of Δ ABC « 1.3 cm. »

6 **Use the sine rule to find the value of x to the nearest tenth :**

(1)



(2)



« 7.4 cm. , 9.2 cm.»

7 ABC is a triangle in which : m (∠ A) = 60° and a = $7\sqrt{3}$ cm.

Find the area and the circumference of the circumcircle of Δ ABC $\left(\pi = \frac{22}{7}\right)$

« 154 cm^2 , 44 cm. »

8 ABC is a triangle in which : a = 13 cm. , m (∠ A) = 53° 8′ , c = 15 cm. Find the radius length of the circumcircle of Δ ABC , then find m (∠ C) « 8.1 cm. , 67° 23′ 9″ or 112° 36′ 51″ »

9 ABC is a triangle in which m (∠ A) = 35° , a = 8 cm. and b = 6 cm.

Find : m (∠ B) « 25° 28′ 45″ »

10 In the triangle ABC , m (∠ A) = 67° 22′ , m (∠ C) = 44° 33′ and b = 100 cm.

Find the perimeter of the triangle ABC and its surface area. « 275 cm. , 3473 cm^2 »

11 ABC is a triangle in which m (∠ B) = 35° , m (∠ C) = 70° and the radius length of the circumcircle of the triangle ABC = 16 cm. Find the area and the perimeter of Δ ABC to the nearest whole number. « 262 cm^2 , 79 cm. »

12 ABC is an isosceles triangle in which : m (∠ A) = 120° and the length of the radius of the circumcircle of Δ ABC is 12 cm.

Find c and calculate the area of Δ ABC « 12 cm. , 62.4 cm^2 »

13 ABC is an isosceles triangle in which : a = b and m (∠ A) = 15° and the perimeter of Δ ABC is 25 cm. Find the area of the circumcircle of Δ ABC « 474 cm^2 »

14 If the perimeter of Δ ABC = 40 cm. , m (∠ A) = 44° and m (∠ B) = 66° ,

Find the lengths of the sides of the triangle ABC « 10.9 cm. , 14.3 cm. , 14.8 cm. »

15 ABC is a triangle in which : c = 12 cm. and m (∠ B) = 3 m (∠ A) = 60°
Find a and the area of Δ ABC to the nearest cm^2 « 4.2 cm. , 22 cm^2 »

---

16 If the area of the triangle ABC is 450 cm^2 , m (∠ B) = 82° and m (∠ C) = 56°
, find the value of a « 27 cm. »

---

17 ABC is an acute-angled triangle in which AC = 12 cm. , sin A = 0.6 and its area is 43.2 cm^2
Find the length of each of $\overline{AB}$ and $\overline{BC}$, also find m (∠ B) « 12 cm. , 7.6 cm. , 71° 34' »

---

18 Find the perimeter of the acute-angled triangle ABC if a = 7 cm. , b = 8 cm.
and m (∠ A) = 60° « 20 cm. »

---

19 **Find the diameter length of the circumcircle of Δ ABC in the two following cases :**
(1) m (∠ A) = 75° , a = 21 cm.
(2) m (∠ B) = 50° , m (∠ C) = 65°, c – b = 6 cm. « 21.7 cm. , 42.8 cm. »

---

20 ABC is triangle in which : b = 5 cm. , tan C = $\frac{4}{3}$ and m (∠ B) = 30°
find a , c and the area of the triangle to the nearest integer. « 10 cm. , 8 cm. , 20 cm^2 »

---

21 XYZ is a triangle in which : sin X + sin Y + sin Z = 2.37 and its perimeter is 56.88 cm.
Find the length of the radius of the circumcircle of Δ XYZ « 12 cm. »

---

22 ABC is a triangle in which sin A : sin B : sin C = 2 : 4 : 5 and c – b = 3 cm.
Find each of a and b « 6 cm. , 12 cm. »

---

23 ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3 , if a = 5 cm.
, then find the perimeter of the triangle ABC « 15.9 cm. »

---

24 ABC is a triangle in which m (∠ A) = $\frac{2}{3}$ m (∠ B) = $\frac{1}{2}$ m (∠ C) , the length of the radius of its circumcircle = 10 cm. Find the area of Δ ABC « 110 cm^2 »

---

25 ABC is a triangle in which 6 sin A = 4 sin B = 3 sin C and its perimeter is 45 cm.
Find each of a and c « 10 cm. , 20 cm. »

---

26 $\overline{AB}$ and $\overline{AC}$ are two chords in a circle. If their lengths are 43.5 cm. and 52.1 cm. respectively and they are drawn in two different sides of the diameter $\overline{AD}$ whose length is 100 cm.
Find : (1) m (∠ BAC) **(2)** The length of $\overline{BC}$
« 122° 49' , 84 cm. »

27 ABCD is a parallelogram in which m (∠ A) = 50° , m (∠ DBC) = 70° and BD = 8 cm.

Find the perimeter of the parallelogram. « 38 cm. »

28 ABCD is a parallelogram in which AB = 18.6 cm. , m (∠ CAB) = 36° 22̀

and m (∠ DBA) = 44° 38̀

Find the length of the diagonal $\overline{AC}$ and the area of the parallelogram. « 26.46 cm. , 292 cm^2 »

29 ABCD is a parallelogram. M is the point of intersection of its two diagonals.

Let AC = 20 cm. , m (∠ AMD) = 130° and m (∠ CAB) = 85°

Find the length of $\overline{BD}$ and the area of the parallelogram ABCD « 28.2 cm. , 216 cm^2 »

30 ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, AD = 20 cm. , m (∠ D) = 120° ,

m (∠ B) = 62° and m (∠ ACB) = 23° 25̀

Find the length of each of $\overline{AC}$ and $\overline{BC}$ to the nearest cm. « 29 cm. , 33 cm. »

31 ABCD is a quadrilateral in which CD = 100 cm. , m (∠ BCA) = 36° ,

m (∠ BDA) = 55° , m (∠ BCD) = 85° and m (∠ CDA) = 87°

Find the lengths of $\overline{BD}$ and $\overline{AC}$ to the nearest centimetre. « 112 cm. , 144 cm. »

32 ABCD is a quadrilateral in which m (∠ ABC) = 90° , m (∠ BAD) = 80°

, AB = AD = 10 cm. , BD = BC. Calculate the area of the quadrilateral ABCD « 102 cm^2 »

33 ABCDE is a regular pentagon, whose side length is 18.26 cm.

Find the length of its diagonal $\overline{AC}$ « 29.5 cm. »

Third Higher skills

Choose the correct answer from the given ones :

(1) If the radius length of the circumcircle of the triangle ABC equals 3 cm.

, then : $\frac{abc}{\sin A \sin B \sin C}$ = ...

(a) 3 (b) 6 (c) 27 (d) 216

(2) If ABC is a triangle , then : a csc A + b csc B + c csc C = ...

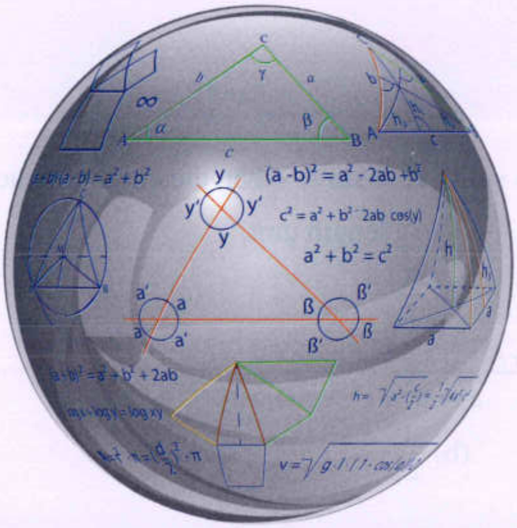
(a) 2 r (b) 4 r (c) 6 r (d) 8 r

(3) If $a = \sin B$, $b = \sin C$, $c = \sin A$, then the circumference of the circumcircle of ΔABC equals .... length unit.

(a) 1 (b) $\frac{\pi}{2}$ (c) π (d) 2π

(4) In ΔABC , $\frac{a \sin A + b \sin B + c \sin C}{a^2 + b^2 + c^2} =$ ....

(a) $\frac{1}{r^2}$ (b) $\frac{1}{2r}$ (c) $2r$ (d) r^2



Exercise 16 The cosine rule

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) In Δ XYZ , the expression $\frac{x^2 + y^2 - z^2}{2xy}$ equals ....

(a) cos X (b) cos Y (c) cos Z (d) sin Z

(2) 📖 In Δ XYZ , $y^2 + z^2 - x^2 = 2yz \times$ ....

(a) cos X (b) sin Z (c) cos Z (d) sin X

(3) In Δ ABC , cos (A + B) = ...

(a) cos C (b) – cos C (c) sin C (d) – sin C

(4) 📖 In any triangle ABC , cos A = ...

(a) – (cos B + cos C) (b) cos B – cos C

(c) cos (B + C) (d) – cos (B + C)

(5) 📖 If $\angle$ A is supplementary to $\angle$ C , then cos A + cos C = ...

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

(6) If ABCD is a cyclic quadrilateral , then cos A + cos C = ...

(a) 1 (b) zero. (c) $\frac{1}{2}$ (d) – 1

(7) In Δ XYZ , $2xy\cos(X + Y)$ = ...

(a) $x^2 + y^2 - z^2$ (b) $y^2 + z^2 - x^2$ (c) $x^2 - z^2 - y^2$ (d) $z^2 - x^2 - y^2$

(15) In triangle ABC , m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 4 , AB = 12 cm. , then the length of $\overline{AC} \simeq$ .... cm.

(a) 10 (b) 11 (c) 16 (d) 18

(16) In triangle ABC , which of the following statements is true ?

(a) sin A + cos B = a + b (b) a sin B = b sin A

(c) a = b sin c (d) $\frac{a}{\sin A} = \frac{\sin B}{b}$

(17) In Δ XYZ , 2 r sin X = ... "where r is the radius length of its circumcircle"

(a) z (b) y (c) x (d) area of Δ XYZ

(18) 📖 If r is the length of the radius of the circumcircle of the triangle XYZ , then $\frac{y}{2 \sin Y}$ = ...

(a) r (b) 2 r (c) $\frac{1}{2}$ r (d) 4 r

(19) 📖 In any triangle LMN , $\frac{\ell}{\sin L}$ = ...

(a) $\frac{m}{\sin N}$ (b) $\frac{n}{\sin M}$ (c) $\frac{m+n}{\sin N + \sin M}$ (d) 3 r

(20) 📖 In Δ ABC , if $\frac{\sin A}{a} = \frac{\sin B}{b}$, then $\frac{2 \sin A - \sin B}{\cdots\cdots} = \frac{\sin A}{a}$

(a) a + b (b) 2 a + b (c) a − 2 b (d) 2 a − b

(21) In acute-angled triangle ABC , $2a = \frac{b}{\sin B}$, then m (∠ A) = ...

(a) 30° (b) 45° (c) 60° (d) 75°

(22) In Δ ABC , sin A = 2 sin C , BC = 6 cm. , then AB = ... cm.

(a) 2 (b) 3 (c) 4 (d) 6

(23) If the radius length of circumcircle of Δ ABC equals 3 cm. and sin A + sin B + sin C = 2, then the perimeter of triangle ABC = ... cm.

(a) 6 (b) 9 (c) 12 (d) 24

(24) 📖 In any triangle ABC , $\frac{\sin (A+B)}{\sin A + \sin B}$ = ...

(a) 1 (b) $\frac{c}{a+b}$ (c) $\frac{a}{b+c}$ (d) $\frac{b}{a+c}$

(25) In Δ ABC , $\frac{a}{a+b} = \frac{\sin A}{\cdots\cdots}$

(a) sin B (b) sin C (c) sin A + sin B (d) sin A + sin C

(26) In Δ XYZ , if 3 sin X = 4 sin Y = 2 sin Z , then $x : y : z =$ ....

(a) 2 : 3 : 4 (b) 6 : 4 : 3 (c) 3 : 4 : 6 (d) 4 : 3 : 6

(27) ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then a : b : c = ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

(28) In Δ ABC : If $\frac{\sin A}{4} = \frac{\sin B}{9} = \frac{\sin C}{7}$, then the greatest angle in measure is ....

(a) $\angle$ A (b) $\angle$ B (c) $\angle$ C (d) right

(29) In triangle ABC , m ($\angle$ A) : m ($\angle$ B) : m ($\angle$ C) = 3 : 5 : 4

, then $c^2 : a^2 =$ ....

(a) $\sqrt{6} : 2$ (b) 2 : 3 (c) 4 : 3 (d) 3 : 2

(30) In Δ ABC , $\frac{a}{b} \times \frac{\sin B}{\sin A} =$ ....

(a) $\frac{c}{\sin C}$ (b) $\frac{\sin C}{c}$ (c) 4r (d) 1

(31) In Δ ABC , if the radius of its circumcircle = 4 cm.

, then $\frac{a + b + c}{\sin A + \sin B + \sin C} =$ ....

(a) 4 (b) 2 (c) 8 (d) 16

(32) If the radius of the circumcircle of Δ ABC equals r , then the perimeter of the triangle = ... (sin A + sin B + sin C)

(a) r (b) 2 r (c) $4 r^2$ (d) $8 r^3$

(33) In Δ ABC , a – b = 4 cm. , sin A = $\frac{3}{2}$ sin B , then a = ... cm.

(a) 4 (b) 6 (c) 8 (d) 12

(34) If the perimeter of Δ ABC is 24 cm. and sin A + sin B = 3 sin C , then C = ... cm.

(a) 4 (b) 6 (c) 8 (d) 9

(35) ABC is a triangle , sin B + sin C = 4 sin A and b + c = 2 a + 10 cm.

, then a = ... cm.

(a) 2 (b) 3 (c) 4 (d) 5

(36) In Δ ABC , AB = 8 cm. , BC = 12 cm. , m ($\angle$ A) – m ($\angle$ C) = 90°

, then tan C = ...

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(37) If r is the radius length of the circumcircle of Δ ABC and a = r

, then m ($\angle$ A) = ...

(a) 30° only. (b) 30° or 120° (c) 150° only. (d) 30° or 150°

(38) If the area of the triangle ABC is Δ and r is the radius length of the circumcircle of the triangle ABC , then : $\frac{4\,r\,\Delta}{abc}$ = ...

(a) 1 (b) 2 (c) 4 (d) 8

(39) In Δ ABC , $\frac{2\,b}{\sin B}$ = ... r (where r is the radius of its circumcircle)

(a) 1 (b) 2 (c) 4 (d) 8

(40) If the triangle ABC is an isosceles right-angled triangle and r is the radius length of the circumcircle of the triangle ABC , then the area of Δ ABC = ... (in terms of r)

(a) $\frac{1}{2}\,r^2$ (b) $2\,r^2$ (c) r^2 (d) $4\,r^2$

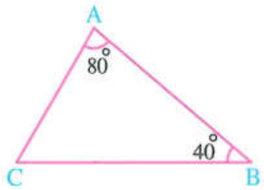
(41) **In the opposite figure :**

If the perimeter of Δ ABC = 20 cm. ,

then the diameter length of its circumcircle ≈ .... cm.

(a) 2 (b) 4

(c) 6 (d) 8



Second Essay questions

1 XYZ is a triangle in which : m ($\angle$ X) = 80° , m ($\angle$ Y) = 60° and z = 10 cm.
, find each of x and y to the nearest cm. « 15 cm. , 13 cm. »

2 ABC is a triangle in which : c = 19 cm. , m ($\angle$ A) = 112° and m ($\angle$ B) = 33°
Find to the nearest hundredth each of b and the length of the radius of the circumcircle of the triangle. « 18.04 cm. , 16.56 cm. »

3 XYZ is a triangle , if y = 68.4 cm. , m ($\angle$ Y) = 100° and m ($\angle$ Z) = 40°
, find : (1) x

(2) The radius length of the circumcircle of the triangle XYZ

(3) The area of the triangle XYZ « 44.64 cm. , 34.73 cm. , 981.34 cm.^{2} »

4 ABC is a triangle in which : b = 10 cm. , m ($\angle$ A) = 40° and m ($\angle$ C) = 80°
Find the length of the greatest side of Δ ABC « 11 cm. »

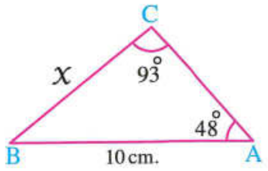
5 ABC is a triangle in which : c = 4.5 cm. , m (∠ A) = 100° and m (∠ B) = 15°

Find the length of the smallest side of Δ ABC « 1.3 cm. »

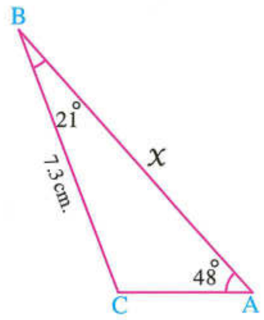
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6 **Use the sine rule to find the value of X to the nearest tenth :**

(1)



(2)



« 7.4 cm. , 9.2 cm.»

---

7 ABC is a triangle in which : m (∠ A) = 60° and a = $7\sqrt{3}$ cm.

Find the area and the circumference of the circumcircle of Δ ABC $\left(\pi = \frac{22}{7}\right)$

« 154 cm^2 , 44 cm. »

---

8 ABC is a triangle in which : a = 13 cm. , m (∠ A) = $53^\circ\ 8'$, c = 15 cm. Find the radius length of the circumcircle of Δ ABC , then find m (∠ C) « 8.1 cm. , $67^\circ\ 23'\ 9''$ or $112^\circ\ 36'\ 51''$ »

---

9 ABC is a triangle in which m (∠ A) = 35° , a = 8 cm. and b = 6 cm.

Find : m (∠ B) « $25^\circ\ 28'\ 45''$ »

---

10 In the triangle ABC , m (∠ A) = $67^\circ\ 22'$, m (∠ C) = $44^\circ\ 33'$ and b = 100 cm.

Find the perimeter of the triangle ABC and its surface area. « 275 cm. , 3473 cm^2 »

---

11 ABC is a triangle in which m (∠ B) = 35° , m (∠ C) = 70° and the radius length of the circumcircle of the triangle ABC = 16 cm. Find the area and the perimeter of Δ ABC to the nearest whole number. « 262 cm^2 , 79 cm. »

---

12 ABC is an isosceles triangle in which : m (∠ A) = 120° and the length of the radius of the circumcircle of Δ ABC is 12 cm.

Find c and calculate the area of Δ ABC « 12 cm. , 62.4 cm^2 »

---

13 ABC is an isosceles triangle in which : a = b and m (∠ A) = 15° and the perimeter of Δ ABC is 25 cm. Find the area of the circumcircle of Δ ABC « 474 cm^2 »

---

14 If the perimeter of Δ ABC = 40 cm. , m (∠ A) = 44° and m (∠ B) = 66° ,

Find the lengths of the sides of the triangle ABC « 10.9 cm. , 14.3 cm. , 14.8 cm. »

15 ABC is a triangle in which : c = 12 cm. and m (∠ B) = 3 m (∠ A) = 60°
Find a and the area of Δ ABC to the nearest cm^2 « 4.2 cm. , 22 cm^2 »

16 If the area of the triangle ABC is 450 cm^2 , m (∠ B) = 82° and m (∠ C) = 56°
, find the value of a « 27 cm. »

17 ABC is an acute-angled triangle in which AC = 12 cm. , sin A = 0.6 and its area is 43.2 cm^2
Find the length of each of $\overline{AB}$ and $\overline{BC}$, also find m (∠ B) « 12 cm. , 7.6 cm. , 71° 34' »

18 Find the perimeter of the acute-angled triangle ABC if a = 7 cm. , b = 8 cm.
and m (∠ A) = 60° « 20 cm. »

19 **Find the diameter length of the circumcircle of Δ ABC in the two following cases :**
(1) m (∠ A) = 75° , a = 21 cm.
(2) m (∠ B) = 50° , m (∠ C) = 65° , c – b = 6 cm. « 21.7 cm. , 42.8 cm. »

20 ABC is triangle in which : b = 5 cm. , tan C = $\frac{4}{3}$ and m (∠ B) = 30°
find a , c and the area of the triangle to the nearest integer. « 10 cm. , 8 cm. , 20 cm^2 »

21 XYZ is a triangle in which : sin X + sin Y + sin Z = 2.37 and its perimeter is 56.88 cm.
Find the length of the radius of the circumcircle of Δ XYZ « 12 cm. »

22 ABC is a triangle in which sin A : sin B : sin C = 2 : 4 : 5 and c – b = 3 cm.
Find each of a and b « 6 cm. , 12 cm. »

23 ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3 , if a = 5 cm.
, then find the perimeter of the triangle ABC « 15.9 cm. »

24 ABC is a triangle in which m (∠ A) = $\frac{2}{3}$ m (∠ B) = $\frac{1}{2}$ m (∠ C) , the length of the radius of its circumcircle = 10 cm. Find the area of Δ ABC « 110 cm^2 »

25 ABC is a triangle in which 6 sin A = 4 sin B = 3 sin C and its perimeter is 45 cm.
Find each of a and c « 10 cm. , 20 cm. »

26 $\overline{AB}$ and $\overline{AC}$ are two chords in a circle. If their lengths are 43.5 cm. and 52.1 cm. respectively and they are drawn in two different sides of the diameter $\overline{AD}$ whose length is 100 cm.

Find : (1) m (∠ BAC) (2) The length of $\overline{BC}$

« 122° 49' , 84 cm. »

27 ABCD is a parallelogram in which m (∠ A) = 50° , m (∠ DBC) = 70° and BD = 8 cm.

Find the perimeter of the parallelogram. « 38 cm. »

28 ABCD is a parallelogram in which AB = 18.6 cm. , m (∠ CAB) = 36° 22′ and m (∠ DBA) = 44° 38′

Find the length of the diagonal $\overline{AC}$ and the area of the parallelogram. « 26.46 cm. , 292 cm.^{2} »

29 ABCD is a parallelogram. M is the point of intersection of its two diagonals.

Let AC = 20 cm. , m (∠ AMD) = 130° and m (∠ CAB) = 85°

Find the length of $\overline{BD}$ and the area of the parallelogram ABCD « 28.2 cm. , 216 cm.^{2} »

30 ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, AD = 20 cm. , m (∠ D) = 120° , m (∠ B) = 62° and m (∠ ACB) = 23° 25′

Find the length of each of $\overline{AC}$ and $\overline{BC}$ to the nearest cm. « 29 cm. , 33 cm. »

31 ABCD is a quadrilateral in which CD = 100 cm. , m (∠ BCA) = 36° , m (∠ BDA) = 55° , m (∠ BCD) = 85° and m (∠ CDA) = 87°

Find the lengths of $\overline{BD}$ and $\overline{AC}$ to the nearest centimetre. « 112 cm. , 144 cm. »

32 ABCD is a quadrilateral in which m (∠ ABC) = 90° , m (∠ BAD) = 80° , AB = AD = 10 cm. , BD = BC. Calculate the area of the quadrilateral ABCD « 102 cm.^{2} »

33 ABCDE is a regular pentagon, whose side length is 18.26 cm.

Find the length of its diagonal $\overline{AC}$ « 29.5 cm. »

Third Higher skills

Choose the correct answer from the given ones :

(1) If the radius length of the circumcircle of the triangle ABC equals 3 cm.

, then : $\frac{abc}{\sin A \sin B \sin C}$ = ...

(a) 3 (b) 6 (c) 27 (d) 216

(2) If ABC is a triangle , then : a csc A + b csc B + c csc C = ...

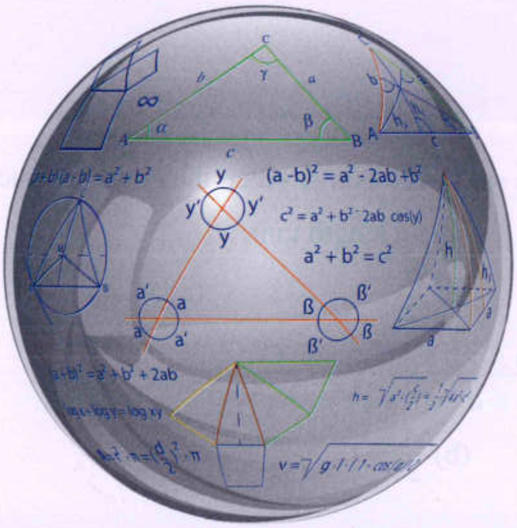
(a) 2 r (b) 4 r (c) 6 r (d) 8 r

(3) If $a = \sin B$, $b = \sin C$, $c = \sin A$, then the circumference of the circumcircle of Δ ABC equals .... length unit.

(a) 1 (b) $\frac{\pi}{2}$ (c) π (d) 2π

(4) In Δ ABC , $\frac{a \sin A + b \sin B + c \sin C}{a^2 + b^2 + c^2} =$ ....

(a) $\frac{1}{r^2}$ (b) $\frac{1}{2r}$ (c) $2r$ (d) r^2



Exercise 16 The cosine rule

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) In Δ XYZ , the expression $\frac{X^2 + y^2 - z^2}{2 X y}$ equals ....

(a) cos X (b) cos Y (c) cos Z (d) sin Z

(2) In Δ XYZ , $y^2 + z^2 - X^2 = 2$ y z × ....

(a) cos X (b) sin Z (c) cos Z (d) sin X

(3) In Δ ABC , cos (A + B) = ...

(a) cos C (b) – cos C (c) sin C (d) – sin C

(4) In any triangle ABC , cos A = ...

(a) – (cos B + cos C) (b) cos B – cos C

(c) cos (B + C) (d) – cos (B + C)

(5) If ∠ A is supplementary to ∠ C , then cos A + cos C = ...

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

(6) If ABCD is a cyclic quadrilateral , then cos A + cos C = ...

(a) 1 (b) zero. (c) $\frac{1}{2}$ (d) – 1

(7) In Δ XYZ , 2 X y cos (X + Y) = ...

(a) $X^2 + y^2 - z^2$ (b) $y^2 + z^2 - X^2$ (c) $X^2 - z^2 - y^2$ (d) $z^2 - X^2 - y^2$

(15) In triangle ABC , $m(\angle A) : m(\angle B) : m(\angle C) = 2 : 3 : 4$, AB = 12 cm. , then the length of $\overline{AC} \simeq$ .... cm.

(a) 10 (b) 11 (c) 16 (d) 18

(16) In triangle ABC , which of the following statements is true ?

(a) $\sin A + \cos B = a + b$ (b) $a \sin B = b \sin A$

(c) $a = b \sin c$ (d) $\frac{a}{\sin A} = \frac{\sin B}{b}$

(17) In ΔXYZ , $2 r \sin X =$ .... "where r is the radius length of its circumcircle"

(a) z (b) y (c) x (d) area of ΔXYZ

(18) 📖 If r is the length of the radius of the circumcircle of the triangle XYZ , then $\frac{y}{2 \sin Y} =$ ....

(a) r (b) 2 r (c) $\frac{1}{2}$ r (d) 4 r

(19) 📖 In any triangle LMN , $\frac{\ell}{\sin L} =$ ....

(a) $\frac{m}{\sin N}$ (b) $\frac{n}{\sin M}$ (c) $\frac{m + n}{\sin N + \sin M}$ (d) 3 r

(20) 📖 In ΔABC , if $\frac{\sin A}{a} = \frac{\sin B}{b}$, then $\frac{2 \sin A - \sin B}{\cdots\cdots} = \frac{\sin A}{a}$

(a) a + b (b) 2 a + b (c) a − 2 b (d) 2 a − b

(21) In acute-angled triangle ABC , $2 a = \frac{b}{\sin B}$, then $m(\angle A) =$ ....

(a) 30° (b) 45° (c) 60° (d) 75°

(22) In ΔABC , $\sin A = 2 \sin C$, BC = 6 cm. , then AB = ... cm.

(a) 2 (b) 3 (c) 4 (d) 6

(23) If the radius length of circumcircle of ΔABC equals 3 cm. and $\sin A + \sin B + \sin C = 2$, then the perimeter of triangle ABC = ... cm.

(a) 6 (b) 9 (c) 12 (d) 24

(24) 📖 In any triangle ABC , $\frac{\sin (A + B)}{\sin A + \sin B} =$ ....

(a) 1 (b) $\frac{c}{a + b}$ (c) $\frac{a}{b + c}$ (d) $\frac{b}{a + c}$

(25) In ΔABC , $\frac{a}{a + b} = \frac{\sin A}{\cdots\cdots}$

(a) sin B (b) sin C (c) sin A + sin B (d) sin A + sin C

(26) In Δ XYZ , if 3 sin X = 4 sin Y = 2 sin Z , then x : y : z = ...

(a) 2 : 3 : 4 (b) 6 : 4 : 3 (c) 3 : 4 : 6 (d) 4 : 3 : 6

(27) ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then a : b : c = ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

(28) In Δ ABC : If $\frac{\sin A}{4} = \frac{\sin B}{9} = \frac{\sin C}{7}$, then the greatest angle in measure is ....

(a) $\angle$ A (b) $\angle$ B (c) $\angle$ C (d) right

(29) In triangle ABC , m ($\angle$ A) : m ($\angle$ B) : m ($\angle$ C) = 3 : 5 : 4
, then $c^2 : a^2$ = ...

(a) $\sqrt{6} : 2$ (b) 2 : 3 (c) 4 : 3 (d) 3 : 2

(30) In Δ ABC , $\frac{a}{b} \times \frac{\sin B}{\sin A}$ = ...

(a) $\frac{c}{\sin C}$ (b) $\frac{\sin C}{c}$ (c) 4r (d) 1

(31) In Δ ABC , if the radius of its circumcircle = 4 cm.
, then $\frac{a + b + c}{\sin A + \sin B + \sin C}$ = ...

(a) 4 (b) 2 (c) 8 (d) 16

(32) If the radius of the circumcircle of Δ ABC equals r , then the perimeter of the triangle = ... (sin A + sin B + sin C)

(a) r (b) 2 r (c) $4 r^2$ (d) $8 r^3$

(33) In Δ ABC , a – b = 4 cm. , sin A = $\frac{3}{2}$ sin B , then a = ... cm.

(a) 4 (b) 6 (c) 8 (d) 12

(34) If the perimeter of Δ ABC is 24 cm. and sin A + sin B = 3 sin C , then C = ... cm.

(a) 4 (b) 6 (c) 8 (d) 9

(35) ABC is a triangle , sin B + sin C = 4 sin A and b + c = 2 a + 10 cm.
, then a = ... cm.

(a) 2 (b) 3 (c) 4 (d) 5

(36) In Δ ABC , AB = 8 cm. , BC = 12 cm. , m ($\angle$ A) – m ($\angle$ C) = 90°
, then tan C = ...

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(37) If r is the radius length of the circumcircle of Δ ABC and a = r

, then m ($\angle$ A) = ...

(a) 30° only. (b) 30° or 120° (c) 150° only. (d) 30° or 150°

(38) If the area of the triangle ABC is Δ and r is the radius length of the circumcircle of the triangle ABC , then : $\frac{4 r \Delta}{abc}$ = ...

(a) 1 (b) 2 (c) 4 (d) 8

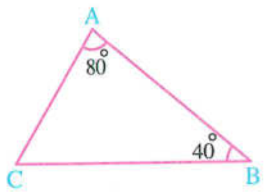
(39) 📖 In Δ ABC , $\frac{2 b}{\sin B}$ = ... r (where r is the radius of its circumcircle)

(a) 1 (b) 2 (c) 4 (d) 8

(40) If the triangle ABC is an isosceles right-angled triangle and r is the radius length of the circumcircle of the triangle ABC , then the area of Δ ABC = ... (in terms of r)

(a) $\frac{1}{2} r^2$ (b) $2 r^2$ (c) r^2 (d) $4 r^2$

(41) In the opposite figure :



If the perimeter of Δ ABC = 20 cm. ,

then the diameter length of its circumcircle ≈ .... cm.

(a) 2 (b) 4

(c) 6 (d) 8

Second Essay questions

1 XYZ is a triangle in which : m ($\angle$ X) = 80° , m ($\angle$ Y) = 60° and z = 10 cm.
, find each of x and y to the nearest cm. « 15 cm. , 13 cm. »

2 ABC is a triangle in which : c = 19 cm. , m ($\angle$ A) = 112° and m ($\angle$ B) = 33°
Find to the nearest hundredth each of b and the length of the radius of the circumcircle of the triangle. « 18.04 cm. , 16.56 cm. »

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, find : (1) x

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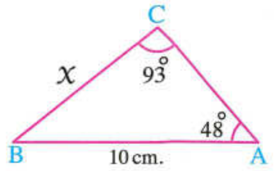
4 ABC is a triangle in which : b = 10 cm. , m ($\angle$ A) = 40° and m ($\angle$ C) = 80°
Find the length of the greatest side of Δ ABC « 11 cm. »

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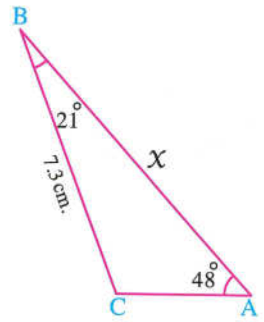
Find the length of the smallest side of Δ ABC « 1.3 cm. »

6 **Use the sine rule to find the value of x to the nearest tenth :**

(1)



(2)



« 7.4 cm. , 9.2 cm.»

7 ABC is a triangle in which : m (∠ A) = 60° and a = $7\sqrt{3}$ cm.

Find the area and the circumference of the circumcircle of Δ ABC $\left(\pi = \frac{22}{7}\right)$

« 154 cm², 44 cm. »

8 ABC is a triangle in which : a = 13 cm. , m (∠ A) = 53° 8' , c = 15 cm. Find the radius length of the circumcircle of Δ ABC , then find m (∠ C) « 8.1 cm. , 67° 23' 9'' or 112° 36' 51'' »

9 ABC is a triangle in which m (∠ A) = 35° , a = 8 cm. and b = 6 cm.

Find : m (∠ B) « 25° 28' 45'' »

10 In the triangle ABC , m (∠ A) = 67° 22' , m (∠ C) = 44° 33' and b = 100 cm.

Find the perimeter of the triangle ABC and its surface area. « 275 cm. , 3473 cm² »

11 ABC is a triangle in which m (∠ B) = 35° , m (∠ C) = 70° and the radius length of the circumcircle of the triangle ABC = 16 cm. Find the area and the perimeter of Δ ABC to the nearest whole number. « 262 cm², 79 cm. »

12 ABC is an isosceles triangle in which : m (∠ A) = 120° and the length of the radius of the circumcircle of Δ ABC is 12 cm.

Find c and calculate the area of Δ ABC « 12 cm. , 62.4 cm² »

13 ABC is an isosceles triangle in which : a = b and m (∠ A) = 15° and the perimeter of Δ ABC is 25 cm. Find the area of the circumcircle of Δ ABC « 474 cm² »

14 If the perimeter of Δ ABC = 40 cm. , m (∠ A) = 44° and m (∠ B) = 66° ,

Find the lengths of the sides of the triangle ABC « 10.9 cm. , 14.3 cm. , 14.8 cm. »

15 ABC is a triangle in which : c = 12 cm. and m (∠ B) = 3 m (∠ A) = 60°
Find a and the area of Δ ABC to the nearest cm^2. « 4.2 cm. , 22 cm^2 »

16 If the area of the triangle ABC is 450 cm^2 , m (∠ B) = 82° and m (∠ C) = 56°
, find the value of a « 27 cm. »

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Find the length of each of $\overline{AB}$ and $\overline{BC}$, also find m (∠ B) « 12 cm. , 7.6 cm. , 71° 34′ »

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20 ABC is triangle in which : b = 5 cm. , tan C = $\frac{4}{3}$ and m (∠ B) = 30°
find a , c and the area of the triangle to the nearest integer. « 10 cm. , 8 cm. , 20 cm^2 »

21 XYZ is a triangle in which : sin X + sin Y + sin Z = 2.37 and its perimeter is 56.88 cm.
Find the length of the radius of the circumcircle of Δ XYZ « 12 cm. »

22 ABC is a triangle in which sin A : sin B : sin C = 2 : 4 : 5 and c – b = 3 cm.
Find each of a and b « 6 cm. , 12 cm. »

23 ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3 , if a = 5 cm.
, then find the perimeter of the triangle ABC « 15.9 cm. »

24 ABC is a triangle in which m (∠ A) = $\frac{2}{3}$ m (∠ B) = $\frac{1}{2}$ m (∠ C) , the length of the radius of its circumcircle = 10 cm. Find the area of Δ ABC « 110 cm^2 »

25 ABC is a triangle in which 6 sin A = 4 sin B = 3 sin C and its perimeter is 45 cm.
Find each of a and c « 10 cm. , 20 cm. »

26 $\overline{AB}$ and $\overline{AC}$ are two chords in a circle. If their lengths are 43.5 cm. and 52.1 cm. respectively and they are drawn in two different sides of the diameter $\overline{AD}$ whose length is 100 cm.
Find : (1) m (∠ BAC) (2) The length of $\overline{BC}$
« 122° 49′ , 84 cm. »

27 ABCD is a parallelogram in which m (∠ A) = 50° , m (∠ DBC) = 70° and BD = 8 cm.

Find the perimeter of the parallelogram. « 38 cm. »

---

28 ABCD is a parallelogram in which AB = 18.6 cm. , m (∠ CAB) = 36° 22'

and m (∠ DBA) = 44° 38'

Find the length of the diagonal $\overline{AC}$ and the area of the parallelogram. « 26.46 cm. , 292 cm^2 »

---

29 ABCD is a parallelogram. M is the point of intersection of its two diagonals.

Let AC = 20 cm. , m (∠ AMD) = 130° and m (∠ CAB) = 85°

Find the length of $\overline{BD}$ and the area of the parallelogram ABCD « 28.2 cm. , 216 cm^2 »

---

30 ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, AD = 20 cm. , m (∠ D) = 120° ,

m (∠ B) = 62° and m (∠ ACB) = 23° 25'

Find the length of each of $\overline{AC}$ and $\overline{BC}$ to the nearest cm. « 29 cm. , 33 cm. »

---

31 ABCD is a quadrilateral in which CD = 100 cm. , m (∠ BCA) = 36° ,

m (∠ BDA) = 55° , m (∠ BCD) = 85° and m (∠ CDA) = 87°

Find the lengths of $\overline{BD}$ and $\overline{AC}$ to the nearest centimetre. « 112 cm. , 144 cm. »

---

32 ABCD is a quadrilateral in which m (∠ ABC) = 90° , m (∠ BAD) = 80°

, AB = AD = 10 cm. , BD = BC. Calculate the area of the quadrilateral ABCD « 102 cm^2 »

---

33 ABCDE is a regular pentagon, whose side length is 18.26 cm.

Find the length of its diagonal $\overline{AC}$ « 29.5 cm. »

Third Higher skills

Choose the correct answer from the given ones :

(1) If the radius length of the circumcircle of the triangle ABC equals 3 cm.

, then : $\frac{abc}{\sin A \sin B \sin C}$ = ...

(a) 3 (b) 6 (c) 27 (d) 216

(2) If ABC is a triangle , then : a csc A + b csc B + c csc C = ...

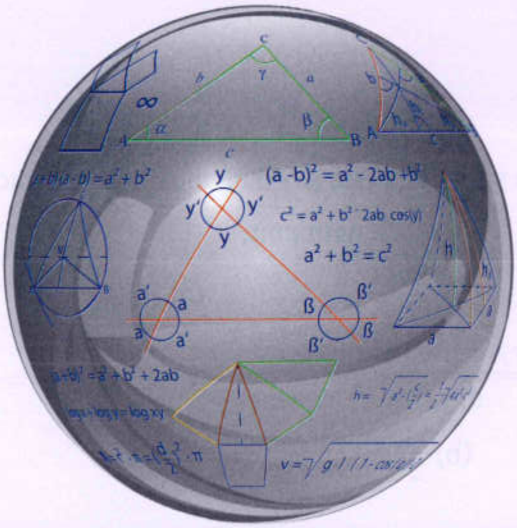
(a) 2 r (b) 4 r (c) 6 r (d) 8 r

(3) If $a = \sin B$, $b = \sin C$, $c = \sin A$, then the circumference of the circumcircle of ΔABC equals .... length unit.

(a) 1 (b) $\frac{\pi}{2}$ (c) π (d) 2π

(4) In ΔABC , $\frac{a \sin A + b \sin B + c \sin C}{a^2 + b^2 + c^2} =$ ....

(a) $\frac{1}{r^2}$ (b) $\frac{1}{2r}$ (c) $2r$ (d) r^2



Exercise 16 — The cosine rule

Test yourself

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First — Multiple choice questions

Choose the correct answer from those given :

(1) In Δ XYZ , the expression $\frac{\mathcal{X}^2 + y^2 - z^2}{2\,\mathcal{X}\,y}$ equals ....

(a) cos X (b) cos Y (c) cos Z (d) sin Z

(2) 📖 In Δ XYZ , $y^2 + z^2 - \mathcal{X}^2 = 2\,y\,z \times$ ....

(a) cos X (b) sin Z (c) cos Z (d) sin X

(3) In Δ ABC , cos (A + B) = ...

(a) cos C (b) – cos C (c) sin C (d) – sin C

(4) 📖 In any triangle ABC , cos A = ...

(a) – (cos B + cos C) (b) cos B – cos C

(c) cos (B + C) (d) – cos (B + C)

(5) 📖 If $\angle$ A is supplementary to $\angle$ C , then cos A + cos C = ...

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

(6) If ABCD is a cyclic quadrilateral , then cos A + cos C = ...

(a) 1 (b) zero. (c) $\frac{1}{2}$ (d) – 1

(7) In Δ XYZ , $2\,\mathcal{X}\,y$ cos (X + Y) = ...

(a) $\mathcal{X}^2 + y^2 - z^2$ (b) $y^2 + z^2 - \mathcal{X}^2$ (c) $\mathcal{X}^2 - z^2 - y^2$ (d) $z^2 - \mathcal{X}^2 - y^2$

(8) In Δ ABC , the expression $\frac{a^2 + b^2 - c^2}{2ab}$ equals zero if ....

(a) m ($\angle$ A) = 60°
(b) m ($\angle$ B) = 90°
(c) m ($\angle$ C) = 120°
(d) m ($\angle$ A) + m ($\angle$ B) = 90°

(9) In Δ LMN , ℓ = 5 cm. , m = 7 cm. , m ($\angle$ N) = 60°
, then n = ... cm. (to the nearest tenth)

(a) 6.2 (b) 5 (c) 4.3 (d) 3.5

(10) In Δ XYZ , x = 5 cm. , y = 3 cm. , m ($\angle$ Z) = $\frac{2}{3}\pi$
, then z = ...

(a) 7 (b) 8 (c) 9 (d) 4

(11) In Δ ABC , if m ($\angle$ A) + m ($\angle$ B) = 120° , a = 2 cm. , b = 3 cm. ,
then c = ... cm.

(a) 4 (b) 3 (c) $\sqrt{7}$ (d) $\sqrt{5}$

(12) In Δ ABC , a = 9 cm. , b = 15 cm. , m ($\angle$ C) = 106°
, then its perimeter ≃ .... cm.

(a) 44 (b) 24 (c) 34 (d) 28

(13) In Δ ABC , b = 2 cm. , c = 2.5 cm. , cos A = $\frac{2}{5}$
, then Δ ABC is ....

(a) a right-angled triangle.
(b) an isosceles triangle.
(c) an equilateral triangle.
(d) a scalene.

(14) In Δ XYZ , if x = y , then cos X = ...

(a) $\frac{2y^2}{z}$ (b) $\frac{z}{2y}$ (c) $\frac{z}{4x}$ (d) $\frac{y}{2x}$

(15) In Δ ABC , cos (A + B) = ...

(a) $\frac{a^2 + b^2 - c^2}{2ab}$ (b) $\frac{a^2 + c^2 - b^2}{2ab}$ (c) $\frac{b^2 + c^2 - a^2}{2bc}$ (d) $\frac{c^2 - a^2 - b^2}{2ab}$

(16) The measure of the greatest angle in triangle the lengths of its sides are 3 cm. , 5 cm.
, 7 cm. equals ....°

(a) 110 (b) 150 (c) 100 (d) 120

(17) In Δ ABC , b = 4 cm. , a + c = 11 cm. , a − c = 1 cm. , then ....

(a) the triangle is an obtuse angled triangle.
(b) the triangle is a right angled triangle.
(c) m ($\angle$ B) = 2 m ($\angle$ A)
(d) m ($\angle$ A) = 2 m ($\angle$ B)

(18) In Δ ABC , $a^2 + b^2 - c^2 + \sqrt{3}\, ab = 0$, then m ($\angle$ C) = ...°

(a) 30 (b) 150 (c) 60 (d) 120

(19) In Δ ABC , if m ($\angle$ C) = 60° , $a^2 + b^2 - c^2 = k\, a\, b$, then k = ...

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) – 1

(20) In Δ ABC , 4 sin A = 3 sin B = 6 sin C , then m ($\angle$ C) = ... (to the nearest degree)

(a) 89° (b) 29° (c) 57° (d) 82°

(21) In Δ ABC , $\frac{1}{2}$ sin A = $\frac{1}{3}$ sin B = $\frac{1}{4}$ sin C , then cos C = ...

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-1}{4}$ (d) $\frac{1}{4}$

(22) If ABC is a triangle in which : 5 sin A sin B = 6 sin B sin C = 9 sin C sin A , then m ($\angle$ C) ≃ ....°

(a) 28 (b) 32 (c) 36 (d) 42

(23) If ABC is a triangle in which : 6 a = 4 b = 3 c , then the measure of the smallest angle in the triangle ≃ ....

(a) 57° 28' (b) 41° 12' (c) 28° 57' (d) 36° 52'

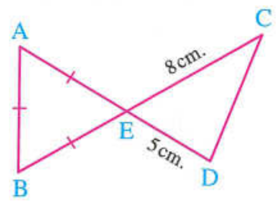
(24) ABC is a triangle in which m ($\angle$ A) = 60° , b : c = 5 : 8 and the area of the circumcircle of the triangle ABC is 147 π cm^2. , then the perimeter of Δ ABC = ... cm.

(a) 21 (b) 34 (c) 54 (d) 60

(25) ABCD is a parallelogram in which AB = 8 cm. , BC = 11 cm. , BD = 9 cm. , then the length of $\overline{AC}$ = ... cm.

(a) 9 (b) 10 (c) 11 (d) 17

(26) **In the opposite figure :**

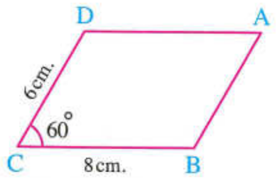


CD = ... cm.

(a) 6 (b) 7

(c) 8 (d) 9

(27) **In the opposite figure :**

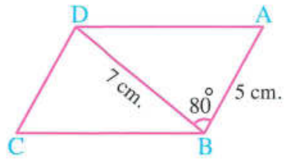


ABCD is a parallelogram , then AC = ... cm.

(a) $2\sqrt{13}$ (b) $2\sqrt{37}$

(c) $2\sqrt{17}$ (d) 148

(28) In the opposite figure :



ABCD is a parallelogram

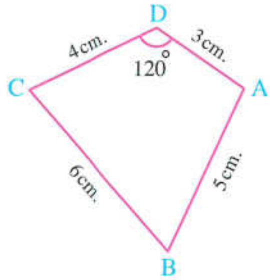
m (∠ ABD) = 80° , BD = 7 cm.

AB = 5 cm. , then the perimeter

of parallelogram = ... to the nearest cm.

(a) 25 (b) 26 (c) 29 (d) 30

(29) In the opposite figure :



cos B = ...

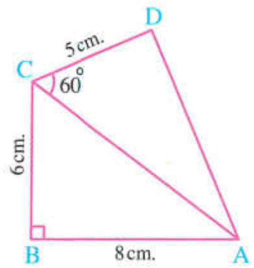
(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

(30) In the opposite figure :



ABCD is a quadrilateral in which AB = 8 cm.

, BC = 6 cm. , m (∠ B) = 90°

, DC = 5 cm. and m (∠ ACD) = 60°

, then the area of the circumcircle of

the triangle ADC = ... cm^2.

(a) 9 π (b) 16 π (c) 25 π (d) 49 π

Second Essay questions

1 XYZ is a triangle in which : m (∠ Z) = 95° , x = 13 cm. , y = 16 cm. Find z « 21.5 cm. »

2 ABC is a triangle in which : a = 3 cm. , c = 5 cm. and m (∠ B) = 36° 21`

Find b to the nearest cm. « 3 cm. »

3 ABC is a triangle in which a = 3 cm. , b = 5 cm. and c = $\sqrt{19}$ cm. , **find :**

(1) m (∠ C) (2) The area of the triangle ABC « 60° , $\frac{15\sqrt{3}}{4}$ cm^2. »

4 If two side lengths of a triangle are $(\sqrt{10} + 2)$ and $(\sqrt{10} - 2)$ and the measure of the included angle = 60° , find the third side length. « $\sqrt{22}$ »

5 ABC is a triangle in which : a = 4 cm. , b = 6 cm. and m (∠ C) = 57°
, find the perimeter of Δ ABC to the nearest cm. « 15 cm. »

6 Find the measures of the angles of the triangle ABC in which a = 7.6 cm.
, b = 5.8 cm. and c = 3.4 cm. « 108° 34̀ , 46° 20̀ , 25° 6̀ »

7 ABC is a triangle in which a = 13 cm. , b = 14 cm. and c = 15 cm. Find m (∠ B) ,
then find the area of the triangle ABC to the nearest cm^2. « 59° 29̀ , 84 cm^2. »

8 Find the measure of the smallest angle in Δ XYZ , where x = 18 cm. , y = 27 cm.
and z = 24 cm. Find also the area of the circumcircle of Δ XYZ « 40° 48̀ , 596 cm^2. »

9 ABC is a triangle in which a = 9 cm. , b = 15 cm. and c = 21 cm. Find the measurement of the largest angle of the triangle and prove that it satisfies the relation :
$\cos C - 5\sqrt{3} \sin C + 8 = 0$ « 120° »

10 The perimeter of the triangle ABC is 52 cm. , a = 13 cm. and b = 17 cm. Find the measure of the greatest angle in the triangle , then find the area of the triangle to the nearest centimetre square. « 93° 22̀ , 110 cm^2. »

11 Find the measure of the greatest angle in Δ XYZ , where x = 24.5 cm. , y = 18 cm.
and z = 10 cm. Find the circumference of the circumcircle of Δ XYZ $\left(\pi = \frac{22}{7}\right)$
« 119° 19̀ , 88 cm. »

12 If the ratio among the lengths of the sides of the triangle XYZ is $x : y : z = 4 : 5 : 6$
, prove that the measure of the smallest angle of the triangle approximately equals 41° 25̀

13 XYZ is a triangle in which sin X : sin Y : sin Z = 7 : 8 : 12
Find the measure of its greatest angle. « 106° 4̀ »

14 ABC is a triangle in which : a = 4 cm. , b = 5 cm. and $\cos C = \frac{-1}{2}$
Find c and the area of Δ ABC « 7.8 cm. , $5\sqrt{3}$ cm^2. »

15 ABC is a triangle in which : 2 sin A = 3 sin B = 4 sin C
Find the measure of the smallest angle. « 26° 23̀ »

16 ABC is a triangle in which $\frac{1}{3} \sin A = \frac{1}{4} \sin B = \frac{1}{5} \sin C$, find m (∠ C) and if the perimeter of the triangle = 24 cm. , find its area. « 90° , 24 cm^2. »

17 ABC is a triangle in which BC = 20 cm. , m (∠ B) = 29° , m (∠ C) = 73° and D is the midpoint of $\overline{BC}$, find the lengths of $\overline{AB}$ and $\overline{AD}$ to the nearest two decimals.

« 19.55 cm. , 11.84 cm. »

---

18 ABC is a triangle in which : a = 8 cm. , b = 7 cm. and c = 9 cm. Let D ∈ $\overline{BC}$ such that BD = 4 cm. Calculate the length of $\overline{AD}$, calculate also the length of the radius of the circumcircle of Δ ABC « 7 cm. , 4.7 cm. »

---

19 ABCD is a parallelogram in which : AC = 16 cm. , DB = 20 cm. and m (∠ AMB) = 50° , where M is the point of intersection of its diagonals.

Find AB and AD to the nearest cm. « 8 cm. , 16 cm. »

---

20 ABCD is a parallelogram in which AB = 9 cm. , BC = 13 cm. and AC = 20 cm.

Find the length of $\overline{BD}$ « 10 cm. »

---

21 ABCD is a trapezium in which : $\overline{AD}$ // $\overline{BC}$, AD = 42 cm. , AB = 30 cm. , BC = 48 cm. and m (∠ A) = 100° **, find the length of each of :** $\overline{BD}$, $\overline{CD}$

« 55.7 cm. , 29.3 cm. »

---

22 ABCD is a quadrilateral in which : AB = 9 cm. , BC = 5 cm. , CD = 8 cm. , DA = 9 cm. and AC = 11 cm.

Prove that : The figure ABCD is a cyclic quadrilateral.

---

23 ABCD is a quadrilateral in which : AB = 6 cm. , BC = 14 cm. , CD = 10 cm. and AC = BD = 16 cm. **Prove that :** ABCD is a cyclic quadrilateral.

---

24 ABCD is a quadrilateral in which : AB = 27 cm. , BC = 12 cm. , CD = 8 cm. , DA = 12 cm. and AC = 18 cm.

Prove that : $\overrightarrow{AC}$ bisects ∠ BAD

---

25 ABCD is a quadrilateral in which : m (∠ DAB) = m (∠ DBC) = 90° , BD = 10 cm. , AD = 8 cm. and m (∠ DCB) = 30° , find AC to the nearest cm. « 22 cm. »

---

26 ABCD is a quadrilateral in which : AB = 15 cm. , BC = 20 cm. , CD = 16 cm. , AC = 25 cm. and m (∠ ACD) = 36° 52′ , find the length of $\overline{AD}$ to the nearest centimetre , then find the area of the quadrilateral ABCD « 16 cm. , 270 cm^2 »

27 ABC is a triangle in which : a = 3 b and m (∠ C) = 60° , find m (∠ B) and m (∠ A)

« 19° 6̀ , 100° 54̀ »

---

28 ABC is a triangle in which : a = 5 cm. , m (∠ B) = 120° and its area is $10\sqrt{3}$ cm^2.

Find each of c and b and also m (∠ A) « 8 cm. , 11.36 cm. , 22° 24̀ »

---

29 ABC is a triangle in which : P – a = 8 cm. , P – b = 6 cm. and P – c = 4 cm. , find the measurement of the largest angle in the triangle where 2 P = a + b + c « 78° 28̀ »

---

30 In the triangle ABC , if P – a = 26 cm. , b = 28 cm. and P + a = 98 cm. where 2 P is the triangle perimeter, find the side lengths of the triangle , then the measurement of the smallest angle. « 36 cm. , 28 cm. , 60 cm. , 17° 51̀ »

---

31 ABC is a triangle whose area is 64 cm^2. , m (∠ A) = 30° , b : c = 3 : 4

Find the perimeter of Δ ABC « 41.8 cm. »

---

32 If sin A : sin B : sin C = 3 : 5 : 7

, **prove that :** cos A : cos B : cos C = 13 : 11 : – 7

---

33 ABC is a triangle whose perimeter is 34 cm. , a = 12 cm. and b – c = 6 cm.

Find the measure of its smallest angle , then calculate its area. « 34° 46̀ 19̋ , 47.9 cm^2. »

---

34 In the triangle XYZ , if $y^2 = (z - x)^2 + z\,x$

, **prove that :** m (∠ Y) = 60°

---

35 **Discover the error :**

In the triangle ABC , if a = 5 cm. , b = 10 cm. , c = 7 cm. and m (∠ A) = 27.66°

Find : m (∠ B)

Ziad's answer

$\because \frac{b}{\sin B} = \frac{a}{\sin A}$

$\therefore \frac{10}{\sin B} = \frac{5}{\sin 27.66°}$

$\therefore \sin B = \frac{10 \sin 27.66°}{5} \simeq 0.9284$

$\therefore$ m (∠ B) ≃ 68.19°

Karim's answer

$\because \cos B = \frac{a^2 + c^2 - b^2}{2\,ac}$

$\therefore \cos B = \frac{(5)^2 + (7)^2 - (10)^2}{2 \times 5 \times 7}$

$\simeq -0.3714$

$\therefore$ m (∠ B) ≃ 111.8°

Which of the two answers is correct ?

Third Higher skills

Choose the correct answer from those given :

(1) If the area of Δ ABC = 12 cm^2 , then $(b^2 + c^2 - a^2) \tan A =$ ....

(a) 12 (b) 24 (c) 48 (d) 96

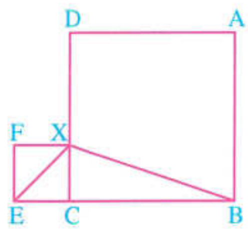
(2) In Δ ABC , if m ($\angle$ A) = 60° , then : $(1 + \frac{a}{c} + \frac{b}{c})(1 + \frac{c}{b} - \frac{a}{b}) =$ ....

(a) zero (b) 1 (c) 2 (d) 3

(3) In Δ ABC , if $\frac{a^3 + b^3 + c^3}{a + b + c} = a^2$, then m ($\angle$ A) = ...

(a) 30° (b) 60° (c) 45° (d) 150°

(4) In the opposite figure :



ABCD , XCEF are two squares

if BC = 3 CE , then

sin ($\angle$ BXE) = ...

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}$

(c) $\frac{-1}{\sqrt{5}}$ (d) $\frac{-2}{\sqrt{5}}$



Exercise 17 Solution of the triangle

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) Solving the triangle means ....

(a) to find the lengths of its sides.

(b) to find the measures of its angles.

(c) to find the relation between the lengths of its sides and the measures of its angles.

(d) to find the lengths of its sides and the measures of its angles.

(2) The perimeter of ΔABC , in which $b = 11$ cm. , $m(\angle A) = 67^\circ$, $m(\angle C) = 46^\circ$ equals .... (to the nearest cm.)

(a) 22 (b) 38 (c) 31 (d) 27

(3) By solving the triangle ABC in which $a = 5$ cm. , $b = 7$ cm. , $m(\angle C) = 65^\circ$, then $c \simeq$ .... cm. (to the nearest tenth)

(a) 4.4 (b) 2.1 (c) 6.7 (d) 8.2

(4) By solving ΔABC in which $a = 2$ cm. , $b = 4\sqrt{2}$ cm. , $c = 2\sqrt{5}$ cm. , then

First : $\cos A =$ ....

(a) $\frac{3}{\sqrt{10}}$ (b) $\frac{4}{5}$ (c) $\frac{2}{\sqrt{10}}$ (d) $\frac{\sqrt{10}}{5}$

Second : $m(\angle C) =$ ....

(a) $32^\circ\ 18'$ (b) $27^\circ\ 43'$ (c) 135° (d) 45°

(5) The number of possible solutions of Δ ABC in which m ($\angle$ C) = 115° , c = 12 cm. , a = 9 cm. is ....

(a) 1 (b) 2 (c) 3 (d) zero.

(6) The number of possible solutions of Δ ABC in which a = 8 cm. , b = 10 cm. , m ($\angle$ A) = 42° is ....

(a) 1 (b) 2 (c) infinite number. (d) zero.

(7) The number of possible solutions of Δ ABC in which m ($\angle$ A) = 60° , b = 3 cm. , a = 5 cm. is ....

(a) 1 (b) 2 (c) 0 (d) infinite number.

(8) The number of possible solutions of Δ XYZ in which x = 5 cm. , y = 6 cm. , m ($\angle$ X) = 70° equals ....

(a) zero. (b) 2 (c) 1 (d) 3

(9) In Δ XYZ , x = 30 cm. , y = 20 cm. , m ($\angle$ X) = 100° , then these conditions verify ....

(a) unique solution. (b) two solutions.

(c) three solutions. (d) no solution.

(10) In Δ ABC , a = 20 cm. , b = 25 cm. , m ($\angle$ A) = 40° , then these conditions verify ....

(a) unique solution. (b) two solutions.

(c) three solutions. (d) no solution.

(11) If Δ XYZ , m ($\angle$ X) = 100° , x = 3 cm. , y = 4 cm. , then these conditions verify ....

(a) unique solution. (b) two solutions.

(c) three solutions. (d) no solution.

Second Essay questions

Exercise on solving a triangle knowing a side length and the measures of two angles

1 Solve the triangle ABC in which : b = 11 cm. , m ($\angle$ A) = 67° and m ($\angle$ C) = 46°

« 11 cm. , 8.6 cm. , 67° »

2 Solve the triangle ABC in which : a = 8 cm. , m ($\angle$ A) = 60° and m ($\angle$ B) = 40°

« 5.94 cm. , 9.1 cm. , 80° »

3 Solve the triangle ABC in which : $m(\angle A) = 49° 11'$, $m(\angle B) = 67° 17'$, $c = 11.22$ cm.

« 9.5 cm. , 11.6 cm. , 63° 32' »

4 Solve the triangle ABC in which : AB = 9 cm. and $m(\angle A) = 2\, m(\angle B) = 80°$, then calculate its area to the nearest cm^2.

« 10.2 cm. , 6.7 cm. , 60° , the area ≃ 30 cm^2 »

5 Solve the triangle XYZ in which : XY = 40 cm. , $m(\angle X) = 75° 12'$ and $m(\angle Y) = 48° 15'$, then find the height of the triangle drawn from Z to $\overline{XY}$

« 46.4 cm. , 35.8 cm. , 56° 33' , the height ≃ 34.6 cm. »

Exercise on solving a triangle knowing the lengths of two sides and the measure of the included angle

6 Solve Δ XYZ in which : $m(\angle Z) = 60°$, XZ = 16 cm. and YZ = 13 cm.

« 14.7 cm. , 49° 51' , 70° 9' »

7 Solve the triangle ABC in which : a = 5 cm. , b = 7 cm. and $m(\angle C) = 65°$

« 6.7 cm. , 71° 50' , 43° 10' »

8 Solve the triangle ABC in which : $m(\angle A) = 153° 12'$ and b = c = 6 cm.

« 11.67 cm. , 13° 24' , 13° 24' »

9 Solve Δ LMN in which : $\ell = 12.5$ cm. , n = 7.25 cm . and $m(\angle M) = 1.2^{rad}$

« 11.96 cm. , 76° 53' , 34° 22' »

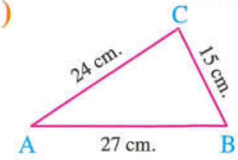
10 Solve Δ LMN in which : LM = 48.5 cm. , MN = 46 cm. and $\cos M = -0.6$

« 84.53 cm. , 25° 48' , 126° 52' , 27° 20' »

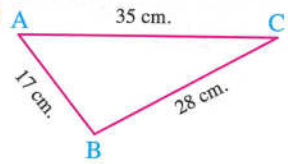
Exercise on solving a triangle knowing the lengths of the three sides

11 **Solve the triangle ABC in each of the following figures :**

(1)



(2)



12 Solve the triangle ABC in which : a = 13 cm. , b = 14 cm. and c = 15 cm.

« 53° 8' , 59° 29' , 67° 23' »

13 Solve the triangle ABC in which : a = 5 cm. and b = 2 c = 8 cm.

« 30° 45′ , 125° 6′ , 24° 9′ »

14 Solve the triangle ABC in which : a = 2 cm. , $b = 4\sqrt{2}$ cm. and $c = 2\sqrt{5}$ cm.

« 18° 26′ , 116° 34′ , 45° »

15 Solve the triangle XYZ in which : XY = 15 cm. , YZ = 25 cm. and XZ = 30 cm.

« 56° 15′ , 93° 49′ , 29° 56′ »

Exercise on the activity

16 Solve the triangle ABC in which : a = 10 cm. , b = 9 cm. and m (∠ B) = 57°

17 Solve the triangle ABC in which : m (∠ A) = 50° , a = 4 cm. and b = 3 cm.

18 Solve the triangle ABC in which : m (∠ C) = 116° , c = 12 cm. and a = 10 cm.

19 **Show if the following conditions satisfy the existence of one triangle or more , or don't satisfy the existence of any triangle at all, then find the possible solutions , approximated the side lengths to the nearest tenth and the angles measures to the nearest degree :**

(1) a = 15 cm. , b = 10 cm. and m (∠ A) = 120°

(2) a = 12 cm. , b = 15 cm. and m (∠ A) = 100°

(3) 📖 a = 20 cm. , b = 28 cm. and m (∠ A) = 42°

(4) 📖 a = 5 cm. , b = 7 cm. and m (∠ A) = 60°

(5) 📖 a = 12 cm. , c = 7 cm. and m (∠ A) = 27°

Miscellaneous exercises

20 Solve the isosceles triangle ABC in which : m (∠ A) = 110° and a = 8 cm.

« 4.9 cm. , 4.9 cm. , 35° , 35° »

21 Solve the triangle ABC in which : a = 21 cm. , $\cos B = \frac{3}{5}$ and $\tan C = \frac{5}{12}$

« 17.3 cm. , 8.3 cm. , 104° 15′ , 53° 8′ , 22° 37′ »

22 Solve the triangle ABC in which : a = 5 cm. , m (∠ B) = 120° and its area is $10\sqrt{3}$ cm².

« 11.36 cm. , 8 cm. , 22° 24′ , 37° 36′ »

23 Solve the triangle ABC in which m (∠ A) : m (∠ B) : m (∠ C) = 4 : 5 : 6 and its perimeter equals 50 cm. « 14.5 cm. , 16.9 cm. , 18.6 cm. , 48° , 60° , 72° »

---

24 Solve the triangle ABC in which sin A : sin B : sin C = 3 : 4 : 6 and its perimeter equals 52 cm. « 12 cm. , 16 cm. , 24 cm. , 26° 23' , 36° 20' , 117° 17' »

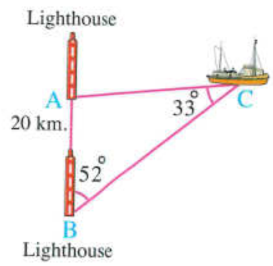
---

25 Solve the acute-angled triangle ABC in which a = 21 cm. , b = 25 cm. and the diameter length of its circumcircle = 28 cm. « 26 cm. , 48° 35' , 63° 14' , 68° 11' »

Life Applications on Unit Four

From the school book

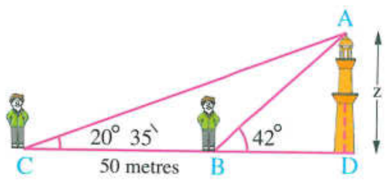
1 The distance between two lighthouses A and B is 20 km. on one line from North to South. If a ship is located at position C where m (∠ ACB) = 33° and m (∠ ABC) = 52° , find the distance between the ship and each lighthouse.



« 29 km. , 36.6 km. »

---

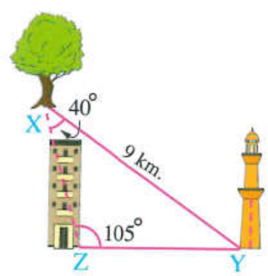
2 Ahmed and Salah stand in front of a minaret and the distance between them is 50 metres as shown in the opposite figure. How high is the minaret to the nearest tenth of metre ?



« 32.2 m. »

---

3 In the opposite figure , there are three geographical positions forming a triangle. If the distance between position X and position Y is 9 km. , the measurement of the angle at position X is 40° and the measurement of the angle at position Z is 105°,



Find : (1) The distance between position X and position Z

(2) The area of the triangle whose vertices are X , Y , Z

« 5.3 km. , 15 km.^{2} »

---

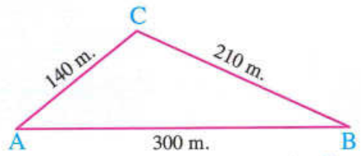
4 **Sports :** Ahmed run a distance of 8 km. in a certain direction , then he turned with an angle of measure 80° to run a distance of 9 km. How long is it from the starting point to the final point ?

« 11 km. »

---

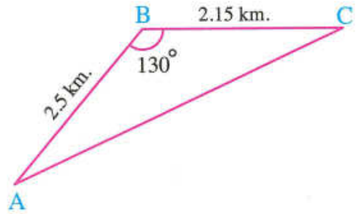
5 **Sports :** In a soccer match , the midfielder was distant from the right wing player for 15 m. If the midfielder turned with an angle of measure 45° and saw the left wing player distant from him 17 m. How far is it from the right wing player to the left one ? « 12.4 m. »

6 **Land survey :** A triangle-like piece of land whose side lengths are 300 m. , 210 m. and 140 m. Use the cosine rule to find the area of the land to the nearest square metre.



« 13148 m^2 »

7 **Transportation :** Ahmed travels by his motorcycle to cover a distance from point A to point B, then to point C with speed of 50 km./h. and he returns directly from point C to point A with speed of 60 km./h. How long does he take in his back and fourth trip to the nearest tenth of minute ?



« 9.8 minutes »

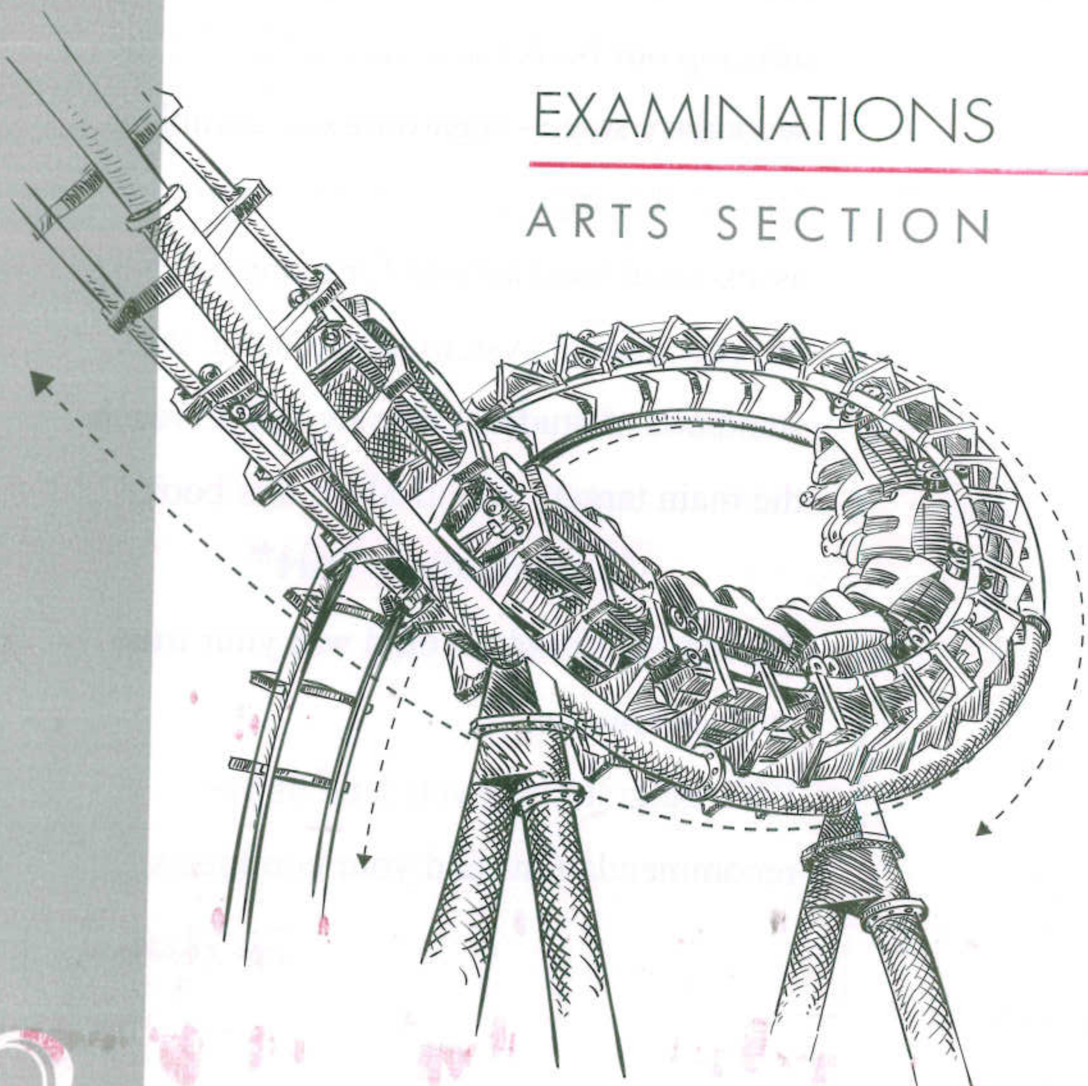
EL-MOASSER

General Mathematics

By a group of supervisors

EXAMINATIONS

ARTS SECTION

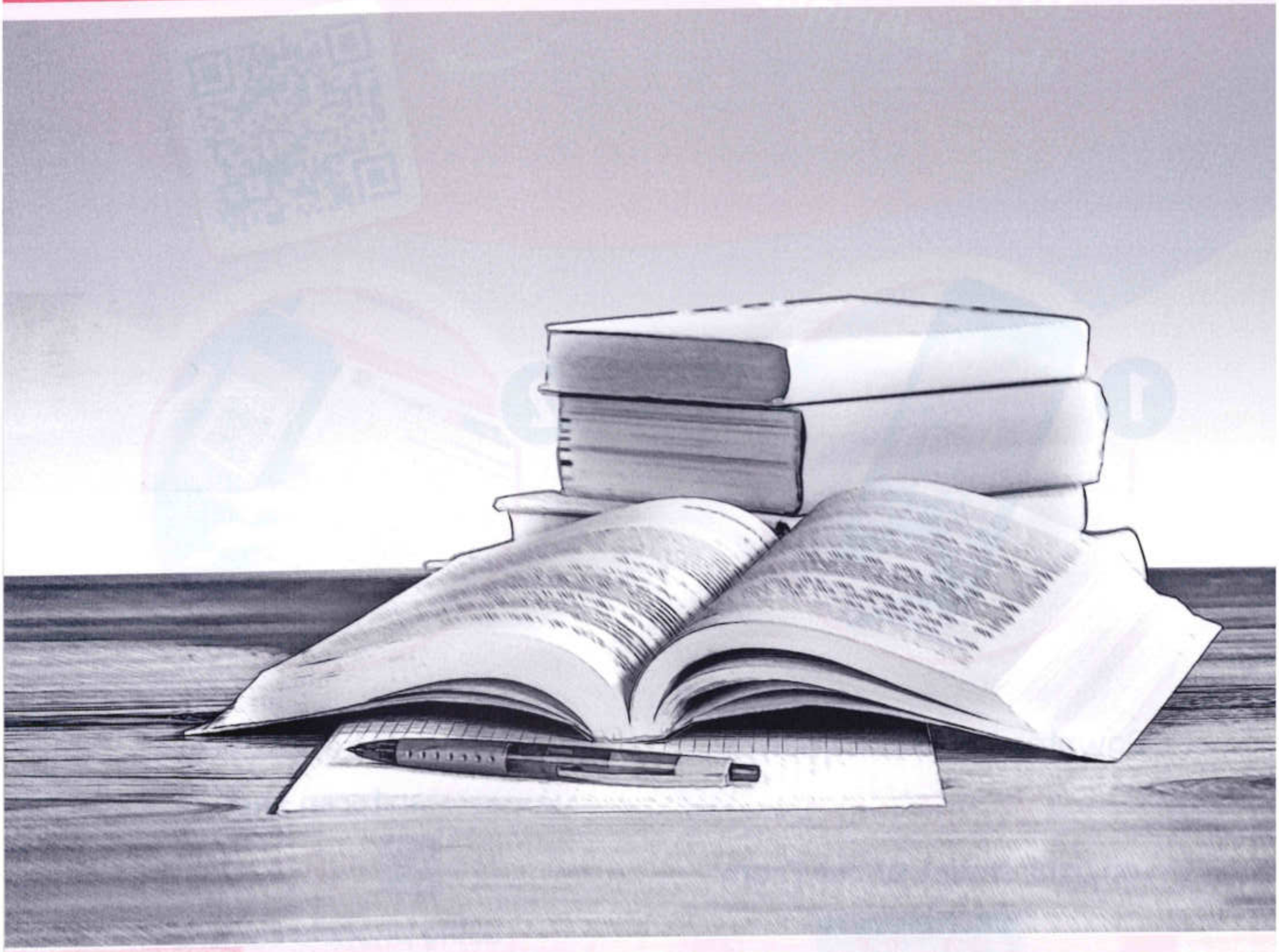


FIRST TERM

2 SEC.

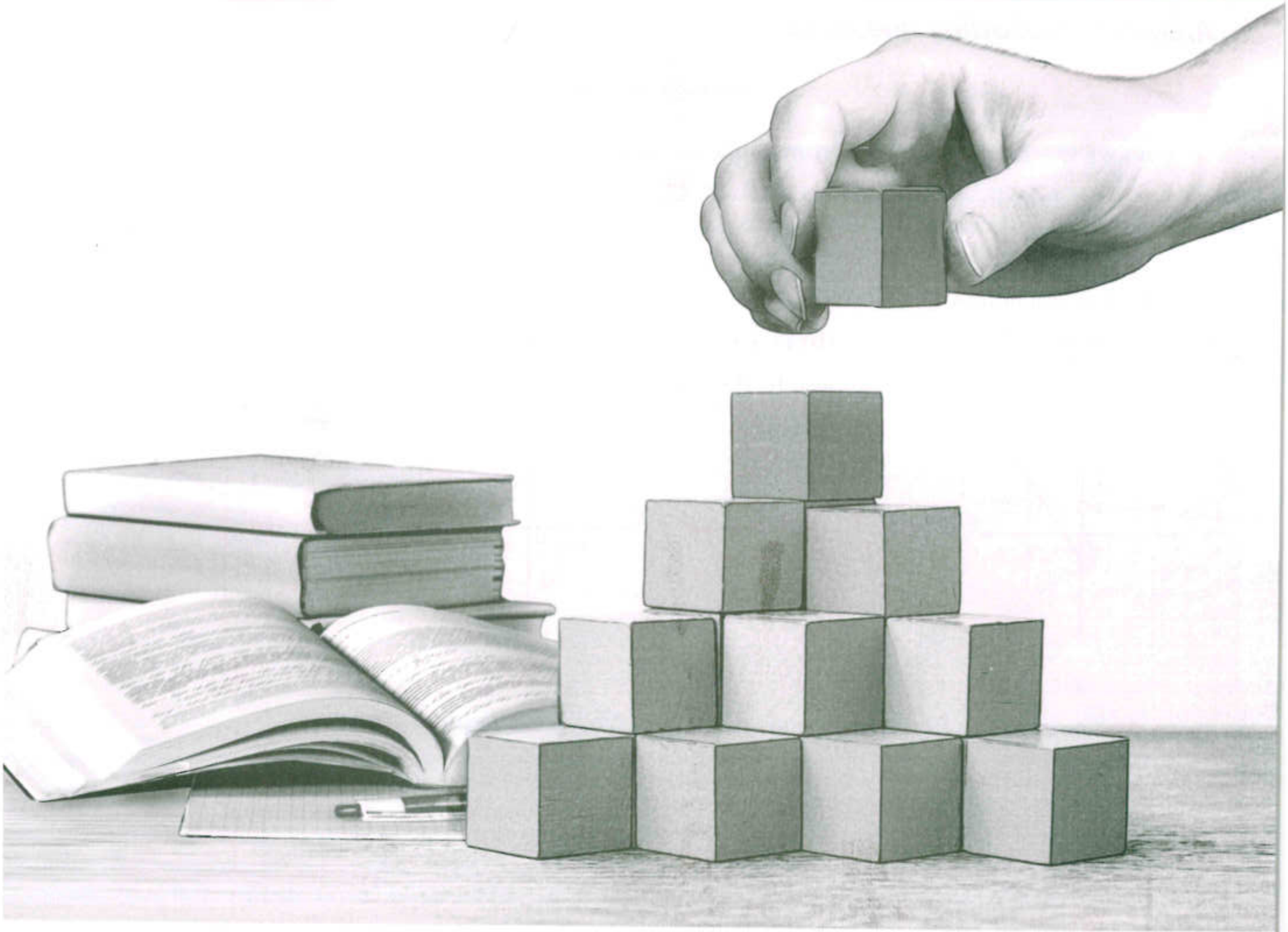
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CONTENTS



- Accumulative quizzes.
- School book examinations.
- Final examinations.
- Guide answers.

Accumulative quizzes



- **First :** Accumulative quizzes on algebra.
- **Second :** Accumulative quizzes on calculs.
- **Third :** Accumulative quizzes on trigonometry.

First | Accumulative quizzes on algebra

Total mark

10

Quiz 1 on lesson 1 – unit 1

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

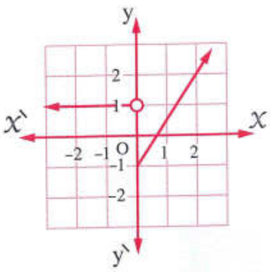
(1) The domain of the function $f : f(x) = \sqrt{x-2}$ is ....

(a) $[2 , \infty[$ (b) $]2 , \infty[$ (c) $]-\infty , 2]$ (d) $]-\infty , 2[$

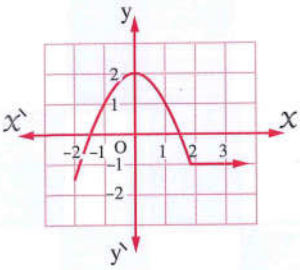
(2) The domain of the function $f : f(x) = \sqrt[3]{x-3}$ is ....

(a) $[3 , \infty[$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $\mathbb{R}$

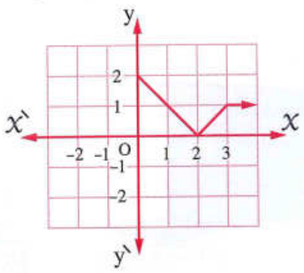
(3) Which of the following graphs does not represent a function in x ?



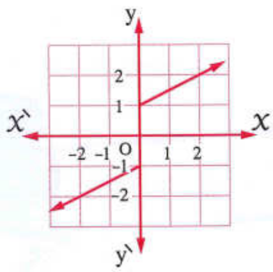
(a)



(b)



(c)



(d)

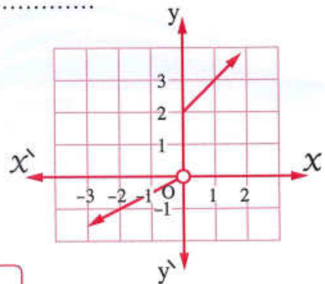
(4) The opposite figure represents a function whose domain is ....

(a) $\mathbb{R}$

(b) $\mathbb{R} - [0 , 2[$

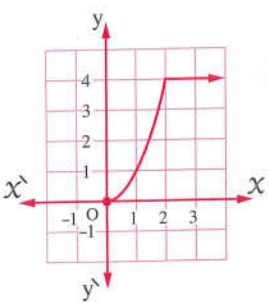
(c) $\mathbb{R} - \{0\}$

(d) $\mathbb{R} -]0 , 2]$

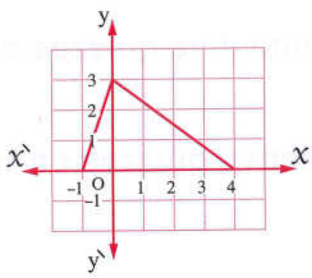


Second question *6 marks* **2 marks for each item**

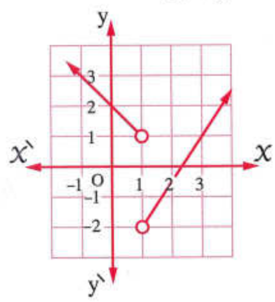
Discuss the monotony of each of the functions represented by the following figures :



(1)



(2)



(3)

Total mark

…… / 10

Quiz 2 till lesson 2 – unit 1

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

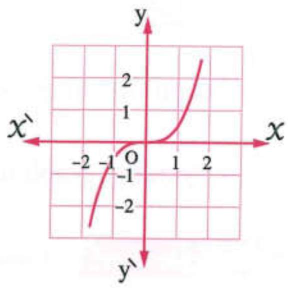
(1) The domain of the function $f : f(x) = \frac{1}{x}$ is ……………

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{1\}$ (d) $\{0\}$

(2) Which of the following functions is even ?

(a) $\sin x$ (b) $x \cos x$ (c) $x \sin x$ (d) $\tan x$

(3) The opposite figure represents a function which is ……………



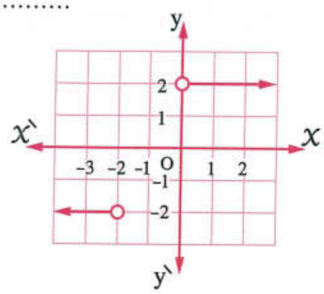
(a) even.

(b) odd.

(c) neither even nor odd.

(d) symmetric about y-axis.

(4) The opposite figure represents a function whose range is ……………



(a) $\mathbb{R}$

(b) $\mathbb{R} - [-2 , 0]$

(c) $]-2 , 2[$

(d) $\{-2 , 2\}$

Second question *6 marks* **2 marks for each item**

Determine the type of the functions defined by the following rules whether they are odd , even or otherwise :

(1) $f(x) = x^4 + x^2$

(2) $f(x) = x^2 \sin x$

(3) $f(x) = \frac{x^3 + 2}{x - 3}$

Total mark

10

Quiz 3 till lesson 3 – unit 1

Answer the following questions :

First question ***4 marks***

Graph the curve of the function $f : f(X) = \begin{cases} |X| & , \quad X \le 0 \\ X^2 & , \quad X > 0 \end{cases}$

, from the graph deduce the range , determine its type whether it is even , odd or otherwise and discuss its monotony.

Second question ***3 marks***

Graph the function $f : f(X) = \begin{cases} -X & , \quad -2 \le X < 0 \\ X & , \quad 0 < X < 2 \end{cases}$

, from the graph deduce the range and discuss its monotony.

Third question ***3 marks***

Graph the function $f : f(X) = \begin{cases} X - 1 & , \quad 2 < X \le 4 \\ -1 & , \quad -2 \le X \le 2 \end{cases}$

, from the graph deduce the range.

Total mark

...... / 10

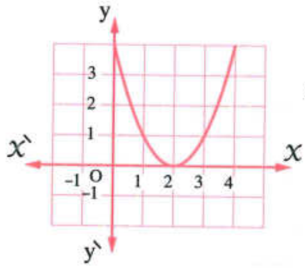
Quiz 4 till lesson 4 – unit 1

Answer the following questions :

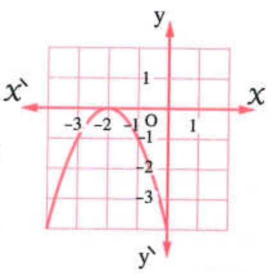
First question ***4 marks*** **1 mark for each item**

Choose the correct answer from those given :

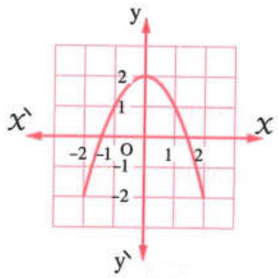
(1) If f is a function where $f(x) = x^2 + 2$, then the graph which represents f is ....



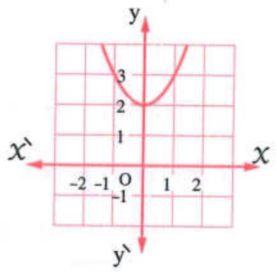
(a)



(b)



(c)



(d)

(2) The range of the function f where $f(x) = \begin{cases} 0 & \text{at } x \leq 0 \\ -1 & \text{at } x > 0 \end{cases}$ is ....

(a) $\{0\}$ (b) $\{-1\}$

(c) $\mathbb{R}$ (d) $\{0, -1\}$

(3) The symmetric point of the function curve of f where $f(x) = x^3$ is ....

(a) $(1, 1)$ (b) $(0, 0)$ (c) $(1, 0)$ (d) $(0, 1)$

(4) The range of the function $f : f(x) = \frac{1}{x-2} - 1$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{-1\}$ (d) $\mathbb{R}$

Second question ***6 marks***

Use the curve of the function $f : f(x) = x^3$ to represent the function $g : g(x) = (x-3)^3$, show the domain of g and its range, discuss its monotony and determine its type whether it is even, odd or otherwise.

Total mark

10

Quiz 5 till lesson 5 – unit 1

Answer the following questions :

First question 4 marks 1 mark for each item

Choose the correct answer from those given :

(1) If $f(X) = 5$, then $f(7) =$ ....

(a) 1 (b) 5 (c) 7 (d) 35

(2) The vertex of the function curve of f where $f(X) = X^2 + 1$ is ....

(a) (1 , 0) (b) (– 1 , 0) (c) (0 , 1) (d) (0 , – 1)

(3) The curve of the function g : g $(X) = X^3 + 2$ is the same as the curve of the function $f : f(X) = X^3$ by translation 2 units in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

(4) The solution set in $\mathbb{R}$ of the equation : $|X| + 1 = 0$ is ....

(a) $\{1\}$ (b) $\{-1\}$ (c) $\{1 , -1\}$ (d) $\varnothing$

Second question 2 marks

Find in $\mathbb{R}$ the solution set of the equation : $\sqrt{X^2 - 2X + 1} = 8$

Third question 4 marks 1 mark for each item

In the opposite figure :

The curve of the function $f : f(X) = |X|$ is drawn , then it is translated in the two directions of coordinate axes ,

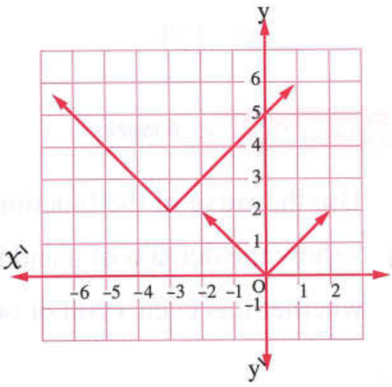
write :

(1) The rule of the resulted function.

(2) The coordinates of its vertex.

(3) Its range and deduce its monotony.

(4) The equation of its symmetric axis.



Total mark

10

Quiz 6 till lesson 6 – unit 1

Answer the following questions :

First question *2 marks* $\frac{1}{2}$ mark for each item

Choose the correct answer from those given :

(1) The solution set of the inequality : $|X| - 1 >$ zero in $\mathbb{R}$ is ....

(a) $\mathbb{R} - [-1 , 1]$ (b) $]-1 , 1[$ (c) $\mathbb{R} -]-1 , 1[$ (d) $[-1 , 1]$

(2) If $f(X) = 2$, then $f(2X) =$ ....

(a) 2 (b) 4 (c) zero (d) 1

(3) If the function $f : f(X) = \frac{1}{X}$, then the symmetric point of the function $g : g(X) = f(X + 1)$ is ....

(a) $(1 , 0)$ (b) $(0 , 1)$ (c) $(-1 , 0)$ (d) $(-1 , 1)$

(4) The range of the function f where $f(X) = |X|$ is ....

(a) $[0 , \infty[$ (b) $]0 , \infty[$ (c) $]-\infty , 0[$ (d) $]-\infty , 0]$

Second question *4 marks* 2 marks for each item

Find in $\mathbb{R}$, the solution set of each of the following :

(1) $|2X - 3| = |X + 1|$

(2) $|3X - 2| \leq 5$

Third question *4 marks*

Graph the curve of the function f where $f(X) = |X - 3|$ and deduce from the graph the range of the function , its monotony and whether it is even , odd or otherwise.

Total mark

......

10

Quiz 7 till lesson 1 – unit 2

Answer the following questions :

First question ***6 marks*** **1 mark for each item**

Choose the correct answer from those given :

(1) The number of roots of the equation : $X^7 = -128$ equals ....

(a) 1 (b) 7 (c) 2 (d) – 2

(2) The range of the function $f : f(X) = \begin{cases} 0 & \text{at} \quad X \leq 0 \\ -1 & \text{at} \quad X > 0 \end{cases}$ is ....

(a) $\{0\}$ (b) $\{-1\}$ (c) $\mathbb{R}$ (d) $\{0 , -1\}$

(3) The solution set of the inequality : $|X| \leq 1$ in $\mathbb{R}$ is ....

(a) $]-\infty , 1]$ (b) $]-1 , 1]$ (c) $[-1 , 1]$ (d) $]-1 , 1[$

(4) The curve of the function g : $g(X) = (X + 4)^2$ is the same as the curve of the function $f : f(X) = X^2$ by translation 4 units in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overleftarrow{Oy}$ (c) $\overleftarrow{OX}$ (d) $\overrightarrow{Oy}$

(5) The solution set of the equation : $X^{\frac{2}{3}} = 25$ in $\mathbb{R}$ is ....

(a) $\{5\}$ (b) $\{5 , -5\}$ (c) $\{125\}$ (d) $\{125 , -125\}$

(6) If $7^{X+1} = 3^{2X+2}$, then $X =$ ....

(a) – 1 (b) 1 (c) 4 (d) zero

Second question ***4 marks*** **2 marks for each item**

Find in $\mathbb{R}$ the solution set of each of the following :

(1) $(X + 1)^{\frac{3}{4}} = 8$

(2) $\sqrt{X^2 - 2X + 1} = 1$

Total mark

.... / 10

Quiz 8 till lesson 2 – unit 2

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) If $f(X) = 2^X$, then $f(-1) =$ ....

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(2) The equation of the symmetric axis of the function f where $f(X) = (X-3)^2 + 2$ is ....

(a) $X = 2$ (b) $X = 3$ (c) $y = 2$ (d) $y = 3$

(3) The function f where $f(X) = a^X$ is decreasing on its domain $\mathbb{R}$ at ....

(a) $a = 1$ (b) $a > 1$ (c) $0 < a < 1$ (d) $a = -1$

(4) $\frac{3^{2X} \times 2^{2X}}{6^{2X-1}} =$ ....

(a) $\frac{1}{6}$ (b) 1 (c) 6 (d) 36

Second question *3 marks*

Graph the curves of the two functions f, g where $f(X) = X + 1$, $g(X) = 1 - X$, from the graph find the area of the triangle bounded by the two intersecting straight lines and X-axis.

Third question *3 marks*

The price of merchandise is increasing at rate 3 % per annum. Given that the original price is 1000 pounds, how much is the price after 3 years ?

Total mark

...... / 10

Quiz 9 till lesson 3 – unit 2

Answer the following questions :

First question *4 marks* 1 mark for each item

Choose the correct answer from those given :

(1) If $\log_2 X = 3$, then $X =$ ....

(a) 2 (b) 3 (c) 8 (d) 9

(2) If $5^{X+1} = 7^{X+1}$, then $3^{X+1} =$ ....

(a) zero (b) 1 (c) 2 (d) 3

(3) The symmetric point of the curve of the function $f : f(X) = \frac{1}{X-3} + 4$ is ....

(a) (4 , 3) (b) (3 , – 4) (c) (– 3 , – 4) (d) (3 , 4)

(4) The solution set of the equation : $\log_3 |X| = 1$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $\{-3\}$ (c) $\{3, -3\}$ (d) $\{1, -1\}$

Second question *3 marks*

If $f(X) = X^2 |X|$, determine the type of the function whether it is even , odd or otherwise , then find in $\mathbb{R}$ the solution set of the equation : $f(X) = 1$

Third question *3 marks*

If $f(X) = 7^X$, **find the value of X such that :** $f(2X - 1) + f(2X + 1) = \frac{50}{49}$

Total mark

....

10

Quiz 10 till lesson 4 – unit 2

Answer the following questions :

First question *2 marks* $\frac{1}{2}$ mark for each item

Choose the correct answer from those given :

(1) The value of x which satisfies the equation : $\log_2 x = -3$ is $x =$ ....

(a) $\frac{1}{8}$ (b) $\sqrt{3}$ (c) 8 (d) 9

(2) If $\log 3 = x$, $\log 7 = y$, then $\log 21 =$ ....

(a) $x y$ (b) $\frac{x}{y}$ (c) $x + y$ (d) $x - y$

(3) $\log_{abc} a + \log_{abc} b + \log_{abc} c =$ ....

(a) a (b) b (c) c (d) 1

(4) The function f where $f(x) = \frac{2|x|}{x}$ is equivalent to $f : f(x) =$ ....

(a) $\begin{cases} 2 & \text{at} \quad x > 0 \\ -2 & \text{at} \quad x < 0 \end{cases}$ (b) 2

(c) -2 (d) $\begin{cases} 2 & \text{at} \quad x \geq 0 \\ -2 & \text{at} \quad x < 0 \end{cases}$

Second question *6 marks* $1\frac{1}{2}$ mark for each item

Find in $\mathbb{R}$ the solution set of each of the following :

(1) $\log_x (x + 2) = 2$

(2) $2^{x+1} = 5$

(3) $3^{x+2} - 3^{x+1} = 18$

(4) $|2x - 3| \leq 7$

Third question *2 marks*

Use the curve of the function f where $f(x) = x^2$ to represent the function g where $g(x) = f(x) + 2$, from the graph find the range of g and prove that it is even.

Second Accumulative quizzes on calculus

Total mark

....

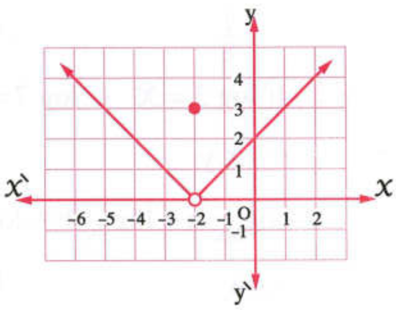
10

Quiz 1 on lesson 1 – unit 3

Answer the following questions :

First question *4 marks* **1 mark for each item**

From the opposite graph , complete each of the following :



(1) $\lim_{x \to -2} f(x) =$ ....

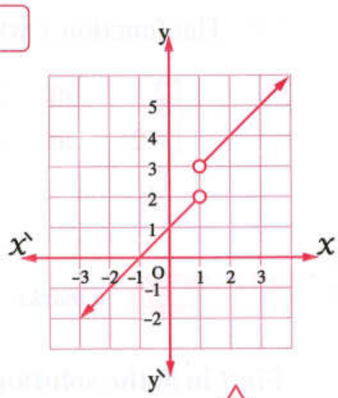
(2) $\lim_{x \to 0} f(x) =$ ....

(3) $f(-2) =$ ....

(4) $f(0) =$ ....

Second question *6 marks* **2 marks for each item**

Choose the correct answer from those given :

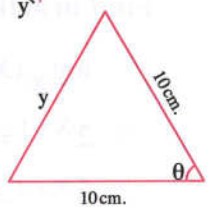


(1) The opposite graph represents the curve of the function f , then $\lim_{x \to 1} f(x) =$ ....

(a) 2 (b) 3

(c) 1 (d) not exist.

(2) **In the opposite figure :**

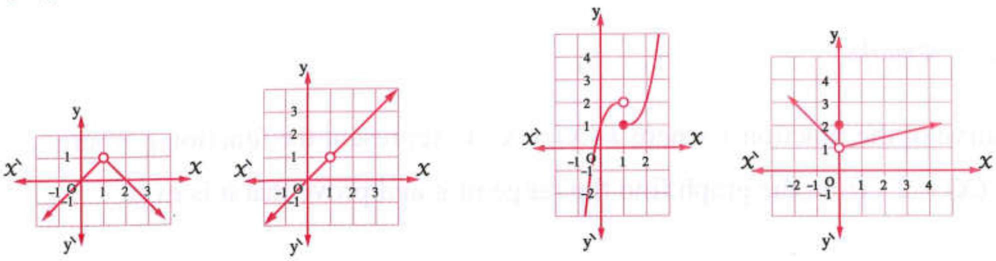


When $\theta \longrightarrow \frac{\pi}{2}$

, then y $\longrightarrow$ .... cm.

(a) 2 (b) 5 (c) 10 (d) $10\sqrt{2}$

(3) Which of the following functions doesn't have a limit at $x = 1$?



(a) (b) (c) (d)

Total mark

....
10

Quiz 2 till lesson 2 – unit 3

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) $\lim_{x \to 1} \left(\frac{3}{4}\right) = \ldots\ldots$

(a) 3 (b) 4 (c) $\frac{3}{4}$ (d) 1

(2) $\lim_{x \to 0} (2x^2 + 3) = \ldots\ldots$

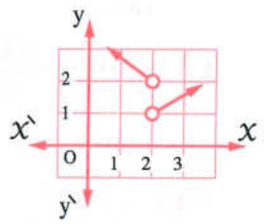
(a) 2 (b) 3 (c) 5 (d) 7

(3) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \ldots\ldots$

(a) 1 (b) 2 (c) -1 (d) not exist.

(4) **From the opposite figure :**

$\lim_{x \to 2} f(x) = \ldots\ldots$



(a) 1 (b) zero

(c) 2 (d) not exist

Second question *4 marks* **2 marks for each item**

Find each of the following :

(1) $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4}$

(2) $\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3}$

Third question *2 marks*

If $\lim_{x \to 2} \frac{a}{x + 1} = 4$, find the value of a

Total mark

....

10

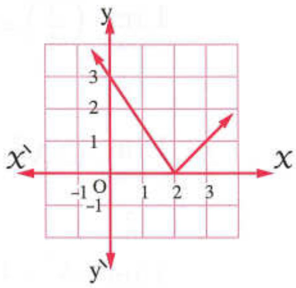
Quiz 3 till lesson 3 – unit 3

Answer the following questions :

First question *2 marks* $\frac{1}{2}$ mark for each item

Choose the correct answer from those given :

(1) The opposite figure represents the curve of the function f, then $\lim\limits_{x \to 2} f(x) =$ ....



(a) 2 (b) 3

(c) zero (d) not exist.

(2) $\lim\limits_{x \to a} \frac{x^n - a^n}{x^m - a^m} =$ ....

(a) $\frac{m}{n}$ (b) $\frac{m}{n}(a)^{m-n}$ (c) $\frac{n}{m}(a)^{m-n}$ (d) $\frac{n}{m}(a)^{n-m}$

(3) $\lim\limits_{x \to -2} \frac{3x^2 - 12}{x + 2} =$ ....

(a) 18 (b) – 3 (c) 12 (d) – 12

(4) $\lim\limits_{x \to 1} \frac{x^5 - 1}{x - 1} =$ ....

(a) 5 (b) 1 (c) 4 (d) 20

Second question *8 marks* 2 marks for each item

Find each of the following :

(1) $\lim\limits_{x \to 1} \frac{x^3 - 2x + 1}{x^2 - 1}$

(2) $\lim\limits_{x \to 2} \frac{x^5 - 32}{x^2 - 4}$

(3) $\lim\limits_{x \to 0} \frac{(x+1)^5 - 1}{x}$

(4) $\lim\limits_{x \to 6} \frac{x - 6}{\sqrt{x - 2} - 2}$

Total mark .... / 10

Quiz 4 till lesson 4 – unit 3

Answer the following questions :

First question *2 marks* $\frac{1}{2}$ mark for each item

Choose the correct answer from those given :

(1) $\lim\limits_{x \to \infty} \frac{\sqrt{x^2}}{x} = \text{...............}$

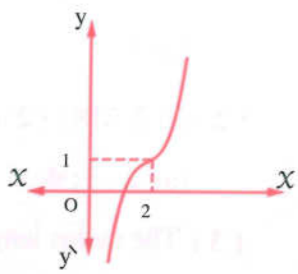
(a) zero (b) 1 (c) 2 (d) – 1

(2) From the opposite figure :

$\lim\limits_{x \to 2} f(x) = \text{...............}$

(a) 2 (b) 1

(c) zero (d) 3



(3) If $\lim\limits_{x \to \infty} \frac{a\,x + 6}{x - 5} = 3$, then a = ... where $a \in \mathbb{R}$

(a) 1 (b) 6 (c) zero (d) 3

(4) $\lim\limits_{x \to 0} \frac{\sqrt{x+1} - 1}{x} = \text{...............}$

(a) zero (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) not exist.

Second question *8 marks* 2 marks for each item

Find each of the following :

(1) $\lim\limits_{x \to \infty} \frac{5\,x^2 + 5\,x + 1}{3\,x^2 - 7}$

(2) $\lim\limits_{x \to 1} \frac{(x + 1)^5 - 32}{x - 1}$

(3) $\lim\limits_{x \to \infty} \left(\sqrt{4\,x^2 + 5\,x} - 2\,x\right)$

(4) $\lim\limits_{x \to 1} \frac{x^3 + x^2 - 2}{x - 1}$

Third Accumulative quizzes on trigonometry

Quiz 1 on lesson 1 – unit 4

Total mark
....
10

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) In Δ ABC , m (∠ A) = 30° , a = 6 cm. , then $\frac{b}{\sin B}$ = ... cm.

(a) 3 (b) 6 (c) $\frac{1}{5}$ (d) 12

(2) In Δ ABC , 2 sin A = 3 sin B = 4 sin C , then a : b : c = ...

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 4 : 6 (d) 6 : 4 : 3

(3) The radius length of the circumcircle of Δ ABC in which m (∠ A) = 30° , a = 10 cm. is .... cm.

(a) 5 (b) 10 (c) 20 (d) 40

(4) In Δ ABC , 2 r sin A = ...

(a) a (b) b (c) c (d) area of Δ ABC

Second question *3 marks*

In Δ ABC , m (∠ A) = 23° , m (∠ B) = 67° , and the radius length of its circumcircle = 10 cm. , find each of a , b to the nearest cm.

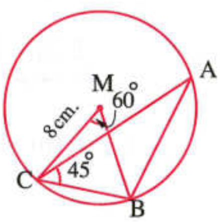
Third question *3 marks*

In the opposite figure :

M is the centre of the circle whose radius length is 8 cm. ,

m (∠ BMC) = 60° , m (∠ ACB) = 45°

Find : the length of each of $\overline{AB}$, $\overline{AC}$



Total mark

...... / 10

Quiz 2 till lesson 2 – unit 4

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) ABC is an equilateral triangle of side length $10\sqrt{3}$ cm. , then the diameter length of its circumcircle equals ....

(a) 5 cm. (b) 20 cm. (c) 15 cm. (d) 10 cm.

(2) In any triangle ABC , $\frac{a^2 + b^2 - c^2}{2\,a\,b} =$ ....

(a) sin A (b) cos A (c) cos C (d) sin C

(3) In Δ ABC , a : b : c = 3 : 4 : 5 , then m ($\angle$ C) = ...

(a) 45° (b) 60° (c) 90° (d) 120°

(4) In Δ ABC , b = c , then cos C = ...

(a) $\frac{a}{2\,b}$ (b) $\frac{b}{2\,c}$ (c) $\frac{2\,b}{c}$ (d) $\frac{2\,b}{a}$

Second question *3 marks*

Find the measure of the greatest angle in Δ ABC in which

a = 7 cm. , b = 5 cm. , c = 3 cm.

Third question *3 marks*

ABCD is a parallelogram , AC = 10 cm. , BD = 8 cm. , $\overline{AC}$ and $\overline{BD}$ intersect at M and m ($\angle$ AMB) = 70° Find the perimeter of the parallelogram.

Total mark

10

Quiz 3 till lesson 3 – unit 4

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) In Δ ABC , a = b = 8 cm. , the perimeter of Δ ABC = 26 cm. , then m ($\angle$ C) ≃ ....

(a) 35.3° (b) 52.3° (c) 77.4° (d) 108°

(2) In Δ LMN , $\frac{\ell}{\sin L}$ = ...

(a) $\frac{m}{\sin N}$ (b) $\frac{n}{\sin M}$ (c) $\frac{m + n}{\sin N + \sin M}$ (d) 3 r

(3) In Δ ABC , the expression $\frac{a^2 + b^2 - c^2}{2ab}$ equals zero if ....

(a) m ($\angle$ A) = 60° (b) m ($\angle$ B) = 90°

(c) m ($\angle$ C) = 120° (d) m ($\angle$ A) + m ($\angle$ B) = 90°

(4) In Δ ABC , cos A = ...

(a) – (cos B + cos C) (b) cos B – cos C

(c) cos (B + C) (d) – cos (B + C)

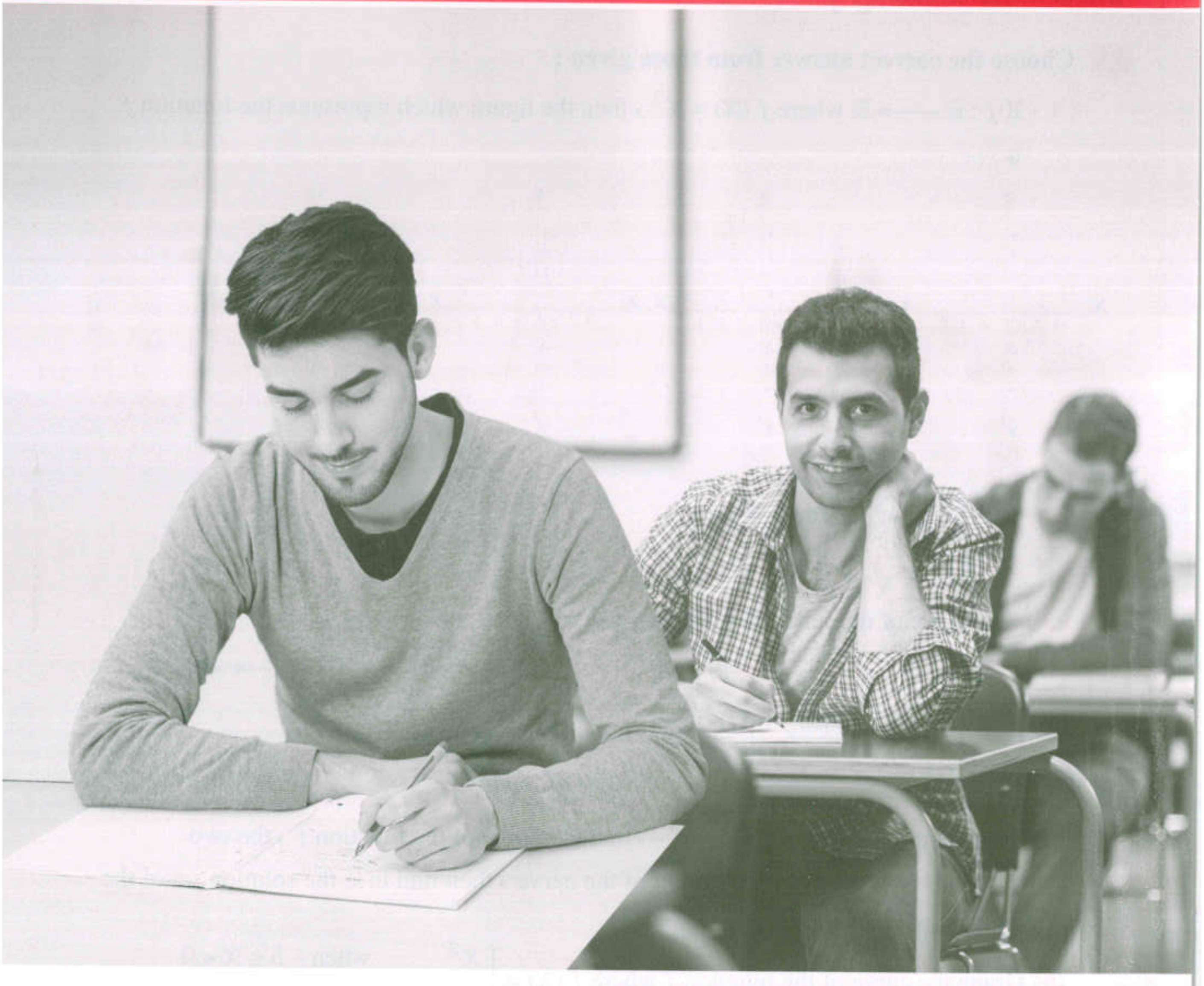
Second question *3 marks*

Solve the triangle ABC in which a = 20 cm. , m ($\angle$ B) = 67° 37̀ , m ($\angle$ C) = 42° 23̀

Third question *3 marks*

ABCD is a quadrilateral , AB = 9 cm. , BC = 5 cm. , CD = 8 cm. , DA = 9 cm. , AC = 11 cm. , **prove that :** ABCD is a cyclic quadrilateral.

School book examinations



▶ **First :** School book examinations in algebra.

▶ **Second :** School book examinations in calculus and trigonometry.

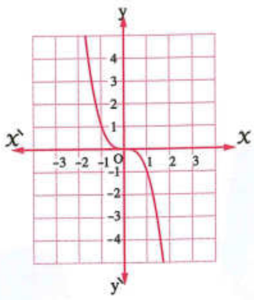
First — School book examinations in algebra

Model 1

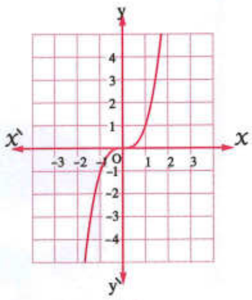
Answer the following questions :

1 **Choose the correct answer from those given :**

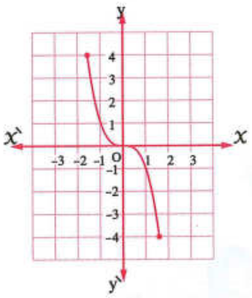
(1) If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^3$, then the figure which represents the function f is ....



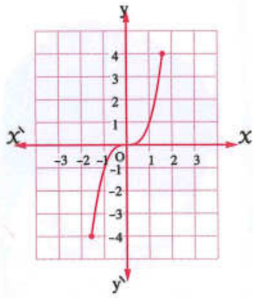
(a)



(b)



(c)



(d)

(2) If $5^{x-3} = 4^{3-x}$, then $x =$ ....

(a) $\frac{5}{4}$ (b) 3 (c) $\frac{4}{5}$ (d) 0

(3) The range of the function f where $f(x) = |x|$ is ....

(a) $[0 , \infty[$ (b) $]0 , \infty[$ (c) $]-\infty , 0]$ (d) $]-\infty , 0[$

(4) If $f(x) = 5^x$, then $f(-2) =$ ....

(a) -2 (b) 5 (c) $\frac{1}{25}$ (d) $\frac{1}{5}$

---

2 (a) If the function f where $f(x) = \frac{1}{x}$, find the range of the function f , the two coordinates of the symmetry point of the curve , then find in $\mathbb{R}$ the solution set of the equation $f\left(\frac{1}{x}\right) = 4$

(b) Graph the curve of the function f where $f(x) = \begin{cases} x^2 & \text{when } -5 \le x < 2 \\ 6 - x & \text{when } 2 \le x \le 8 \end{cases}$

From the graph , determine the range of the function and investigate its monotony.

---

3 (a) Graph the curve of the function f where $f(x) = |x - 3|$, deduce the range and monotony of the function and tell whether it is even , odd or otherwise.

(b) **Find the solution set for each of the following in $\mathbb{R}$:**

(1) $|x - 3| \ge 5$ (2) $|x - 3| = 0$

4 (a) **Find the solution set for each of the following in $\mathbb{R}$:**

(1) $\log X = \log 3 + \log 10$ (2) $9^{X} - 3 \times 3^{X} = 0$

(b) **Reduce :**

(1) $\frac{4^{2n+1} \times 2^{1-n}}{8^{n+2}}$ (2) $\log_6 54 - \log_6 9$

5 (a) **Without using the calculator , find in the simplest form the value of :**

$$\frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30}$$

(b) **Tell whether each of the functions defined by the following rules is odd or even :**

(1) $f(X) = X + \sin X$ (2) $f(X) = X^3 - 2X^2$

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

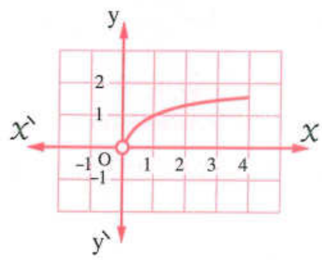
(1) The solution set of the inequality $|X| - 1 >$ zero in $\mathbb{R}$ is ....

(a) $\mathbb{R} - [-1, 1]$ (b) $]-1, 1[$ (c) $\mathbb{R} -]-1, 1[$ (d) $[-1, 1]$

(2) If $4 = \log_2 X$, then the equivalent exponential form is ....

(a) $X^2 = 4$ (b) $X^4 = 2$ (c) $X = 16$ (d) $X = 8$

(3) The domain of the function in the figure opposite is ....



(a) $[0, \infty[$ (b) $]0, \infty[$

(c) $[0, 1]$ (d) $]0, 2[$

(4) Which of the following functions represents an increasing exponential function on its domain $\mathbb{R}$?

(a) $y = 3(1.05)^{X}$ (b) $y = 3\left(\frac{1}{1.05}\right)^{X}$ (c) $y = 3 + (0.5)^{X}$ (d) $y = (0.05)^{X}$

2 (a) If $f(x) = |x - 3| + |x + 2|$, **prove that :** $f(2) = f(-1)$

(b) **Use the curve of the function f where $f(x) = x^2$ to graph the following functions :**

(1) $f_1 : f_1(x) = x^2 - 3$ (2) $f_2 : f_2(x) = (x + 1)^2$

---

3 (a) **Find the solution set of each of the following equations in $\mathbb{R}$:**

(1) $\log_2 x + \log_2 (x + 1) = 1$ (2) $3^x + 3^{1+x} = 36$

(b) (1) **Find the solution set of the following equation in $\mathbb{R}$:** $4^x + 2^{x+1} = 8$

(2) **Without using the calculator , prove that :** $\log_6 8 + \log_6 27 = \log_3 27$

---

4 (a) **Find the solution set of the inequality :** $|x| + 1 < 2$ in $\mathbb{R}$

(b) Graph the function f where $f(x) = \frac{1}{x} - 1$ From the graph , find the domain and the range , then investigate its monotony and tell whether it is even , odd or otherwise.

---

5 (a) Graph the curve of the function f where $f(x) = \begin{cases} x + 1 & , \ -1 \le x < 2 \\ 5 - x & , \ 2 \le x \le 5 \end{cases}$

From the graph , deduce the range of this function , investigate its monotony and tell whether it is even , odd or otherwise.

(b) **If $f(x) = 2^{x+1}$, find in $\mathbb{R}$ the solution set of :**

(1) $f(x) = 32$ (2) $f(x - 2) = \frac{1}{8}$

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) In Δ ABC , if a = b = 8 cm. and the perimeter of Δ ABC = 26 cm. , then m ($\angle$C) $\simeq$ ....

(a) 35.3° (b) 52.3° (c) 77.4° (d) 108°

(2) $\lim\limits_{x \to 1} \frac{x^2 - 1}{x - 1} = $ ....

(a) 0 (b) 1 (c) 2 (d) 3

(3) In Δ ABC , if m ($\angle$ A) = 30° and a = 6 cm. , then $\frac{b}{\sin B}$ = ... cm.

(a) 3 (b) 6 (c) $\frac{1}{5}$ (d) 12

(4) $\lim\limits_{x \to 1} \frac{x^5 - 1}{x - 1} = $ ....

(a) 5 (b) 1 (c) 4 (d) 20

---

2 (a) **Find :**

(1) $\lim\limits_{x \to \infty} \frac{5x^4 + 3x^2 - 6}{2x + x^4}$ (2) $\lim\limits_{x \to -2} \frac{x + 2}{x - 3}$

(b) If ABC is a triangle in which $\frac{1}{2} \sin A = \frac{1}{3} \sin B = \frac{1}{4} \sin C$, find the measure of its largest angle.

---

3 (a) **Find :** (1) $\lim\limits_{x \to \infty} \frac{4 - 3x^2}{\sqrt{x^4 + 5}}$ (2) $\lim\limits_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3}$

(b) Find the perimeter of Δ ABC in which a = 8 cm. , b = 6 cm. , m ($\angle$ C) = 48°

---

4 (a) **Find :** (1) $\lim\limits_{x \to 3} \frac{x^2 - 6x + 9}{x - 3}$ (2) $\lim\limits_{x \to 2} \frac{2x^2 - 8}{x - 2}$

(b) **Find the diameter length of the circumcircle of Δ ABC in each of the following cases :**

(1) m ($\angle$ A) = 75° , a = 21 cm.

(2) m ($\angle$ B) = 50° , m ($\angle$ C) = 65° , c − b = 6 cm.

---

5 (a) **Find the value of each of the following :**

(1) $\lim\limits_{x \to 3} \frac{(x - 6)^2 - 9}{x^2 - 9}$ (2) $\lim\limits_{x \to -1} \frac{2x^3 - x^2 - 2x + 1}{x^3 + 1}$

(b) ABC is a triangle in which m ($\angle$ A) = 36° , m ($\angle$ C) = 45° and b = 9 cm.

Find the area of the circumcircle of the triangle.

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) In any triangle LMN , $\frac{\ell}{\sin L}$ = ...

(a) $\frac{m}{\sin N}$ (b) $\frac{n}{\sin M}$ (c) $\frac{m+n}{\sin N + \sin M}$ (d) 3 r

(2) $\lim\limits_{x \to \infty} \frac{\sqrt{4x^2+1}}{x-2}$ = ...

(a) 4 (b) 5 (c) $\frac{5}{-2}$ (d) 2

(3) $\lim\limits_{x \to 0} (2x^2+3)$ = ...

(a) 2 (b) 3 (c) 5 (d) 7

(4) In Δ ABC , if 2 sin A = 3 sin B = 4 sin C , then a : b : c = ...

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 4 : 6 (d) 6 : 4 : 3

2 (a) **Find :**

(1) $\lim\limits_{x \to 2} \frac{x^5-32}{x-2}$ (2) $\lim\limits_{x \to 1} \frac{(x-2)^4-1}{x-1}$

(b) ABCD is a parallelogram in which AB = 7 cm. , the two diagonals $\overline{AC}$ and $\overline{BD}$ form two angles of measurements 65° and 28° with $\overline{AB}$ respectively, find the lengths of $\overline{BD}$ and $\overline{AC}$

3 (a) **Find :**

(1) $\lim\limits_{x \to 3} \frac{x^3-27}{x^2-9}$ (2) $\lim\limits_{x \to \infty} \frac{4x^2+1}{x^2-2}$

(b) ABCD is a quadrilateral in which AB = 9 cm. , BC = 5 cm. , CD = 8 cm. , DA = 9 cm. and AC = 11 cm. , prove that ABCD is a cyclic quadrilateral.

4 (a) **Find :**

(1) $\lim\limits_{x \to 1} \frac{x^2+5x-6}{x^2-1}$ (2) $\lim\limits_{x \to 1} \frac{(x+1)^5-32}{x-1}$

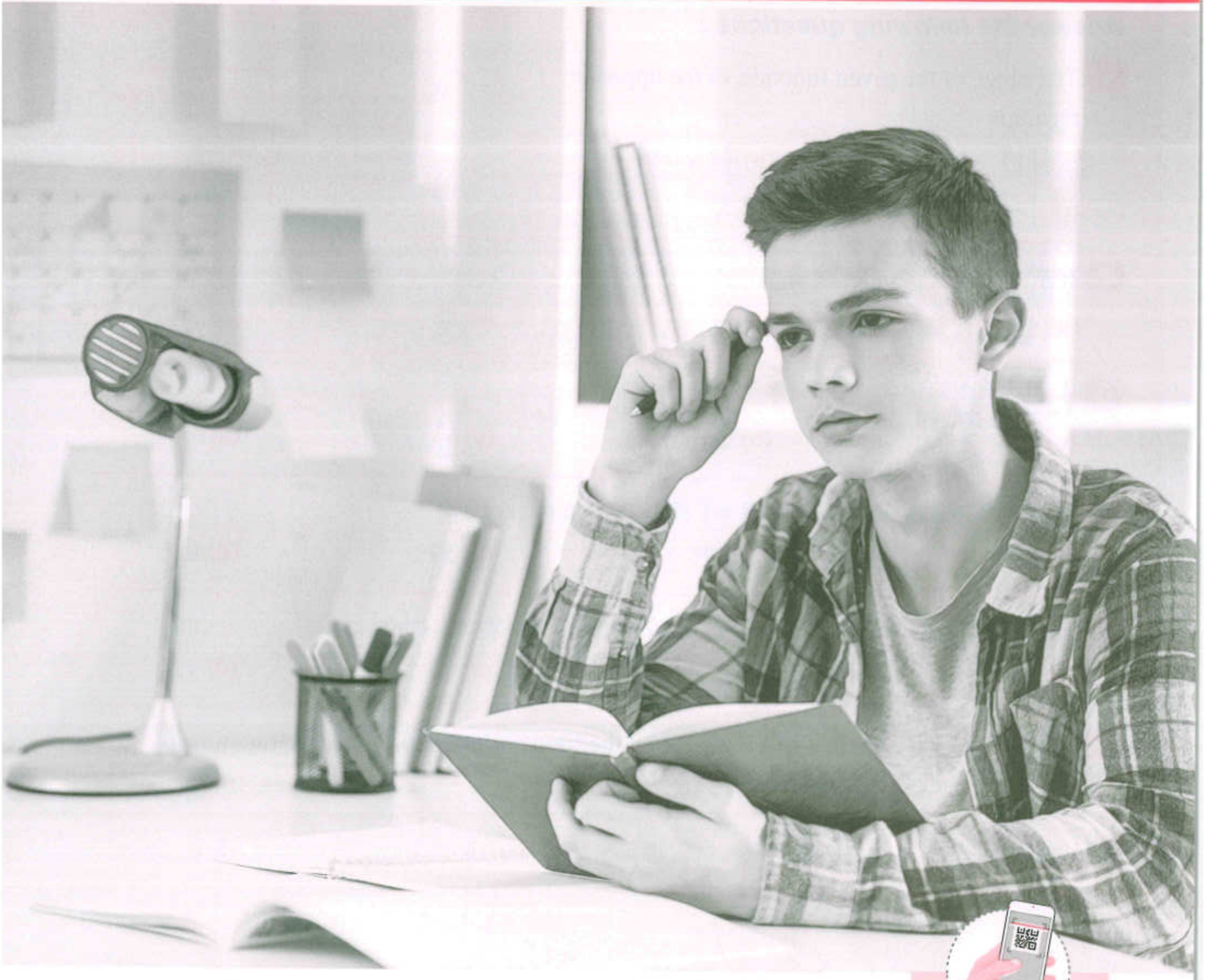
(b) ABC is a triangle in which cos A = $\frac{2}{5}$, b = $2\frac{1}{2}$ cm. and c = 2 cm. Prove that the triangle is isosceles.

5 (a) **Find :**

(1) $\lim\limits_{x \to 1} \frac{x^3-2x+1}{x^2-1}$ (2) $\lim\limits_{x \to 1} \left(\frac{1}{x}+3\right)$

(b) ABC is a triangle in which m (∠ B) = 35° , m (∠ C) = 70° , and the radius length of the circumcircle of the triangle = 16 cm. , find the area and the perimeter of the triangle ABC to the nearest integer.

Final examinations



▶ **First :** **Examination models.**

▶ **Second :** **Multiple choice examinations.**

Scan the QR codes to solve interactive tests

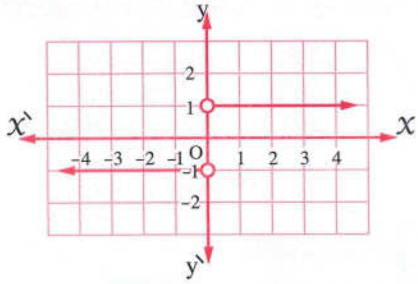
First — Examination models

Model 1

Interactive test 1

Answer the following questions :

1 The range of the given function in the opposite figure is ....



(a) $\{1\}$ (b) $\{1, -1\}$

(c) $\{-1\}$ (d) $\mathbb{R}$

---

2 If $5^{x-3} = 4^{3-x}$, then $x =$ ....

(a) $\frac{5}{4}$ (b) 3 (c) $\frac{4}{5}$ (d) zero

---

3 $\lim\limits_{x \to \infty} \frac{2x+3}{5x^2+4} =$ ....

(a) 2 (b) zero (c) $\frac{3}{4}$ (d) $\frac{2}{5}$

---

4 In Δ ABC, if $4 \sin A = 3 \sin B = 6 \sin C$, then $m(\angle C) \simeq$ ....

(a) 89° (b) 29° (c) 57° (d) 82°

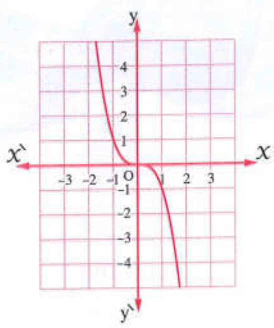
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5 In Δ ABC, $m(\angle C) = 96° 23'$, $a = 7$ cm. , $b = 9$ cm. , then $c \simeq$ .... cm.

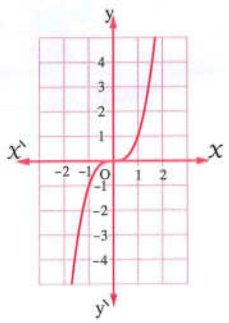
(a) 7 (b) 9 (c) 13 (d) 12

---

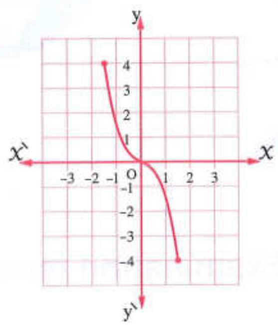
6 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^3$, then the figure which represents the function f is ....



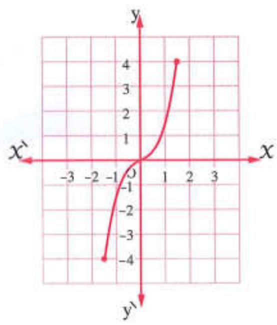
(a)



(b)



(c)



(d)

---

7 The solution set in $\mathbb{R}$ of the equation : $2^{2x} - 12 \times 2^{x} + 2^5 = 0$ equals ....

(a) $\{4, 8\}$ (b) $\{2, 3\}$ (c) $\{16, 2\}$ (d) $\{1, 4\}$

8 Use the curve of the function f where $f(x) = \frac{1}{x}$ to represent the function $g : g(x) = f(x-2) + 2$ and from the graph determine the range and discuss its monotony.

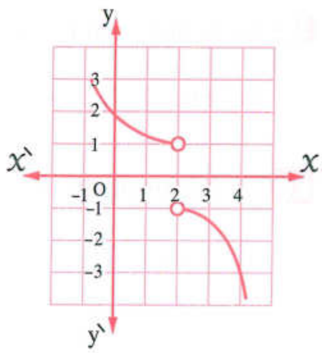
9 $\lim\limits_{x \to 0} \frac{(x+2)^5 - 32}{x} = \dots\dots\dots$

(a) 25 (b) 64 (c) 80 (d) 100

10 The opposite figure represents the curve of the function f, then $\lim\limits_{x \to 2} f(x) = \dots\dots\dots$

(a) 1 (b) − 1

(c) 2 (d) does not exist.



11 Solve the triangle ABC in which a = 21 cm., $\cos B = \frac{3}{5}$, $\tan C = \frac{5}{12}$, approximating the side lengths to the nearest cm.

12 The function $f : f(x) = a^x$ is increasing if $\dots\dots\dots$

(a) a > 0 (b) a > 1 (c) a = 1 (d) 0 < a < 1

13 If $x = 5 + 2\sqrt{6}$, then $\log\left(x + \frac{1}{x}\right) = \dots\dots\dots$

(a) 1 (b) $5 - 2\sqrt{6}$ (c) 10 (d) $5 + 2\sqrt{6}$

14 ABC is an equilateral triangle, its side length = $5\sqrt{3}$ cm., then the diameter length of its circumcircle equals $\dots\dots\dots$ cm.

(a) $5\sqrt{3}$ (b) $10\sqrt{3}$ (c) 10 (d) 5

15 $\log_5 49 \times \log_8 5 \times \log_9 8 \times \log_7 9 = \dots\dots\dots$

(a) log 100 (b) log 7 (c) log 5 (d) log 2

16 If $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(x) = (a+1)x + b - 2$ and $f(x)$ maps each real number to itself, then (a, b) = $\dots\dots\dots$

(a) (0, 3) (b) (0, − 3) (c) (0, 2) (d) (− 1, 2)

17 $\lim\limits_{x \to 1} \frac{x^7 - 1}{x - 1} = \text{..............}$

(a) 35 (b) 7 (c) 42 (d) 1

---

18 The solution set of the equation : $\log_3 x \times \log_2 3 = 5$ in $\mathbb{R}$ is ....

(a) $\{32\}$ (b) $\{5\}$ (c) $\{3\}$ (d) $\{2\}$

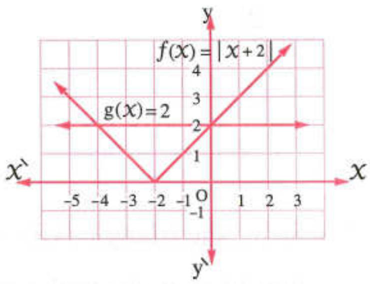
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19 In Δ ABC , m ($\angle$ A) : m ($\angle$ B) : m ($\angle$ C) = 3 : 5 : 4 , then $c^2 : a^2 = \text{..............}$

(a) $\sqrt{6} : 2$ (b) 2 : 3 (c) 4 : 3 (d) 3 : 2

---

20 **In the opposite figure :**



The solution set of the inequality $f(x) < g(x)$ in $\mathbb{R}$ is ....

(a) $\{-4 , 0\}$ (b) $[-4 , 0]$

(c) $\mathbb{R} - [-4 , 0]$ (d) $]-4 , 0[$

---

21 **Find :** $\lim\limits_{x \to \infty} \frac{\sqrt{4x^2 + 1}}{5x + 2}$

---

22 The type of the function $f : f(x) = \frac{\sin x}{x}$ is ....

(a) even. (b) odd.

(c) neither odd nor even. (d) both odd and even.

---

23 $\lim\limits_{x \to \frac{\pi}{4}} \frac{\tan x}{x} = \text{..............}$

(a) $\frac{\pi}{4}$ (b) 1 (c) $\frac{4}{\pi}$ (d) does not exist.

---

24 If $x^{\frac{3}{2}} = 8$, then $x = \text{..............}$

(a) 2 (b) 4 (c) 8 (d) 9

---

25 $\lim\limits_{x \to 2} \frac{x^3 - 7x + 6}{3x^2 - 8x + 4} = \text{..............}$

(a) $\frac{4}{5}$ (b) $\frac{2}{3}$ (c) $\frac{5}{4}$ (d) $\frac{3}{2}$

26 If $\lim\limits_{x \to 1} \frac{2x+a}{x+1} = 5$, then a = ...

(a) 2 (b) 5 (c) 8 (d) 10

---

27 In any triangle XYZ , $x^2 + y^2 - 2xy \cos Z =$ ....

(a) x^2 (b) y^2 (c) z^2 (d) z

---

28 **Find the solution set in ℝ of the inequality :** $\sqrt{4x^2 - 12x + 9} \leq 9$

---

29 The number of possible solutions for the triangle ABC where : m (∠ A) = 60° , b = 3 cm. , a = 5 cm. is ....

(a) 1 (b) 2

(c) zero (d) infinite number.

---

30 If $\left(\frac{1}{2}\right)^{a^2 - a - 2} = 1$ where a > 0 , then a = ...

(a) 1 (b) − 3 (c) 2 (d) 3

---

31 If $f(x) = \frac{\sqrt{x^2 - 2x + 1}}{x - 1}$, then the range of the function f is ....

(a) {1} (b) ℝ (c) [− 1 , 1[(d) {− 1 , 1}

---

32 $\lim\limits_{x \to \infty} \frac{\sqrt{x+5} - \sqrt{5}}{\sqrt{x} - \sqrt{5}} =$ ....

(a) 1 (b) − 1 (c) ∞ (d) − ∞

Model 2

Interactive test 2

Answer the following questions :

1 The range of the function $f : f(x) = |x|$ is ....

(a) [0 , ∞[(b)]0 , ∞[(c)]− ∞ , 0] (d)]− ∞ , 0[

---

2 $\lim\limits_{x \to \infty} \left(\frac{3}{5}\right)^{\frac{1}{x}} =$ ....

(a) 1 (b) − 1 (c) $\frac{3}{5}$ (d) ∞

3 ABCD is a quadrilateral in which AB = 27 cm. , BC = 12 cm. , CD = 8 cm. , DA = 12 cm. , AC = 18 cm. **Prove that :** $\overrightarrow{AC}$ bisects ∠ BAD

4 $\lim_{x \to 1} \frac{x^5 - 1}{x - 1} = \dots\dots$

(a) 5 (b) 1 (c) 4 (d) 20

5 In Δ ABC , $\frac{a}{\sin A} = 6$ cm. , then the radius length of its circumcircle = ... cm.

(a) 2 (b) 3 (c) 5 (d) 6

6 If f is an odd function and $x f(x) + x^3 f(-x) = 2$, then $f(2) = \dots\dots$

(a) 3 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) -3

7 If $x = 5 + 2\sqrt{6}$, find in the simplest form the value of $\log\left(\frac{1}{x} + x\right)$ without using calculator.

8 In Δ XYZ , $\frac{x^2 + y^2 - z^2}{2xy} = \dots\dots$

(a) cos X (b) cos Y (c) cos Z (d) sin Z

9 $\lim_{x \to 1} \frac{x^2 - x^{-2}}{x - x^{-1}} = \dots\dots$

(a) zero (b) 1 (c) 2 (d) -2

10 If $f(x) = 3^x$, then the solution set in $\mathbb{R}$ of the equation $f(2x) - 28 f(x) + f(3) = $ zero equals ....

(a) $\{1 , 27\}$ (b) $\{1 , 3\}$ (c) $\{0 , 3\}$ (d) $\{3\}$

11 The logarithmic form that equivalent to the exponential form : $2^7 = 128$ is ....

(a) $\log_2 128 = 7$ (b) $\log_2 7 = 128$

(c) $\log_7 128 = 2$ (d) $\log_7 2 = 128$

12 The curve of the even function is symmetric about the straight line ....

(a) $y = x$ (b) $\overleftrightarrow{yy}$ (c) $\overleftrightarrow{xx}$ (d) $y = -x$

13 In Δ LMN , $\frac{\sin L}{3} = \frac{2 \sin M}{3} = \frac{\sin N}{4}$, then $\ell : m : n = \dots\dots$

(a) 6 : 8 : 3 (b) 3 : 6 : 8 (c) 8 : 3 : 6 (d) 6 : 3 : 8

14 In Δ ABC , c = 7 cm. , m ($\angle$ A) = 70° , m ($\angle$ B) = 40° , then b $\simeq$ .... cm.

(a) 3.7 (b) 4.8 (c) 8.4 (d) 7.3

---

15 If $\lim\limits_{X \to a} \frac{aX}{3} = 12$, then a = ...

(a) ± 12 (b) ± 6 (c) 3 (d) − 3

---

16 Use the curve of the function $f : f(X) = \frac{1}{X}$ to graph the curve of the function $g : g(X) = \frac{1}{X-2} + 3$ from the graph state the domain and range of g and the monotony and its type whether it is even , odd or otherwise.

---

17 The range of the function $f : f(X) = \frac{X-2}{2-X}$ equals ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\{-1\}$

---

18 If log 3 = X , log 7 = y , then log 21 = ...

(a) Xy (b) X + y (c) X − y (d) $\frac{X}{y}$

---

19 $\log_3 5 \times \log_2 3 \times \log_5 16$ = ...

(a) 30 (b) 15 (c) log 10000 (d) $\log_{30} 240$

---

20 The curve of the function $g : g(X) = X^2 + 4$ is the same as the curve of $f : f(X) = X^2$ by translation 4 units in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

---

21 $\lim\limits_{X \to 5} \frac{\sqrt{X-1} - 2}{X-5}$ = ...

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) 4 (d) $\frac{1}{4}$

---

22 The function f where $f(X) = a^X$ is decreasing on its domain if ....

(a) a = 1 (b) a > 1 (c) 0 < a < 1 (d) a = − 1

---

23 $\lim\limits_{X \to 0} \frac{(4X+1)^9 - 1}{3X}$ = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 9 (d) 12

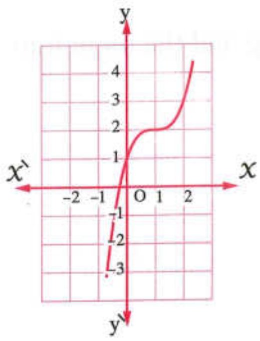
24 $\lim\limits_{x \to \infty} \frac{\sqrt{x^2}}{x} = \ldots\ldots\ldots\ldots$

(a) zero (b) 2 (c) -1 (d) 1

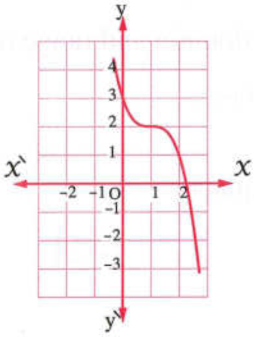
25 The solution set in $\mathbb{R}$ of the equation : $|x-7| = 2$ is ....

(a) $\{9, 5\}$ (b) $\{7, 3\}$ (c) $\varnothing$ (d) $\{3, -3\}$

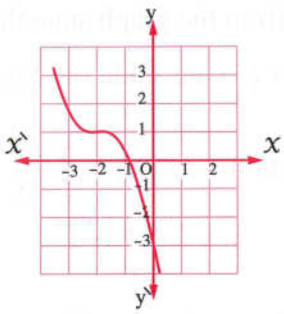
26 If $f(x) = 2 - (x-1)^3$, then the figure that represents the function f is ....



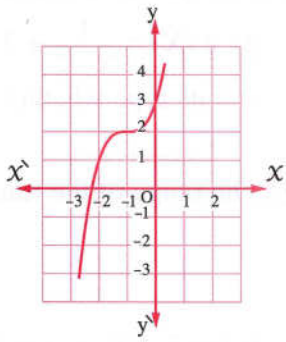
(a)



(b)



(c)



(d)

27 If the perimeter of Δ ABC = 33 cm. , $\sin A + \sin C = \frac{2}{3}$, $\sin B = \frac{1}{4}$
, then AC = ... cm.

(a) 6 (b) 9 (c) 12 (d) 15

28 **Find :** $\lim\limits_{x \to 1} \frac{(x+2)^4 - 81}{x-1}$

29 If $3^{(2x+7)} = 11^x$, then the value of x to the nearest 1 decimal place equals ....

(a) 28.3 (b) 38.3 (c) 3.8 (d) 28.4

30 $\lim\limits_{x \to \infty} (5 + 3x^2 + x) = \ldots\ldots\ldots\ldots$

(a) not exist (b) 5 (c) ∞ (d) 9

31 ABCD is a parallelogram , m ($\angle$ A) = 50° , m ($\angle$ DBC) = 70° , BD = 8 cm.
, then the perimeter of the parallelogram ABCD to the nearest cm. = ... cm.

(a) 38 (b) 30 (c) 19 (d) 48

32 The solution set of the inequality $|x-1| \le 3$ is ....

(a) $[-2, 4]$ (b) $]-2, 4[$ (c) $]-2, 4]$ (d) $\mathbb{R} - [-2, 4]$

Model 3

Interactive test 3

Answer the following questions :

1 If $f(x) = 7^{x+1}$, then the solution set of the equation : $f(2x-1) + f(x-2) = 50$ in $\mathbb{R}$ equals ....

(a) $\{1\}$ (b) $\{1, -1\}$ (c) $\{1, -50\}$ (d) $\{7, -50\}$

---

2 If $\log 3 = x$, $\log 5 = y$, then $\log 15 =$ ....

(a) $x\,y$ (b) $\frac{x}{y}$ (c) $x + y$ (d) $x - y$

---

3 $\lim\limits_{x \to \infty} \frac{5 + x^{-2}}{1 + 3x^{-2}} =$ ....

(a) $\frac{1}{3}$ (b) $\frac{5}{4}$ (c) $\frac{5}{3}$ (d) 5

---

4 If $f(x) = 5^{x}$, then $f(-2) =$ ....

(a) -2 (b) 5 (c) $\frac{1}{25}$ (d) $\frac{1}{5}$

---

5 The domain of the function $f : f(x) = \log_3 (x - 2)$ is $x >$ ....

(a) 3 (b) 5 (c) 1 (d) 2

---

6 $\lim\limits_{x \to 16} \frac{\sqrt[4]{x} - 1}{x - 16} =$ ....

(a) zero (b) $\frac{1}{2}$ (c) 1 (d) does not exist.

---

7 $\lim\limits_{x \to \infty} \frac{2x + 7}{\sqrt{9x^2 + 5}} =$ ....

(a) $\frac{2}{9}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{7}{3}$

---

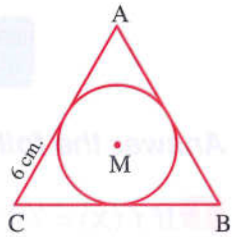
8 $\log 25 + \frac{\log 8 \times \log 16}{\log 64} =$ ....

(a) $\log_2 16$ (b) $\log_5 25$ (c) $\log 4$ (d) $\log 10$

9 **In the opposite figure :**

If the perimeter of Δ ABC = 42 cm. ,

the circle touches the sides of the triangle internally , then : m ($\angle$ B) = ...



(a) $53° \grave{8}$ (b) $67° \grave{23}$ (c) $36° \grave{53}$ (d) $32° \grave{37}$

---

10 $\lim_{x \to 2} \frac{x-2}{x+1} =$ ....

(a) zero (b) 1 (c) 2 (d) ∞

---

11 $\lim_{x \to \infty} \frac{\sqrt{x^2}}{x} =$ ....

(a) zero (b) 2 (c) ∞ (d) 1

---

12 $\lim_{x \to 5} \frac{x^2 - 8x + 15}{x^2 - 10x + 25} =$ ....

(a) does not exist. (b) zero (c) 2 (d) 3

---

13 The included area between the curves of the two functions $f : f(x) = |x + 3| - 2$, $g : g(x) =$ zero is .... square units.

(a) 2 (b) 3 (c) 4 (d) 5

---

14 If $\log_3 y = x$, then the exponential form is ....

(a) $y = x^3$ (b) $x = y^3$ (c) $x = 3^y$ (d) $y = 3^x$

---

15 If f is an odd function on $[-x , x]$, then $f(-x) + f(x) =$ ....

(a) $2x$ (b) undefined. (c) $-2x$ (d) zero

---

16 In Δ ABC , if 2 sin A = 3 sin B = 4 sin C , then a : b : c = ...

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 4 : 6 (d) 6 : 4 : 3

---

17 $\lim_{x \to 2} \frac{x^5 - 32}{x^2 + 3x - 10} =$ ....

(a) $\frac{16}{7}$ (b) $\frac{80}{7}$ (c) $\frac{7}{80}$ (d) $\frac{7}{16}$

18 The radius length of the circumcircle of the triangle ABC in which m (∠ A) = 30° , a = 10 cm. equals ....

(a) 10 cm. (b) 20 cm. (c) 5 cm. (d) 40 cm.

19 The function $f : f(X) = \begin{cases} X^2 & , \quad X > 2 \\ -X^2 & , \quad X \le 2 \end{cases}$ is decreasing on the interval ....

(a)]0 , 2[(b)]– ∞ , 0[(c) ℝ – [0 , 2[(d)]0 , ∞[

20 The solution set of the equation : $\log_X 81 = 4$ in ℝ is ....

(a) {– 3} (b) {3} (c) {3 , – 3} (d) {9}

21 **Prove that :** $\frac{2^X \times 9^{X+1}}{3 \times (18)^X} = 3$

22 Graph the function $f : f(X) = \begin{cases} |X| & , \quad X \le 0 \\ X^3 & , \quad X > 0 \end{cases}$, from the graph state the range of the function and discuss its montony.

23 The solution set of the equation : | X + 2 | = – 2 in ℝ is ....

(a) ∅ (b) ℝ (c)]– ∞ , – 2[(d)]– ∞ , – 2]

24 The measure of the greatest angle in the triangle whose side lengths are 3 cm. , 5 cm. , 7 cm. equals ....

(a) 150° (b) 120° (c) 60° (d) 30°

25 In Δ ABC , a = 5 cm. , b = 7 cm. , m (∠ A) = 40°

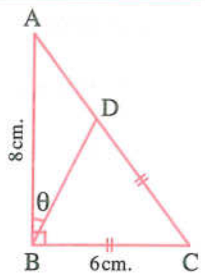
Find : m (∠ B) and the area of Δ ABC in each case.

26 **In the opposite figure :**

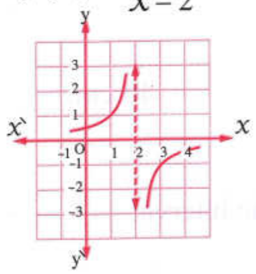
If CD = CB = 6 cm. , then tan θ = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

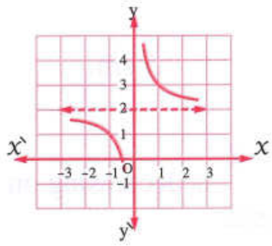
(c) $\frac{1}{2}$ (d) 2



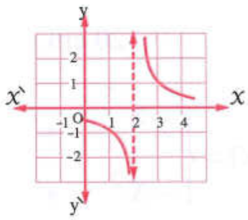
27 If $f(x) = \frac{1}{x-2}$, then the graph that represents the function f is ....



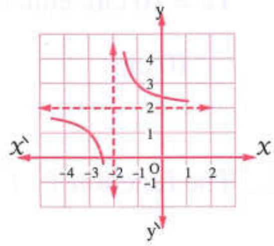
(a)



(b)



(c)



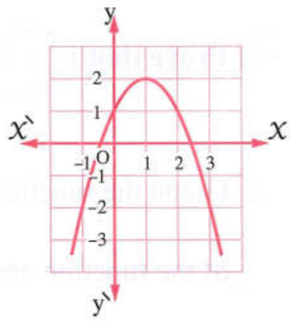
(d)

---

28 **Find :** $\lim_{x \to \infty} \frac{(x+1)(5x-3)}{x^2+3}$

---

29 The rule of the function shown in the opposite figure is $f(x) =$ ....



(a) $(x-2)^2 + 1$

(b) $-(x-2)^2 + 1$

(c) $-(x-1)^2 + 2$

(d) $(-x+1)^2 + 1$

---

30 In Δ ABC , $a^2 + b^2 - c^2 =$ ....

(a) cos A

(b) a b cos C

(c) cos C

(d) 2 a b cos C

---

31 The solution set of the equation : $x^{\frac{2}{3}} = 25$ in $\mathbb{R}$ is ....

(a) $\{5\}$

(b) $\{5 , -5\}$

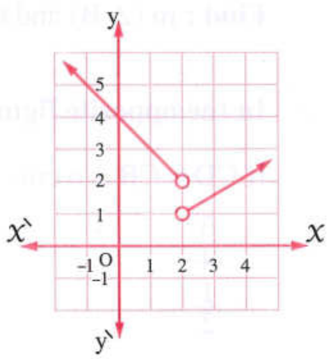
(c) $\{125\}$

(d) $\{125 , -125\}$

---

32 **In the opposite figure :**

$\lim_{x \to 2} f(x) =$ ....



(a) zero

(b) not exist

(c) 2

(d) 1

26 If $\lim\limits_{x \to 1} \frac{2x+a}{x+1} = 5$, then a = ...

(a) 2 (b) 5 (c) 8 (d) 10

27 In any triangle XYZ , $x^2 + y^2 - 2xy \cos Z =$ ....

(a) x^2 (b) y^2 (c) z^2 (d) z

28 **Find the solution set in $\mathbb{R}$ of the inequality :** $\sqrt{4x^2 - 12x + 9} \leq 9$

29 The number of possible solutions for the triangle ABC where : m (∠ A) = 60° , b = 3 cm. , a = 5 cm. is ....

(a) 1 (b) 2

(c) zero (d) infinite number.

30 If $\left(\frac{1}{2}\right)^{a^2 - a - 2} = 1$ where a > 0 , then a = ...

(a) 1 (b) – 3 (c) 2 (d) 3

31 If $f(x) = \frac{\sqrt{x^2 - 2x + 1}}{x - 1}$, then the range of the function f is ....

(a) {1} (b) $\mathbb{R}$ (c) [– 1 , 1[(d) {– 1 , 1}

32 $\lim\limits_{x \to \infty} \frac{\sqrt{x+5} - \sqrt{5}}{\sqrt{x} - \sqrt{5}} =$ ....

(a) 1 (b) – 1 (c) ∞ (d) – ∞

Model 2

Interactive test 2

Answer the following questions :

1 The range of the function $f : f(x) = |x|$ is ....

(a) [0 , ∞[(b)]0 , ∞[(c)]– ∞ , 0] (d)]– ∞ , 0[

2 $\lim\limits_{x \to \infty} \left(\frac{3}{5}\right)^{\frac{1}{x}} =$ ....

(a) 1 (b) – 1 (c) $\frac{3}{5}$ (d) ∞

3 ABCD is a quadrilateral in which AB = 27 cm. , BC = 12 cm. , CD = 8 cm. , DA = 12 cm. , AC = 18 cm. **Prove that :** $\overrightarrow{AC}$ bisects ∠ BAD

4 $\lim\limits_{x \to 1} \frac{x^5 - 1}{x - 1} =$ ....

(a) 5 (b) 1 (c) 4 (d) 20

5 In Δ ABC , $\frac{a}{\sin A} = 6$ cm. , then the radius length of its circumcircle = ... cm.

(a) 2 (b) 3 (c) 5 (d) 6

6 If f is an odd function and $x f(x) + x^3 f(-x) = 2$, then $f(2) =$ ....

(a) 3 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) -3

7 If $x = 5 + 2\sqrt{6}$, find in the simplest form the value of $\log\left(\frac{1}{x} + x\right)$ without using calculator.

8 In Δ XYZ , $\frac{x^2 + y^2 - z^2}{2xy} =$ ....

(a) cos X (b) cos Y (c) cos Z (d) sin Z

9 $\lim\limits_{x \to 1} \frac{x^2 - x^{-2}}{x - x^{-1}} =$ ....

(a) zero (b) 1 (c) 2 (d) -2

10 If $f(x) = 3^x$, then the solution set in ℝ of the equation $f(2x) - 28 f(x) + f(3) =$ zero equals ....

(a) $\{1 , 27\}$ (b) $\{1 , 3\}$ (c) $\{0 , 3\}$ (d) $\{3\}$

11 The logarithmic form that equivalent to the exponential form : $2^7 = 128$ is ....

(a) $\log_2 128 = 7$ (b) $\log_2 7 = 128$

(c) $\log_7 128 = 2$ (d) $\log_7 2 = 128$

12 The curve of the even function is symmetric about the straight line ....

(a) $y = x$ (b) $\overleftrightarrow{yy}$ (c) $\overleftrightarrow{xx}$ (d) $y = -x$

13 In Δ LMN , $\frac{\sin L}{3} = \frac{2 \sin M}{3} = \frac{\sin N}{4}$, then $\ell : m : n =$ ....

(a) 6 : 8 : 3 (b) 3 : 6 : 8 (c) 8 : 3 : 6 (d) 6 : 3 : 8

14 In Δ ABC , c = 7 cm. , m ($\angle$ A) = 70° , m ($\angle$ B) = 40° , then b $\simeq$ .... cm.

(a) 3.7 (b) 4.8 (c) 8.4 (d) 7.3

15 If $\lim\limits_{x \to a} \frac{a x}{3} = 12$, then a = ...

(a) ± 12 (b) ± 6 (c) 3 (d) − 3

16 Use the curve of the function $f : f(x) = \frac{1}{x}$ to graph the curve of the function $g : g(x) = \frac{1}{x-2} + 3$ from the graph state the domain and range of g and the monotony and its type whether it is even , odd or otherwise.

17 The range of the function $f : f(x) = \frac{x-2}{2-x}$ equals ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\{-1\}$

18 If log 3 = x , log 7 = y , then log 21 = ...

(a) x y (b) x + y (c) x − y (d) $\frac{x}{y}$

19 $\log_3 5 \times \log_2 3 \times \log_5 16$ = ...

(a) 30 (b) 15 (c) log 10000 (d) $\log_{30} 240$

20 The curve of the function $g : g(x) = x^2 + 4$ is the same as the curve of $f : f(x) = x^2$ by translation 4 units in the direction of ....

(a) $\overrightarrow{Ox}$ (b) $\overleftarrow{Ox}$ (c) $\overrightarrow{Oy}$ (d) $\overleftarrow{Oy}$

21 $\lim\limits_{x \to 5} \frac{\sqrt{x-1} - 2}{x-5}$ = ...

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) 4 (d) $\frac{1}{4}$

22 The function f where $f(x) = a^x$ is decreasing on its domain if ....

(a) a = 1 (b) a > 1 (c) 0 < a < 1 (d) a = − 1

23 $\lim\limits_{x \to 0} \frac{(4x+1)^9 - 1}{3x}$ = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 9 (d) 12

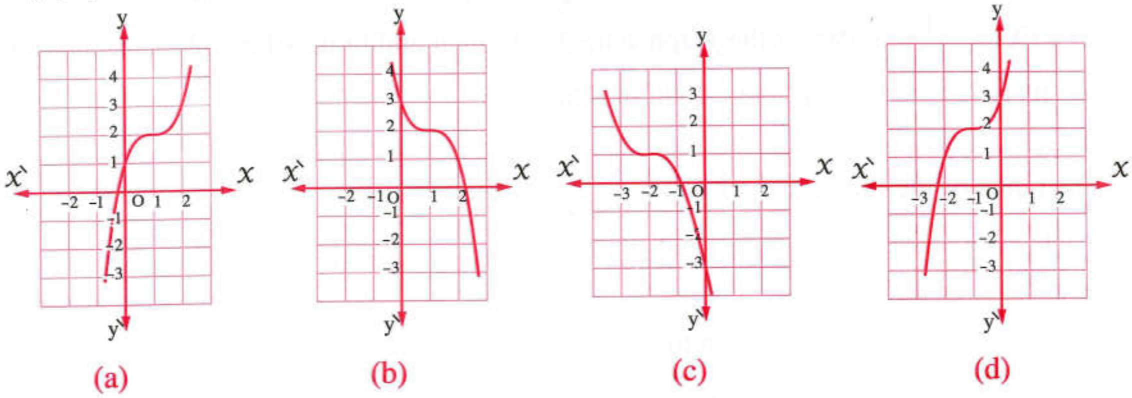
24 $\lim\limits_{x \to \infty} \frac{\sqrt{x^2}}{x} = \ldots\ldots\ldots\ldots$

(a) zero (b) 2 (c) -1 (d) 1

25 The solution set in $\mathbb{R}$ of the equation : $|x-7| = 2$ is ……………

(a) $\{9, 5\}$ (b) $\{7, 3\}$ (c) $\varnothing$ (d) $\{3, -3\}$

26 If $f(x) = 2 - (x-1)^3$, then the figure that represents the function f is ……………



27 If the perimeter of $\Delta ABC = 33$ cm. , $\sin A + \sin C = \frac{2}{3}$, $\sin B = \frac{1}{4}$

, then AC = …………… cm.

(a) 6 (b) 9 (c) 12 (d) 15

28 **Find :** $\lim\limits_{x \to 1} \frac{(x+2)^4 - 81}{x-1}$

29 If $3^{(2x+7)} = 11^x$, then the value of x to the nearest 1 decimal place equals ……………

(a) 28.3 (b) 38.3 (c) 3.8 (d) 28.4

30 $\lim\limits_{x \to \infty} (5 + 3x^2 + x) = \ldots\ldots\ldots\ldots$

(a) not exist (b) 5 (c) ∞ (d) 9

31 ABCD is a parallelogram , m ($\angle$ A) = 50° , m ($\angle$ DBC) = 70° , BD = 8 cm.

, then the perimeter of the parallelogram ABCD to the nearest cm. = …………… cm.

(a) 38 (b) 30 (c) 19 (d) 48

32 The solution set of the inequality $|x-1| \le 3$ is ……………

(a) $[-2, 4]$ (b) $]-2, 4[$ (c) $]-2, 4]$ (d) $\mathbb{R} - [-2, 4]$

Model 3

Interactive test 3

Answer the following questions :

1 If $f(x) = 7^{x+1}$, then the solution set of the equation : $f(2x-1) + f(x-2) = 50$ in $\mathbb{R}$ equals ……………

(a) $\{1\}$ (b) $\{1, -1\}$ (c) $\{1, -50\}$ (d) $\{7, -50\}$

2 If $\log 3 = x$, $\log 5 = y$, then $\log 15 =$ ……………

(a) xy (b) $\frac{x}{y}$ (c) $x + y$ (d) $x - y$

3 $\lim\limits_{x \to \infty} \frac{5 + x^{-2}}{1 + 3x^{-2}} =$ ……………

(a) $\frac{1}{3}$ (b) $\frac{5}{4}$ (c) $\frac{5}{3}$ (d) 5

4 If $f(x) = 5^{x}$, then $f(-2) =$ ……………

(a) -2 (b) 5 (c) $\frac{1}{25}$ (d) $\frac{1}{5}$

5 The domain of the function $f : f(x) = \log_3 (x-2)$ is $x >$ ……………

(a) 3 (b) 5 (c) 1 (d) 2

6 $\lim\limits_{x \to 16} \frac{\sqrt{x} - 1}{x - 16} =$ ……………

(a) zero (b) $\frac{1}{2}$ (c) 1 (d) does not exist.

7 $\lim\limits_{x \to \infty} \frac{2x + 7}{\sqrt{9x^2 + 5}} =$ ……………

(a) $\frac{2}{9}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{7}{3}$

8 $\log 25 + \frac{\log 8 \times \log 16}{\log 64} =$ ……………

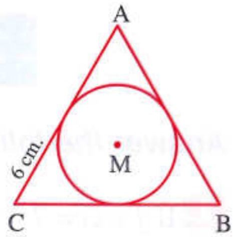
(a) $\log_2 16$ (b) $\log_5 25$ (c) $\log 4$ (d) $\log 10$

9 **In the opposite figure :**

If the perimeter of Δ ABC = 42 cm. ,

the circle touches the sides of the triangle

internally , then : m ($\angle$ B) = ...



(a) $53° \grave{8}$ (b) $67° \grave{23}$ (c) $36° \grave{53}$ (d) $32° \grave{37}$

10 $\lim_{x \to 2} \frac{x-2}{x+1} =$ ....

(a) zero (b) 1 (c) 2 (d) ∞

11 $\lim_{x \to \infty} \frac{\sqrt{x^2}}{x} =$ ....

(a) zero (b) 2 (c) ∞ (d) 1

12 $\lim_{x \to 5} \frac{x^2 - 8x + 15}{x^2 - 10x + 25} =$ ....

(a) does not exist. (b) zero (c) 2 (d) 3

13 The included area between the curves of the two functions $f : f(x) = |x + 3| - 2$, g : g (x) = zero is .... square units.

(a) 2 (b) 3 (c) 4 (d) 5

14 If $\log_3 y = x$, then the exponential form is ....

(a) $y = x^3$ (b) $x = y^3$ (c) $x = 3^y$ (d) $y = 3^x$

15 If f is an odd function on $[-x , x]$, then $f(-x) + f(x) =$ ....

(a) $2x$ (b) undefined. (c) $-2x$ (d) zero

16 In Δ ABC , if 2 sin A = 3 sin B = 4 sin C , then a : b : c = ...

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 4 : 6 (d) 6 : 4 : 3

17 $\lim_{x \to 2} \frac{x^5 - 32}{x^2 + 3x - 10} =$ ....

(a) $\frac{16}{7}$ (b) $\frac{80}{7}$ (c) $\frac{7}{80}$ (d) $\frac{7}{16}$

18 The radius length of the circumcircle of the triangle ABC in which m (∠ A) = 30° , a = 10 cm. equals ....

(a) 10 cm. (b) 20 cm. (c) 5 cm. (d) 40 cm.

---

19 The function $f : f(x) = \begin{cases} x^2 & , \quad x > 2 \\ -x^2 & , \quad x \le 2 \end{cases}$ is decreasing on the interval ....

(a) $]0 , 2[$ (b) $]-\infty , 0[$ (c) $\mathbb{R} - [0 , 2[$ (d) $]0 , \infty[$

---

20 The solution set of the equation : $\log_x 81 = 4$ in $\mathbb{R}$ is ....

(a) $\{-3\}$ (b) $\{3\}$ (c) $\{3 , -3\}$ (d) $\{9\}$

---

21 **Prove that :** $\frac{2^x \times 9^{x+1}}{3 \times (18)^x} = 3$

---

22 Graph the function $f : f(x) = \begin{cases} |x| & , \quad x \le 0 \\ x^3 & , \quad x > 0 \end{cases}$, from the graph state the range of the function and discuss its montony.

---

23 The solution set of the equation : $|x + 2| = -2$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $]-\infty , -2[$ (d) $]-\infty , -2]$

---

24 The measure of the greatest angle in the triangle whose side lengths are 3 cm. , 5 cm. , 7 cm. equals ....

(a) 150° (b) 120° (c) 60° (d) 30°

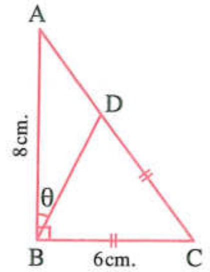
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25 In Δ ABC , a = 5 cm. , b = 7 cm. , m (∠ A) = 40°

Find : m (∠ B) and the area of Δ ABC in each case.

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26 **In the opposite figure :**

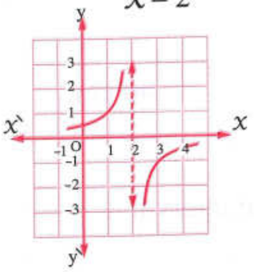


If CD = CB = 6 cm. , then tan θ = ...

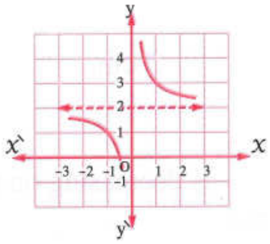
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{1}{2}$ (d) 2

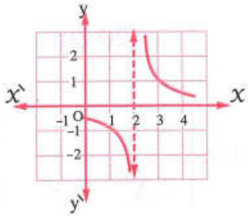
27 If $f(x) = \frac{1}{x-2}$, then the graph that represents the function f is ....



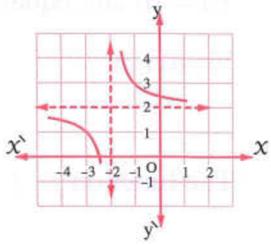
(a)



(b)



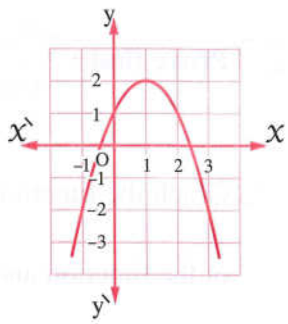
(c)



(d)

28 **Find :** $\lim\limits_{x \to \infty} \frac{(x+1)(5x-3)}{x^2+3}$

29 The rule of the function shown in the opposite figure is $f(x) =$ ....



(a) $(x-2)^2+1$

(b) $-(x-2)^2+1$

(c) $-(x-1)^2+2$

(d) $(-x+1)^2+1$

30 In Δ ABC , $a^2+b^2-c^2 =$ ....

(a) cos A

(b) a b cos C

(c) cos C

(d) 2 a b cos C

31 The solution set of the equation : $x^{\frac{2}{3}} = 25$ in $\mathbb{R}$ is ....

(a) $\{5\}$

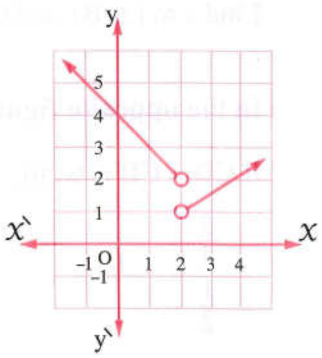
(b) $\{5 , -5\}$

(c) $\{125\}$

(d) $\{125 , -125\}$

32 **In the opposite figure :**

$\lim\limits_{x \to 2} f(x) =$ ....



(a) zero

(b) not exist

(c) 2

(d) 1

15 The domain of the function $f : f(x) = \sqrt{9 - x}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{9\}$ (c) $]-\infty , 9]$ (d) $[9 , \infty[$

16 **Find :** $\lim\limits_{x \to 5} \frac{x^2 - 5x}{\sqrt{x + 4} - 3}$

17 In Δ ABC , c = 19 cm. , m ($\angle$ A) = 112° , m ($\angle$ B) = 33° , then the area of Δ ABC to the nearest cm². equals .... cm².

(a) 64 (b) 128 (c) 185 (d) 159

18 The solution set of the inequality : $|x| - 1 > 0$ in $\mathbb{R}$ is ....

(a) $\mathbb{R} - [-1 , 1]$ (b) $]-1 , 1[$ (c) $\mathbb{R} -]-1 , 1[$ (d) $[-1 , 1]$

19 $\lim\limits_{x \to -2} \left| \frac{1}{x} \right| =$ ....

(a) 1 (b) -1 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

20 $\lim\limits_{x \to \infty} \left(\frac{3x^2 + 2x + 1}{x^2 - 3x + 2} \right)^4 =$ ....

(a) 3 (b) 9 (c) 27 (d) 81

21 Graph the curve of the function $f : f(x) = (x + 2)^3 + 1$ and from the graph deduce the range and its monotony and its type whether it is even , odd or otherwise.

22 Which of the following does not equal $\left(\sqrt[5]{x^4}\right)$?

(a) $\left(\sqrt[5]{x}\right)^4$ (b) $\sqrt[4]{x^5}$ (c) $x^{\frac{4}{5}}$ (d) $\left(x^{\frac{1}{5}}\right)^4$

23 If the function f is even in $[c , d]$, then c + d = ...

(a) 2 c (b) 2 d (c) c – d (d) zero

24 $\lim\limits_{x \to -2} \frac{x^5 + 32}{x^6 - 64} =$ ....

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{-12}{5}$ (d) $\frac{-5}{12}$

25 If $\sqrt[3]{x^2} = 9$, then $x \in$ ....

(a) $\{27\}$ (b) $\{27 , -27\}$ (c) $\{1\}$ (d) $\varnothing$

26 $\lim_{x \to \infty} \left(\frac{5}{x} + 1\right) = \dots\dots$

(a) zero (b) 1 (c) 5 (d) 6

27 If $\left(\frac{1}{2}\right)^{a^2 - a - 2} = 1$, where a > zero , then a = ……

(a) 1 (b) – 3 (c) 2 (d) 3

28 Solve Δ ABC in which m (∠ C) = 116° , c = 12 cm. , a = 10 cm.

29 Which of the functions defined by the following rules represents an exponential function increasing on its domain ℝ ?

(a) $y = 3\,(1.05)^x$ (b) $y = \frac{1}{3}\left(\frac{1}{1.5}\right)^x$ (c) $y = 3 + (0.5)^x$ (d) $y = (0.5)^x$

30 In Δ ABC , if 2 sin A = 3 sin B = 4 sin C , then a : b : c = ……

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 4 : 6 (d) 6 : 4 : 3

31 If $\lim_{x \to a} \frac{a x}{3} = 12$, then a = ……

(a) ± 12 (b) ± 6 (c) 4 (d) $\frac{1}{6}$

32 If $|x| + |x - 3| = 3$, then $x(x - 3)$ …… zero

(a) < (b) > (c) ≤ (d) ≥

Model 9

Interactive test 9

Answer the following questions :

1 The solution set of the equation : $\log_{(x+3)} 125 = 3$ in ℝ is ……

(a) {5} (b) {3} (c) ∅ (d) {2}

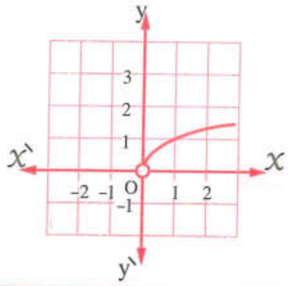
2 Δ LMN in which m (∠ L) = 30° , m = 9 cm. has two solutions when ℓ = …… cm.

(a) 6 (b) 10 (c) 11 (d) 2

3 If $4 = \log_2 x$, then the equivalent exponential form is ……

(a) $x^2 = 4$ (b) $x^4 = 2$ (c) $x = 2^4$ (d) $2^x = 4$

4 The domain of the function represented by the opposite figure is ....



(a) $[0, \infty[$ (b) $]0, \infty[$
(c) $]-\infty, 0[$ (d) $]0, 3[$

5 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x+1) - f(x) = x - 1$, then $f(10) - f(9) =$ ....

(a) 1 (b) 9 (c) 8 (d) 18

6 $\lim\limits_{x \to 0} \frac{x^2 + x}{x^3 + x} =$ ....

(a) $\frac{2}{3}$ (b) 1 (c) zero (d) does not exist.

7 The image of the curve $y = |x| - 5$ by translation 3 units in the direction of $\overrightarrow{Ox}$ and 5 units in the direction of $\overrightarrow{Oy}$ is ....

(a) $y = |x - 3| + 5$ (b) $y = |x - 3|$ (c) $y = |x - 3| - 10$ (d) $y = |x + 3|$

8 $\lim\limits_{x \to \infty} \frac{\sqrt{4x^2 + 7} + 3x}{2x + 9} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{5}{4}$ (d) $\frac{5}{9}$

9 $\lim\limits_{x \to \infty} \frac{\sqrt{x + 5} - \sqrt{5}}{\sqrt{x} - \sqrt{5}} =$ ....

(a) 1 (b) -1 (c) ∞ (d) $-\infty$

10 The solution set of the inequality : $|x - 1| < -2$ in $\mathbb{R}$ is ....

(a) $]-1, 3[$ (b) $\mathbb{R} - [-1, 3]$ (c) $]-2, 2[$ (d) $\varnothing$

11 In Δ ABC , c (a cos B + b cos A) = ...

(a) a^2 (b) b^2 (c) c^2 (d) $2c^2$

12 ABCD is a parallelogram in which : AB = 9 cm. , BC = 13 cm. , AC = 20 cm. , then the length of $\overline{BD}$ equals .... cm.

(a) 10 (b) 5 (c) 18.5 (d) 20

13 If the domain of the function $f : f(x) = \frac{2}{x^2 - 6x + k}$ is $\mathbb{R} - \{3\}$, then k = ...

(a) 3 (b) -3 (c) 9 (d) ± 9

14 $\lim\limits_{x \to 4} \frac{x^3 - 64}{x - 4} = \dots\dots$

(a) 96 (b) 48 (c) 32 (d) 16

15 If $f(x) = \frac{\sqrt{x^2 - 2x + 1}}{x - 1}$, then the range of the function f is $\dots\dots$

(a) $\{1\}$ (b) $\mathbb{R}$ (c) $[-1, 1[$ (d) $\{-1, 1\}$

16 Graph the function $f : f(x) = \sqrt{x^2 - 4x + 4}$ and determine its range and discuss its monotony.

17 The solution set of the following equation in $\mathbb{R}$: $\log_2 x - \frac{3}{\log_2 x} = 2$ equals $\dots\dots$

(a) $\{\frac{1}{2}\}$ (b) $\{8, 2\}$ (c) $\{8, \frac{1}{2}\}$ (d) $\{2\}$

18 $\lim\limits_{h \to 0} \frac{(x + h)^9 - x^9}{h} = \dots\dots$

(a) x^9 (b) $9x^8$ (c) zero (d) does not exist.

19 $\log_3 15 - \log_3 5 = \dots\dots$

(a) 3 (b) 1 (c) zero (d) -3

20 If ABC is a triangle in which $a = 4$ cm. , $b = 4\sqrt{3}$ cm. , $c = 8$ cm. , then sine of its smallest angle equals $\dots\dots$

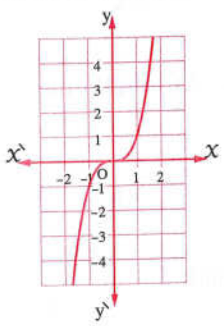
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) zero

21 Solve the acute-angled triangle ABC in which $a = 21$ cm. , $b = 25$ cm. , and the diameter length of its circumcircle is 28 cm.

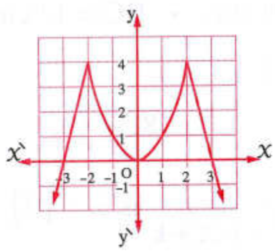
22 If $x = 5 + 2\sqrt{6}$, then $\log\left(\frac{1}{x} + x\right) = \dots\dots$

(a) 1 (b) $5 - 2\sqrt{6}$ (c) 10 (d) $5 + 2\sqrt{6}$

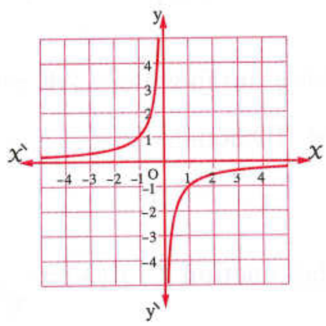
23 Which of the functions represented graphically as follows is neither even nor odd ?



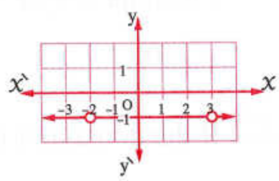
(a)



(b)



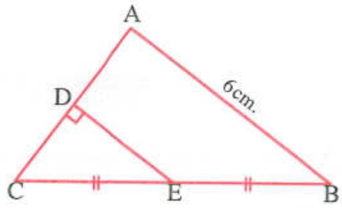
(c)



(d)

24 **In the opposite figure :**

If $\tan (\angle DEC) = \frac{3}{4}$, then the radius length of the circumcircle of $\Delta ABC = \cdots\cdots\cdots$ cm.



(a) 9 (b) 5.7

(c) $4\frac{3}{4}$ (d) 3.75

---

25 Graph the curve of the function $f : f(x) = x^3 - 5$ and from the graph discuss the monotonicity of the function and show its type whether it is even , odd or otherwise

---

26 $\lim\limits_{x \to 2} \frac{5x - 10}{4x - 8} = \cdots\cdots\cdots$

(a) $\frac{5}{4}$ (b) zero (c) 2 (d) $\frac{4}{5}$

---

27 The solution set of the equation : $\frac{1}{\log_2 x} + \frac{1}{\log_3 x} = 2$ is $\cdots\cdots\cdots$

(a) $\{\sqrt{6}\}$ (b) $\{-\sqrt{6}\}$ (c) $\{\sqrt{6}, -\sqrt{6}\}$ (d) $\{6\}$

---

28 **Find :** $\lim\limits_{x \to 0} \frac{\sqrt{9x + 16} - 4}{x}$

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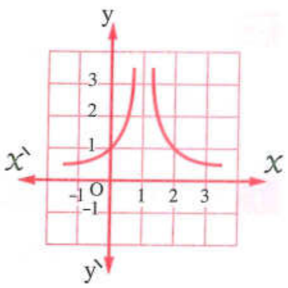
29 If $\sqrt[3]{x^2} = 9$, then $x \in \cdots\cdots\cdots$

(a) $\{-81, 81\}$ (b) $\{-27, 27\}$ (c) $\{-9, 9\}$ (d) $]3, 7[$

---

30 **In the opposite figure :**

$f(x) = \cdots\cdots\cdots$



(a) $\frac{1}{x - 1}$ (b) $\frac{1}{|x - 1|}$

(c) $|x^2 - 1|$ (d) $|x - 1|^2$

---

31 $\lim\limits_{x \to \infty} \left(x - \sqrt{x^2 + 5x}\right) = \cdots\cdots\cdots$

(a) 2 (b) 3 (c) $\frac{-5}{2}$ (d) $\frac{1}{4}$

---

32 In ΔXYZ , if $\sin X = 2 \sin Z$, YZ = 6 cm. , then the length of $\overline{XY} = \cdots\cdots\cdots$ cm.

(a) 12 (b) 2 (c) 6 (d) 3

Model 10

Interactive test 10

Answer the following questions :

1 The solution set of the equation $\log_3 (x-4) + \log_3 (x+4) = 2$ in $\mathbb{R}$ is ....

(a) $\{5\}$ (b) $\{5, -5\}$ (c) $\{3, -3\}$ (d) $\{3, 5\}$

2 $\lim\limits_{x \to 0} \dfrac{(x+2)^2 - 4}{x^2 + x} =$ ....

(a) zero (b) 2 (c) 4 (d) 8

3 If the ratio among the measures of the angles of a triangle is 8 : 3 : 1 , then the ratio between the longest two sides in the triangle is ....

(a) $\sqrt{3} : 2$ (b) $\sqrt{6} : 2$ (c) 8 : 3 (d) 8 : 5

4 $\lim\limits_{x \to -3} \dfrac{\sqrt{x+7} - 2}{x+3} =$ ....

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) 4

5 If $3^a = 4^b$, then : $9^{\frac{a}{b}} + 16^{\frac{b}{a}} =$ ....

(a) 7 (b) 12 (c) 20 (d) 25

6 If $\lim\limits_{x \to \infty} \dfrac{3k|x|}{4x+3} = 6$, then k = ...

(a) 6 (b) $\frac{3}{4}$ (c) 8 (d) 3

7 If $f(x) = x^3$, then the image of the curve of f by reflection in x-axis and translation 3 units in the direction of $\overrightarrow{Ox}$ and two units in the direction of $\overrightarrow{Oy}$ is ....

(a) $-(x-3)^3 - 2$ (b) $-(x+3)^3 + 2$

(c) $-(x+3)^3 - 2$ (d) $-[(x+3)^3 + 2]$

8 **Find :** $\lim\limits_{x \to 2} \dfrac{(x-1)^6 - 1}{x - 2}$

9 The perimeter of Δ ABC is 70 cm. , a = 26 cm. , m ($\angle$ A) = 60° , find its area.

10 If $2^{x-3} = 1$, then $x =$ ....

(a) -3 (b) 3 (c) 1 (d) zero

11 If $a \in \mathbb{R}^+ - \{1\}$, x , $y \in \mathbb{R}^+$, $\log_a y \neq 0$, then $\frac{\log_a x}{\log_a y} =$ ....

(a) $\log_a \frac{x}{y}$ (b) $\log_a (x - y)$ (c) $\log_a x - \log_a y$ (d) $\log_y x$

12 $\frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30} =$ ....

(a) 1 (b) $\log_6 5$ (c) $\log 30$ (d) 30

13 In ΔABC , $m(\angle A) = 112°$, $m(\angle B) = 33°$, $c = 19$ cm.
, then b to the nearest cm. = ... cm.

(a) 16 (b) 17 (c) 18 (d) 20

14 If $2^x = 20$, $n < x < n + 1$, n is an integer , then n = ...

(a) 4 (b) 5 (c) 6 (d) 10

15 In ΔXYZ , $y^2 + z^2 - x^2 = 2yz \times$ ....

(a) cos X (b) sin Z (c) cos Z (d) sin X

16 $\lim_{x \to 1} \frac{x^2 + 5x - 6}{x^2 - 1} =$ ....

(a) 1 (b) 5 (c) 6 (d) 3.5

17 **Without using the calculator prove that :**

$\log_5 \frac{15}{7} + \log_5 \frac{35}{3} - \log_5 \frac{1}{5} = \log_2 8$

18 The exponential function whose base is a , is increasing if ....

(a) $a > 0$ (b) $a > 1$ (c) $0 < a < 1$ (d) $a = 1$

19 $\lim_{x \to 2} \frac{x^5 - 32}{x^4 - 16} =$ ....

(a) 2 (b) 20 (c) $\frac{5}{4}$ (d) $\frac{5}{2}$

20 If f is an odd function , a $\in$ the domain of f , then $f(a) + f(-a) =$ ....

(a) $2f(a)$ (b) $2f(-a)$ (c) zero (d) $f(a)$

21 The solution set in $\mathbb{R}$ of the equation : $|x - 3| = |9 - 2x|$ equals ....

(a) $\{4\}$ (b) $\{4 , 6\}$ (c) $\{6\}$ (d) $\{2 , 6\}$

22 Determine the type of the function $f : f(x) = x^2 + \sin x$ whether it is even , odd or otherwise.

23 $\lim\limits_{x \to \infty} \dfrac{5x^2 - 3x + 6}{1 + 4x - 7x^2} = \text{.............}$

(a) $\frac{5}{7}$ (b) $\frac{-5}{7}$ (c) 6 (d) 35

24 The range of the function $f : f(x) = \begin{cases} 2x + 3 & , \quad x > 3 \\ 9 & , \quad x < 3 \end{cases}$ is ....

(a) $\{3\}$ (b) $\mathbb{R}$ (c) $]9 , \infty[$ (d) $[9 , \infty[$

25 In Δ ABC , if m ($\angle$ B) = 60° , m ($\angle$ C) = 30° , c = 4 cm. , then b = ... cm.

(a) 4 (b) 8 (c) $2\sqrt{3}$ (d) $4\sqrt{3}$

26 $\lim\limits_{x \to 0} \dfrac{(x+1)^{12} - 1}{x} = \text{.............}$

(a) 1 (b) 6 (c) zero (d) 12

27 If the area of Δ ABC is "x" and the radius length of its circumcircle is "r"

, then $\dfrac{4 r x}{a b c} = \text{.............}$

(a) $\dfrac{a}{\sin A}$ (b) cos A (c) 1 (d) r

28 If $f(x) = 7^{x+1}$, then the value of x which satisfies : $f(2x - 1) + f(x - 2) = 50$ equals ....

(a) 1 (b) 7 (c) zero (d) 2

29 If L , M are the roots of the equation : $x^2 - 4x + 4 = 0$, then $\log_2 L + \log_2 M = \text{.............}$

(a) 2 (b) -2 (c) -4 (d) 4

30 If $\sqrt{x^2 - 2x + 1} > 4$, then $x \in$ ....

(a) $[-3 , 5]$ (b) $]-3 , 5[$ (c) $\mathbb{R} -]-3 , 5[$ (d) $\mathbb{R} - [-3 , 5]$

31 $\lim\limits_{x \to 1} \dfrac{1 - x^9}{x^7 - 1} = \text{.............}$

(a) 2 (b) $\frac{9}{7}$ (c) $\frac{-9}{7}$ (d) $\frac{1}{7}$

32 In triangle ABC , a = 4 cm. , b = 7 cm. , m ($\angle$ A) = 112° , then the number of triangles satisfy these conditions equals ....

(a) 1 (b) 2 (c) 0 (d) infinite number.

15 The domain of the function $f : f(x) = \sqrt{9 - x}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{9\}$ (c) $]-\infty , 9]$ (d) $[9 , \infty[$

16 **Find :** $\lim\limits_{x \to 5} \frac{x^2 - 5x}{\sqrt{x + 4} - 3}$

17 In Δ ABC , c = 19 cm. , m (∠ A) = 112° , m (∠ B) = 33° , then the area of Δ ABC to the nearest cm^2 equals .... cm^2.

(a) 64 (b) 128 (c) 185 (d) 159

18 The solution set of the inequality : $|x| - 1 > 0$ in $\mathbb{R}$ is ....

(a) $\mathbb{R} - [-1 , 1]$ (b) $]-1 , 1[$ (c) $\mathbb{R} -]-1 , 1[$ (d) $[-1 , 1]$

19 $\lim\limits_{x \to -2} \left| \frac{1}{x} \right| =$ ....

(a) 1 (b) -1 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

20 $\lim\limits_{x \to \infty} \left(\frac{3x^2 + 2x + 1}{x^2 - 3x + 2} \right)^4 =$ ....

(a) 3 (b) 9 (c) 27 (d) 81

21 Graph the curve of the function $f : f(x) = (x + 2)^3 + 1$ and from the graph deduce the range and its monotony and its type whether it is even , odd or otherwise.

22 Which of the following does not equal $\left(\sqrt[5]{x^4}\right)$?

(a) $\left(\sqrt[5]{x}\right)^4$ (b) $\sqrt[4]{x^5}$ (c) $x^{\frac{4}{5}}$ (d) $\left(x^{\frac{1}{5}}\right)^4$

23 If the function f is even in $[c , d]$, then c + d = ...

(a) 2 c (b) 2 d (c) c − d (d) zero

24 $\lim\limits_{x \to -2} \frac{x^5 + 32}{x^6 - 64} =$ ....

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{-12}{5}$ (d) $\frac{-5}{12}$

25 If $\sqrt[3]{x^2} = 9$, then $x \in$ ....

(a) $\{27\}$ (b) $\{27 , -27\}$ (c) $\{1\}$ (d) $\varnothing$

26 $\lim_{x \to \infty} \left(\frac{5}{x} + 1\right) = \dots\dots$

(a) zero (b) 1 (c) 5 (d) 6

27 If $\left(\frac{1}{2}\right)^{a^2 - a - 2} = 1$, where a > zero , then a = ...

(a) 1 (b) – 3 (c) 2 (d) 3

28 Solve Δ ABC in which m (∠ C) = 116° , c = 12 cm. , a = 10 cm.

29 Which of the functions defined by the following rules represents an exponential function increasing on its domain ℝ ?

(a) $y = 3\,(1.05)^{x}$ (b) $y = \frac{1}{3}\left(\frac{1}{1.5}\right)^{x}$ (c) $y = 3 + (0.5)^{x}$ (d) $y = (0.5)^{x}$

30 In Δ ABC , if 2 sin A = 3 sin B = 4 sin C , then a : b : c = ...

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 4 : 6 (d) 6 : 4 : 3

31 If $\lim_{x \to a} \frac{a x}{3} = 12$, then a = ...

(a) ± 12 (b) ± 6 (c) 4 (d) $\frac{1}{6}$

32 If $|x| + |x - 3| = 3$, then $x(x - 3)$ .... zero

(a) < (b) > (c) ≤ (d) ≥

Model 9

Interactive test 9

Answer the following questions :

1 The solution set of the equation : $\log_{(x+3)} 125 = 3$ in ℝ is ....

(a) {5} (b) {3} (c) ∅ (d) {2}

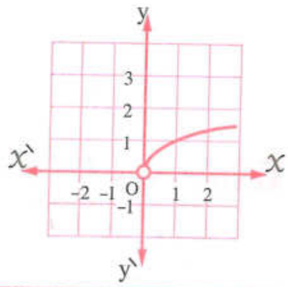
2 Δ LMN in which m (∠ L) = 30° , m = 9 cm. has two solutions when ℓ = ... cm.

(a) 6 (b) 10 (c) 11 (d) 2

3 If $4 = \log_2 x$, then the equivalent exponential form is ....

(a) $x^2 = 4$ (b) $x^4 = 2$ (c) $x = 2^4$ (d) $2^{x} = 4$

4 The domain of the function represented by the opposite figure is ....



(a) $[0, \infty[$ (b) $]0, \infty[$
(c) $]-\infty, 0[$ (d) $]0, 3[$

5 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x+1) - f(x) = x - 1$, then $f(10) - f(9) =$ ....

(a) 1 (b) 9 (c) 8 (d) 18

6 $\lim\limits_{x \to 0} \frac{x^2 + x}{x^3 + x} =$ ....

(a) $\frac{2}{3}$ (b) 1 (c) zero (d) does not exist.

7 The image of the curve $y = |x| - 5$ by translation 3 units in the direction of $\overrightarrow{Ox}$ and 5 units in the direction of $\overrightarrow{Oy}$ is ....

(a) $y = |x - 3| + 5$ (b) $y = |x - 3|$ (c) $y = |x - 3| - 10$ (d) $y = |x + 3|$

8 $\lim\limits_{x \to \infty} \frac{\sqrt{4x^2 + 7} + 3x}{2x + 9} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{5}{4}$ (d) $\frac{5}{9}$

9 $\lim\limits_{x \to \infty} \frac{\sqrt{x+5} - \sqrt{5}}{\sqrt{x} - \sqrt{5}} =$ ....

(a) 1 (b) -1 (c) ∞ (d) $-\infty$

10 The solution set of the inequality : $|x - 1| < -2$ in $\mathbb{R}$ is ....

(a) $]-1, 3[$ (b) $\mathbb{R} - [-1, 3]$ (c) $]-2, 2[$ (d) $\varnothing$

11 In Δ ABC, c (a cos B + b cos A) = ...

(a) a^2 (b) b^2 (c) c^2 (d) $2c^2$

12 ABCD is a parallelogram in which : AB = 9 cm. , BC = 13 cm. , AC = 20 cm. , then the length of $\overline{BD}$ equals .... cm.

(a) 10 (b) 5 (c) 18.5 (d) 20

13 If the domain of the function $f : f(x) = \frac{2}{x^2 - 6x + k}$ is $\mathbb{R} - \{3\}$, then k = ...

(a) 3 (b) -3 (c) 9 (d) ± 9

14 $\lim\limits_{x \to 4} \dfrac{x^3 - 64}{x - 4} =$ ....

(a) 96 (b) 48 (c) 32 (d) 16

15 If $f(x) = \dfrac{\sqrt{x^2 - 2x + 1}}{x - 1}$, then the range of the function f is ....

(a) $\{1\}$ (b) $\mathbb{R}$ (c) $[-1, 1[$ (d) $\{-1, 1\}$

16 Graph the function $f : f(x) = \sqrt{x^2 - 4x + 4}$ and determine its range and discuss its monotony.

17 The solution set of the following equation in $\mathbb{R}$: $\log_2 x - \dfrac{3}{\log_2 x} = 2$ equals ....

(a) $\{\frac{1}{2}\}$ (b) $\{8, 2\}$ (c) $\{8, \frac{1}{2}\}$ (d) $\{2\}$

18 $\lim\limits_{h \to 0} \dfrac{(x + h)^9 - x^9}{h} =$ ....

(a) x^9 (b) $9x^8$ (c) zero (d) does not exist.

19 $\log_3 15 - \log_3 5 =$ ....

(a) 3 (b) 1 (c) zero (d) -3

20 If ABC is a triangle in which a = 4 cm. , $b = 4\sqrt{3}$ cm. , c = 8 cm. , then sine of its smallest angle equals ....

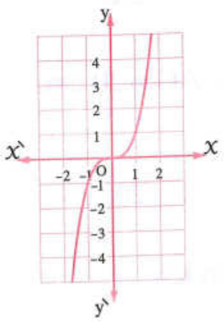
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) zero

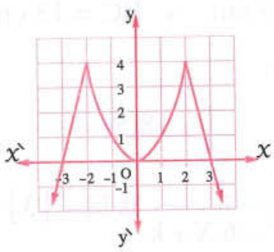
21 Solve the acute-angled triangle ABC in which a = 21 cm. , b = 25 cm. , and the diameter length of its circumcircle is 28 cm.

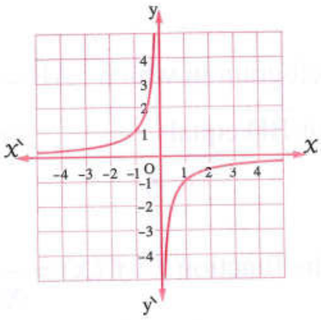
22 If $x = 5 + 2\sqrt{6}$, then $\log\left(\frac{1}{x} + x\right) =$ ....

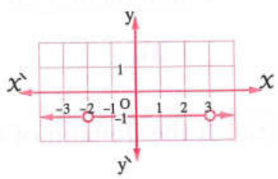
(a) 1 (b) $5 - 2\sqrt{6}$ (c) 10 (d) $5 + 2\sqrt{6}$

23 Which of the functions represented graphically as follows is neither even nor odd ?

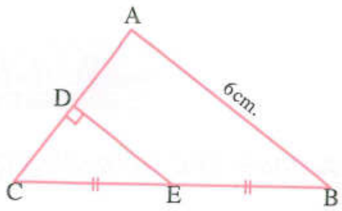

(a)


(b)


(c)


(d)

24 **In the opposite figure :**



If $\tan(\angle DEC) = \frac{3}{4}$, then the radius length of the circumcircle of ΔABC = ... cm.

(a) 9 (b) 5.7

(c) $4\frac{3}{4}$ (d) 3.75

25 Graph the curve of the function $f : f(X) = X^3 - 5$ and from the graph discuss the monotonicity of the function and show its type whether it is even , odd or otherwise

26 $\lim\limits_{X \to 2} \frac{5X - 10}{4X - 8}$ = ...

(a) $\frac{5}{4}$ (b) zero (c) 2 (d) $\frac{4}{5}$

27 The solution set of the equation : $\frac{1}{\log_2 X} + \frac{1}{\log_3 X} = 2$ is ....

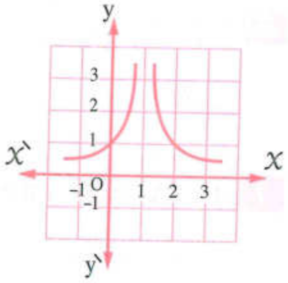
(a) $\{\sqrt{6}\}$ (b) $\{-\sqrt{6}\}$ (c) $\{\sqrt{6}, -\sqrt{6}\}$ (d) $\{6\}$

28 **Find :** $\lim\limits_{X \to 0} \frac{\sqrt{9X + 16} - 4}{X}$

29 If $\sqrt[3]{X^2} = 9$, then $X \in$ ....

(a) $\{-81, 81\}$ (b) $\{-27, 27\}$ (c) $\{-9, 9\}$ (d) $]3, 7[$

30 **In the opposite figure :**



$f(X)$ = ...

(a) $\frac{1}{X - 1}$ (b) $\frac{1}{|X - 1|}$

(c) $|X^2 - 1|$ (d) $|X - 1|^2$

31 $\lim\limits_{X \to \infty} \left(X - \sqrt{X^2 + 5X}\right)$ = ...

(a) 2 (b) 3 (c) $\frac{-5}{2}$ (d) $\frac{1}{4}$

32 In ΔXYZ , if $\sin X = 2 \sin Z$, YZ = 6 cm. , then the length of $\overline{XY}$ = ... cm.

(a) 12 (b) 2 (c) 6 (d) 3

Model 10

Interactive test 10

Answer the following questions :

1 The solution set of the equation $\log_3 (x-4) + \log_3 (x+4) = 2$ in $\mathbb{R}$ is ....

(a) $\{5\}$ (b) $\{5, -5\}$ (c) $\{3, -3\}$ (d) $\{3, 5\}$

2 $\lim\limits_{x \to 0} \frac{(x+2)^2 - 4}{x^2 + x} =$ ....

(a) zero (b) 2 (c) 4 (d) 8

3 If the ratio among the measures of the angles of a triangle is 8 : 3 : 1 , then the ratio between the longest two sides in the triangle is ....

(a) $\sqrt{3} : 2$ (b) $\sqrt{6} : 2$ (c) 8 : 3 (d) 8 : 5

4 $\lim\limits_{x \to -3} \frac{\sqrt{x+7} - 2}{x+3} =$ ....

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) 4

5 If $3^a = 4^b$, then : $9^{\frac{a}{b}} + 16^{\frac{b}{a}} =$ ....

(a) 7 (b) 12 (c) 20 (d) 25

6 If $\lim\limits_{x \to \infty} \frac{3k|x|}{4x+3} = 6$, then k = ...

(a) 6 (b) $\frac{3}{4}$ (c) 8 (d) 3

7 If $f(x) = x^3$, then the image of the curve of f by reflection in x-axis and translation 3 units in the direction of $\overrightarrow{Ox}$ and two units in the direction of $\overrightarrow{Oy}$ is ....

(a) $-(x-3)^3 - 2$ (b) $-(x+3)^3 + 2$

(c) $-(x+3)^3 - 2$ (d) $-[(x+3)^3 + 2]$

8 **Find :** $\lim\limits_{x \to 2} \frac{(x-1)^6 - 1}{x-2}$

9 The perimeter of Δ ABC is 70 cm. , a = 26 cm. , m ($\angle$ A) = 60° , find its area.

10 If $2^{x-3} = 1$, then $x =$ ....

(a) -3 (b) 3 (c) 1 (d) zero

11 If $a \in \mathbb{R}^+ - \{1\}$, x , $y \in \mathbb{R}^+$, $\log_a y \neq 0$, then $\frac{\log_a x}{\log_a y} =$ ....

(a) $\log_a \frac{x}{y}$ (b) $\log_a (x - y)$ (c) $\log_a x - \log_a y$ (d) $\log_y x$

12 $\frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30} =$ ....

(a) 1 (b) $\log_6 5$ (c) $\log 30$ (d) 30

13 In ΔABC , $m(\angle A) = 112°$, $m(\angle B) = 33°$, $c = 19$ cm.

, then b to the nearest cm. = ... cm.

(a) 16 (b) 17 (c) 18 (d) 20

14 If $2^{x} = 20$, $n < x < n + 1$, n is an integer , then n = ...

(a) 4 (b) 5 (c) 6 (d) 10

15 In ΔXYZ , $y^2 + z^2 - x^2 = 2yz \times$ ....

(a) cos X (b) sin Z (c) cos Z (d) sin X

16 $\lim\limits_{x \to 1} \frac{x^2 + 5x - 6}{x^2 - 1} =$ ....

(a) 1 (b) 5 (c) 6 (d) 3.5

17 **Without using the calculator prove that :**

$\log_5 \frac{15}{7} + \log_5 \frac{35}{3} - \log_5 \frac{1}{5} = \log_2 8$

18 The exponential function whose base is a , is increasing if ....

(a) $a > 0$ (b) $a > 1$ (c) $0 < a < 1$ (d) $a = 1$

19 $\lim\limits_{x \to 2} \frac{x^5 - 32}{x^4 - 16} =$ ....

(a) 2 (b) 20 (c) $\frac{5}{4}$ (d) $\frac{5}{2}$

20 If f is an odd function , a $\in$ the domain of f , then $f(a) + f(-a) =$ ....

(a) $2f(a)$ (b) $2f(-a)$ (c) zero (d) $f(a)$

21 The solution set in $\mathbb{R}$ of the equation : $|x - 3| = |9 - 2x|$ equals ....

(a) $\{4\}$ (b) $\{4, 6\}$ (c) $\{6\}$ (d) $\{2, 6\}$

22 Determine the type of the function $f : f(x) = x^2 + \sin x$ whether it is even , odd or otherwise.

23 $\lim\limits_{x \to \infty} \dfrac{5x^2 - 3x + 6}{1 + 4x - 7x^2} =$ ....

(a) $\frac{5}{7}$ (b) $\frac{-5}{7}$ (c) 6 (d) 35

24 The range of the function $f : f(x) = \begin{cases} 2x + 3 & , \quad x > 3 \\ 9 & , \quad x < 3 \end{cases}$ is ....

(a) $\{3\}$ (b) $\mathbb{R}$ (c) $]9 , \infty[$ (d) $[9 , \infty[$

25 In Δ ABC , if m ($\angle$ B) = 60° , m ($\angle$ C) = 30° , c = 4 cm. , then b = ... cm.

(a) 4 (b) 8 (c) $2\sqrt{3}$ (d) $4\sqrt{3}$

26 $\lim\limits_{x \to 0} \dfrac{(x + 1)^{12} - 1}{x} =$ ....

(a) 1 (b) 6 (c) zero (d) 12

27 If the area of Δ ABC is "x" and the radius length of its circumcircle is "r" , then $\dfrac{4rx}{abc} =$ ....

(a) $\dfrac{a}{\sin A}$ (b) cos A (c) 1 (d) r

28 If $f(x) = 7^{x+1}$, then the value of x which satisfies : $f(2x - 1) + f(x - 2) = 50$ equals ....

(a) 1 (b) 7 (c) zero (d) 2

29 If L , M are the roots of the equation : $x^2 - 4x + 4 = 0$, then $\log_2 L + \log_2 M =$ ....

(a) 2 (b) -2 (c) -4 (d) 4

30 If $\sqrt{x^2 - 2x + 1} > 4$, then $x \in$ ....

(a) $[-3 , 5]$ (b) $]-3 , 5[$ (c) $\mathbb{R} -]-3 , 5[$ (d) $\mathbb{R} - [-3 , 5]$

31 $\lim\limits_{x \to 1} \dfrac{1 - x^9}{x^7 - 1} =$ ....

(a) 2 (b) $\frac{9}{7}$ (c) $\frac{-9}{7}$ (d) $\frac{1}{7}$

32 In triangle ABC , a = 4 cm. , b = 7 cm. , m ($\angle$ A) = 112° , then the number of triangles satisfy these conditions equals ....

(a) 1 (b) 2 (c) 0 (d) infinite number.

Model 1

Choose the correct answer from the given ones :

1 The point of symmetry of curve of the function $f : f(X) = X^3$ is ....

(a) (1 , 1) (b) (0 , 0) (c) (1 , 0) (d) (0 , 1)

2 If $2^{X-5} = 3^{5-X}$, then $X =$ ....

(a) zero (b) $\frac{2}{3}$ (c) 5 (d) $\frac{3}{2}$

3 The domain of the function $f : f(X) = \sqrt{X-5}$ equals ....

(a) $]-\infty , 5]$ (b) $[5 , \infty[$ (c) $]-\infty , -5[$ (d) $]5 , \infty[$

4 The range of the function $f : f(X) = |X|$ equals ....

(a) $[0 , \infty[$ (b) $]0 , \infty[$ (c) $]-\infty , 0]$ (d) $]-\infty , 0[$

5 If $f(X) = 2^X$, then $f(-1) =$ ....

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

6 If $4 = \log_2 X$, then the exponential form that equivalent to it is ....

(a) $X^2 = 4$ (b) $X^4 = 2$ (c) $X = 16$ (d) $X = 8$

7 The curve of the function g : $g(X) = |X + 3|$ is the same as the curve of the function $f : f(X) = |X|$ by translation of 3 units in direction ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{OY}$ (d) $\overrightarrow{OY'}$

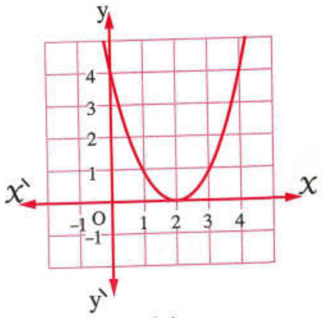
8 If $\log 3 = X$, $\log 4 = y$, then $\log 12 =$ ....

(a) $\log X + \log y$ (b) $X y$

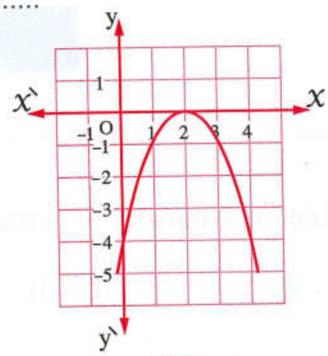
(c) $X - y$ (d) $X + y$

9 If f is a function where $f(X) = X^2 + 2$

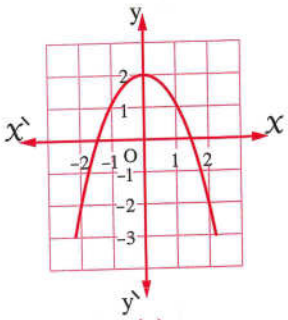
, then the graph that represents the function f is ....



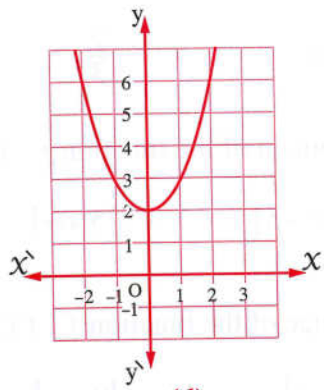
(a)



(b)



(c)



(d)

10 The domain of the function $f : f(X) = \frac{2X}{X^2 - 4}$ is ....

(a) $\mathbb{R} - \{-2, 2\}$ (b) $\mathbb{R} - \{-2, 0, 2\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{4\}$

11 Which of the following function is even function ....

(a) $\sin X$ (b) $X \cos X$ (c) $X \sin X$ (d) $\tan X$

12 The curve of the even function is symmetric about the straight line ....

(a) $y = 0$ (b) $X = 0$ (c) $y = X$ (d) $y + X = 0$

13 The solution set of the inequality : $|X| - 2 > 0$ is ....

(a) $\mathbb{R} - [-2, 2]$ (b) $]-2, 2[$ (c) $\mathbb{R} -]-2, 2[$ (d) $[-2, 2]$

14 If $7^{X+2} = 5$, then $X =$ ....

(a) $(\log_7 5) + 2$ (b) $(\log_7 5) - 2$ (c) $2 \log_7 5$ (d) $(\log_5 7) + 2$

15 $\lim_{x \to 0} \frac{x^2 + x}{2x} = \ldots\ldots$

(a) -1 (b) 1 (c) zero (d) $\frac{1}{2}$

16 $\lim_{x \to \infty} \frac{\sqrt{x^2}}{x} = \ldots\ldots$

(a) 1 (b) zero (c) 2 (d) -1

17 $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} = \ldots\ldots$

(a) $\frac{1}{2}$ (b) 2 (c) zero (d) 1

18 $\lim_{x \to \infty} x^{-5} = \ldots\ldots$

(a) ∞ (b) -5 (c) 5 (d) zero

19 $\lim_{x \to 1} \frac{x^5 - 1}{x - 1} = \ldots\ldots$

(a) 5 (b) 1 (c) 4 (d) 20

20 $\lim_{x \to 3} \frac{a}{x + 1} = 2$, then $a = \ldots\ldots$

(a) 6 (b) 2 (c) 4 (d) 8

21 $\lim_{x \to 3} \frac{x^2 - 3x}{2x - 6} = \ldots\ldots$

(a) 3 (b) 1.5 (c) $\frac{2}{3}$ (d) zero

22 $\lim_{x \to 0} \frac{(x + 3)^4 - 81}{x} = \ldots\ldots$

(a) 4 (b) 27 (c) $\frac{27}{4}$ (d) 108

23 In any ΔXYZ : $x^2 + y^2 - 2xy \cos Z = \ldots\ldots$

(a) x^2 (b) y^2 (c) z^2 (d) z

24 In ΔABC , $m(\angle A) = 30°$, $a = 6$ cm. , then $\frac{b}{\sin B} = \ldots\ldots$

(a) $\frac{1}{5}$ (b) 3 (c) 6 (d) 12

25 In ΔABC if $a = b$, then $\cos A = \ldots\ldots$

(a) $\frac{2b}{c}$ (b) $\frac{c}{2b}$ (c) $\frac{c}{4a}$ (d) $\frac{b}{2a}$

26 ABC is a triangle in which a = 8 cm. , b = 7 cm. , $\cos C = \frac{1}{2}$

, then the area of Δ ABC equals .... cm^2.

(a) 14 (b) $14\sqrt{3}$ (c) 8 (d) $28\sqrt{3}$

27 In any Δ ABC : 2 r sin A = ...

(a) a (b) b (c) c (d) area of Δ ABC

28 Measure of the greatest angle in triangle ABC where a = 3 cm. , b = 4 cm. , c = 5 cm. equals ....

(a) 30° (b) 60° (c) 90° (d) 120°

Model 2

Choose the correct answer from the given ones :

1 $\lim_{x \to 0} \frac{x^2 + x}{2x} =$ ....

(a) − 1 (b) 1 (c) 0 (d) $\frac{1}{2}$

2 The domain of the function $f(x) = \sqrt{x - 5}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$ (c) $[5, \infty[$ (d) $]-\infty, 5]$

3 If $f(x) = 2^x$, then $f(3) =$ ....

(a) 2 (b) 8 (c) $\frac{1}{8}$ (d) $\frac{1}{2}$

4 The symmetric point of the function $f : f(x) = \frac{3x - 2}{x}$ is ....

(a) (2 , 3) (b) (3 , 0) (c) (0 , 3) (d) (− 2 , 3)

5 The solution set of the equation $|x - 1| = 4$ is ....

(a) {5} (b) {− 3} (c) {5 , − 3} (d) ∅

6 The range of the function $f(x) = \frac{1}{x - 2}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{-2\}$

7 In the triangle XYZ , if $a < \frac{y^2 + z^2 - x^2}{yz} < b$, then (a , b) = ...

(a) (− 1 , 1) (b) (0 , 2) (c) (− 2 , 2) (d) $\left(\frac{-1}{2}, \frac{1}{2}\right)$

8 $\lim_{x \to \infty} \frac{\sqrt[3]{8x^3 - 2x + 1}}{5x}$ ....

(a) $\frac{5}{2}$ (b) $\frac{8}{5}$ (c) $\frac{2}{5}$ (d) zero

9 The measure of smallest angle of the triangle Δ ABC whose a = 8 cm. , b = 10 cm. , c = 7 cm. to nearest degree is ....

(a) 35° (b) 83° (c) 54° (d) 44°

10 $\lim\limits_{x \to 1} \frac{x^5 - 1}{x - 1} =$ ....

(a) 5 (b) 1 (c) 4 (d) 20

11 If $3^{x-7} = 5^{x-7}$, then $x =$ ....

(a) 4 (b) 5 (c) 6 (d) 7

12 $\log_5 2 \times \log_3 5 \times \log_2 3 =$ ....

(a) 30 (b) 1 (c) 0 (d) log 3

13 The range of the function $f : f(x) = |x|$ is ....

(a) $[0 , \infty[$ (b) $]0 , \infty[$ (c) $]-\infty , 0]$ (d) $]-\infty , 0[$

14 The solution set of the equation : $\log_3 x + \log_3 (x + 2) = 1$ is ....

(a) $\{1 , -3\}$ (b) $\{1\}$ (c) $\{-3\}$ (d) $\varnothing$

15 The triangle ABC whose m ($\angle$ A) = 30° , m ($\angle$ B) = 70° , c = 15 cm. , then a $\simeq$ .... cm.

(a) 6.7 (b) 7.6 (c) 8.4 (d) 5.6

16 The function $f : f(x) = a^x$ is descending on its domain $\mathbb{R}$ when ....

(a) a = 1 (b) a > 1 (c) 0 < a < 1 (d) a = − 1

17 $\lim\limits_{x \to 3} \frac{x^2 - 7x + 12}{x - 3} =$ ....

(a) 1 (b) − 1 (c) − 2 (d) 3

18 In the triangle Δ ABC , then $\frac{a}{a + b} = \frac{\sin A}{\text{...............}}$

(a) sin B (b) sin C (c) sin A + sin B (d) sin A + sin C

19 $\lim\limits_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3} =$ ....

(a) 4 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$

20 The function $f : f(x) = x^2 - 2$ is descending in the interval ....

(a) $]-\infty , -2[$ (b) $]-2 , \infty[$ (c) $]-\infty , 0[$ (d) $]0 , \infty[$

21 If $(x) = x^2 + 5$, $g(x) = x + 1$, then $(f + g)(3) =$ ....

(a) 12 (b) 14 (c) 16 (d) 18

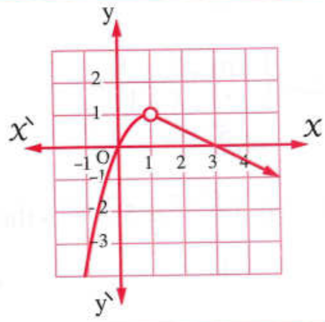
22 In the opposite figure

represent the curve of the function $f(x)$

, then $\lim_{x \to 1} f(x) =$ ....

(a) 0 (b) 1

(c) 2 (d) undefined



23 The curve of the function $g : g(x) = x^2 + 5$ is the same of the curve of the function $f : f(x) = x^2$ by a translation of magnitude 5 units in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{OY}$ (d) $\overrightarrow{OY'}$

24 $\lim_{x \to \infty} 3^{\frac{1}{x}} =$ ....

(a) 3 (b) zero (c) 1 (d) ∞

25 If the function f is odd function , then $\frac{5 f(x) + 3 f(-x)}{4 f(x)} =$ ....

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

26 In the triangle ΔABC , if $m(\angle A) : m(\angle B) : m(\angle C) = 3 : 5 : 4$, then $a^4 : c^4 =$ ....

(a) 9 : 4 (b) 9 : 16 (c) 4 : 9 (d) 81 : 256

27 $\lim_{x \to 4} \frac{x^3 - 64}{x - 4} =$ ....

(a) 96 (b) 48 (c) 32 (d) 16

28 In the triangle ΔABC , $\frac{\sin A}{3} = \frac{\sin B}{2} = \frac{\sin C}{4}$, then $m(\angle A) =$ ....

(a) $28^\circ\ 57'$ (b) $104^\circ\ 29'$ (c) $34^\circ\ 36'$ (d) $46^\circ\ 34'$

Model 3

Choose the correct answer from the given ones :

1 If f is an odd function , $a \in$ the domain of f , then $f(a) + f(-a) =$ ....

(a) $2 f(a)$ (b) $2 f(-a)$ (c) zero (d) $f(a)$

2 $\lim\limits_{x \to 2} \frac{x^5 - 32}{x^4 - 16} =$ ....

(a) 2 (b) 20 (c) $\frac{5}{4}$ (d) $\frac{5}{2}$

3 The solution set of the equation : $\log_{(x+3)} 125 = 3$ is ....

(a) $\{5\}$ (b) $\{3\}$ (c) $\varnothing$ (d) $\{2\}$

4 Number of possible solution of Δ ABC where m ($\angle$ A) = 60° , b = 3 cm. , a = 5 cm. is ....

(a) 1
(b) 2
(c) no solution
(d) infinite number of triangle

5 The domain of the function $f : f(x) = \sqrt{9 - x}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{9\}$ (c) $]-\infty , 9]$ (d) $[9 , \infty[$

6 The type of the function $f : f(x) = \frac{\sin x}{x}$ is ....

(a) even.
(b) odd.
(c) neither odd nor even.
(d) both odd and even.

7 $\lim\limits_{x \to 1} \frac{x^2 + 5x - 6}{x^2 - 1} =$ ....

(a) 1 (b) 5 (c) 6 (d) 3.5

8 $\frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30} =$ ....

(a) 1 (b) $\log_6 5$ (c) log 30 (d) 30

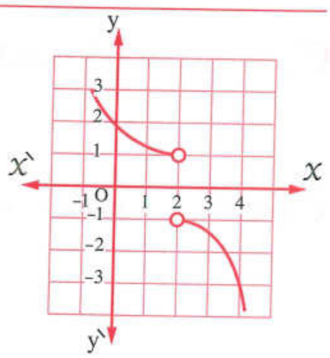
9 In Δ DEF , m ($\angle$ D) = 80° , m ($\angle$ E) = 60° , if f = 12 cm. , then d = ... cm.

(a) $\frac{12 \sin 80°}{\sin 40°}$ (b) $\frac{12 \sin 80°}{\sin 60°}$ (c) $\frac{12 \sin 40°}{\sin 80°}$ (d) $\frac{12 \cos 80°}{\cos 40°}$

10 The vertex point of the curve of the function $f : f(x) = x^2 + 3$ is ....

(a) (3 , 0) (b) (0 , 3) (c) (– 3 , 0) (d) (0 , – 3)

11 The opposite figure represents the curve of the function f , then $\lim\limits_{x \to 2} |f(x)| =$ ....



(a) 1
(b) – 1
(c) 2
(d) does not exits.

12 In Δ ABC , if $4 \sin A = 3 \sin B = 6 \sin C$, then m ($\angle$ C) $\simeq$ .... (to nearest degree)

(a) 89° (b) 29° (c) 57° (d) 82°

13 The function $f : f(x) = a^x$ is increasing if ....

(a) $a > 0$ (b) $a > 1$ (c) $a = 1$ (d) $0 < a < 1$

14 The solution set of the equation : $x^{\frac{4}{3}} = 81$ in $\mathbb{R}$ is ....

(a) $\{27 , -27\}$ (b) $\{9 , -9\}$ (c) $\{9\}$ (d) $\{27\}$

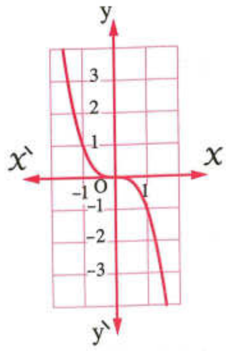
15 ABC is an equilateral triangle , its side length = $5\sqrt{3}$ cm. , then the diameter length of its circumcircle equals .... cm.

(a) $5\sqrt{3}$ (b) $10\sqrt{3}$ (c) 10 (d) 5

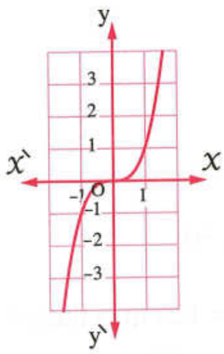
16 In Δ ABC : $a^2 + b^2 - c^2 =$ ....

(a) cos A (b) a b cos C (c) cos C (d) 2 a b cos C

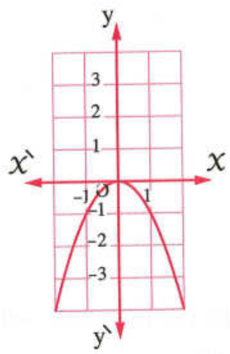
17 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^3$, then the figure which represents the function f is ....



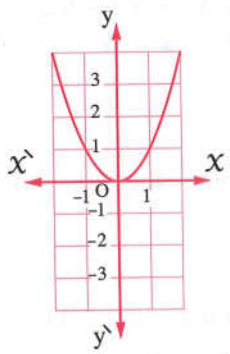
(a)



(b)



(c)



(d)

18 $\lim_{x \to 1} \frac{2x + a}{x + 1} = 5$, then a = ...

(a) 2 (b) 5 (c) 8 (d) 10

19 The range of the function $f : f(x) = \frac{5}{x} + 2$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\{2\}$ (d) $\mathbb{R} - \{0\}$

20 If ABC is a triangle in which a = 4 cm. , b = $4\sqrt{3}$ cm. , c = 8 cm. , then sine of its smallest angle = ...

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) zero

21 The solution set of the inequality : $|3 - 2x| \le 1$ in $\mathbb{R}$ is ....

(a) $[1 , 2]$ (b) $]1 , 2[$ (c) $\mathbb{R} -]1 , 2[$ (d) $\mathbb{R} - [1 , 2]$

22 $\lim_{x \to \infty} \frac{(12)^{\frac{1}{x}}}{x + 7} =$ ....

(a) $\frac{12}{7}$ (b) ∞ (c) 1 (d) zero

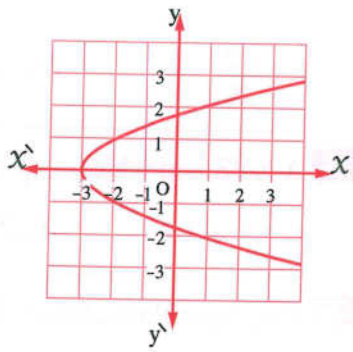
23 The domain of the function $f : f(x) = \frac{1}{|x| - 3}$ is ....

(a) $\{3 , -3\}$ (b) $[-3 , 3]$ (c) $\mathbb{R} - [-3 , 3]$ (d) $\mathbb{R} - \{-3 , 3\}$

24 $\lim_{x \to \infty} \frac{5x^2 - 3x + 6}{1 + 4x - 7x^2} =$ ....

(a) $\frac{5}{7}$ (b) $\frac{-5}{7}$ (c) 6 (d) 35

25 The curve represented in the opposite figure is symmetric about the straight line whose equation is ....



(a) $x = 0$ (b) $y = 0$

(c) $y = -2$ (d) $x = 2$

26 $\lim_{x \to 0} \frac{(2x + 1)^2 - 1}{x} =$ ....

(a) 4 (b) -3 (c) -4 (d) -2

27 $\log_a (x + 2) - \log_a (x - 1) = \log_a 4$, then $x =$ ....

(a) -2 (b) 2 (c) 1 (d) -1

28 $\lim_{x \to 0} \frac{x^2 + x}{x^3 + x} =$ ....

(a) $\frac{2}{3}$ (b) 1 (c) zero (d) does not exist.

Model 4

Choose the correct answer from the given ones :

1 If $4^{x+2} = 5^{x+2}$, then $6^{x+1} =$ ....

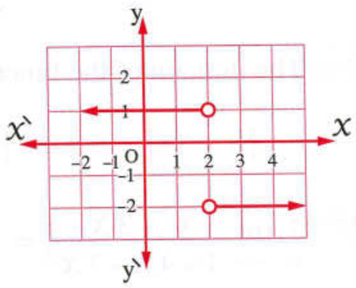
(a) -6 (b) -2 (c) zero (d) $\frac{1}{6}$

2 **From the opposite figure :**

Range of the function is ....

(a) $\{2\}$ (b) $]-\infty , \infty[$

(c) $\{-2 , 1\}$ (d) $]-2 , 1[$



3 $\lim\limits_{x \to \pi} \frac{2x}{\cos x} =$ ....

(a) $\frac{-2}{\pi}$ (b) $\frac{-\pi}{2}$ (c) -2π (d) 2π

4 Δ XYZ , in which , $\frac{1}{3} \sin X = \frac{1}{6} \sin Y = \frac{1}{7} \sin Z$, then $x : y : z =$ ....

(a) $3 : 6 : 7$ (b) $2 : 1 : 7$ (c) $7 : 6 : 3$ (d) $7 : 3 : 6$

5 The logarithmic from of $x^3 = 125$ is ....

(a) $\log_x 3 = 125$ (b) $\log_3 x = 125$ (c) $\log_x 125 = 3$ (d) $\log_3 125 = x$

6 $\lim\limits_{x \to 3} \frac{kx}{x-4} = 6$, then $k =$ ....

(a) -6 (b) -2 (c) 2 (d) 3

7 In Δ DHE If $\frac{d^2 + e^2 - h^2}{2\,de} = 0$, then = ...

(a) $m(\angle D) = 90°$ (b) $m(\angle H) = 150°$ (c) $m(\angle E) = 90°$ (d) $m(\angle H) = 90°$

8 If $\sqrt[3]{x^2} = 4$, then $x \in$ ....

(a) $\{-4 , 4\}$ (b) $\{8\}$ (c) $\{-8 , 8\}$ (d) $\{16\}$

9 If $\log_3 27^x - \log_9 9^x = 8$, then $x =$ ....

(a) 3 (b) 4 (c) 6 (d) 8

10 Which of the following is an even function :

(a) $f(x) = \tan 4x$ (b) $f(x) = x^2 + 2x$

(c) $f(x) = 7$ (d) $f(x) = x^3 \cos x$

11 **From the opposite figure :**

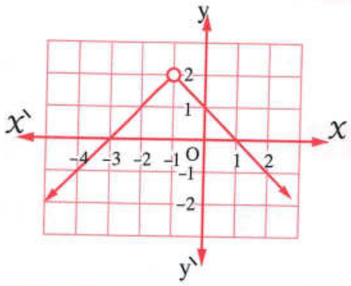
$\lim_{x \to 0} f(x) =$ ....

(a) -1

(b) zero

(c) 1

(d) 2



12 Δ XYZ , in which , y = 6 cm. , z = 10 cm. , m ($\angle$ X) = 120°
, then the perimeter of the triangle = ... cm.

(a) 24.72 (b) 26.3 (c) 28.88 (d) 30

13 Curve of the function $g(x) = x^3 + 2$, is the same as curve of the function $f(x) = x^3$ by translation 2 units in direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OY}$ (c) $\overrightarrow{OX`}$ (d) $\overrightarrow{OY`}$

14 $\lim_{x \to -2} \frac{x^2 - 4}{x^2 - 3x - 10} =$ ....

(a) zero (b) $-\frac{4}{7}$ (c) $\frac{4}{7}$ (d) does not exist.

15 If curve of the function $f(x) = \log_3 x$ passes through point (27 , C) , then C = ...

(a) 3 (b) 9 (c) 27 (d) 81

16 $\lim_{x \to \infty} \frac{2x^{-2} + 3}{5x^{-2} - 4} =$ ....

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $-\frac{1}{2}$ (d) $-\frac{3}{4}$

17 S.S. of the inequality : $|2x - 1| > 1$ is .... in $\mathbb{R}$

(a) $]0 , 1[$ (b) $\mathbb{R} -]0 , 1[$ (c) $[0 , 1]$ (d) $\mathbb{R} - [0 , 1]$

18 If function : $f(x) = 2 - (x - 3)^2$, then the function is decrease on the interval ....

(a) $]-\infty , -3[$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]2 , \infty[$

19 If $\lim_{x \to \infty} \frac{K x^2}{3x^2 + 1} = 4$, then K = ...

(a) 16 (b) 12 (c) 7 (d) $\frac{4}{3}$

20 Area of the circumcircle of Δ DHE in which h = 10 sin H equals .... cm^2.

(a) 10π (b) 20π (c) 25π (d) 100π

21 If $3^{x+1} - 3^{x} = 54$, then $x =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

22 Δ LMN , L = 13 cm. , M = 14 cm. , N = 15 cm. , then sin L = ...

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{5}{3}$

23 If $f : \{-2 , -1 , 0 , 1 , 2\} \longrightarrow \mathbb{R}$, $f(x) = x^2 + 2$

, then range of the function is ....

(a) $\mathbb{R}$ (b) $[2 , 6]$ (c) $[2 , \infty[$ (d) $\{2 , 3 , 6\}$

24 $\lim\limits_{x \to \infty} \left(\frac{1}{3x^2} + \frac{4x}{5+x}\right) =$ ....

(a) zero (b) $\frac{4}{5}$ (c) 4 (d) ∞

25 If $f(x) = 3^{x+2}$, then $f(x+1) \times f(-x) =$ ....

(a) 27 (b) 81 (c) 243 (d) 729

26 In any Δ XYZ , cos (X + Y) = ...

(a) cos X + cos Y (b) cos Z (c) – cos X – cos Y (d) – cos Z

27 If point (2 , – 1) ∈ to the curve of an odd function , then point .... ∈ to the same function.

(a) (2 , 1) (b) (– 2 , 1) (c) (– 1 , 2) (d) (1 , – 2)

28 $\lim\limits_{h \to 0} \frac{(x+h)^7 - x^7}{7h} =$ ....

(a) 0 (b) 1 (c) x^7 (d) x^6

Model 5

Choose the correct answer from the given ones :

1 The point of the vertex of the curve of the function $f(x) = (x-2)^2 + 3$ is ....

(a) $(2, 3)$ (b) $(2, -3)$ (c) $(-2, 3)$ (d) $(-2, -3)$

2 If $3^{x-2} = 2^{x-2}$, then $x =$ ....

(a) 3 (b) -2 (c) 0 (d) 2

3 If f is an odd function on $[-x, x]$, then $f(-x) + f(x) =$ ....

(a) $2x$ (b) not defined (c) $-2x$ (d) 0

4 If $\log 3 = x$, $\log 5 = y$, then $\log 15 =$ ....

(a) $x + y$ (b) $x - y$ (c) xy (d) $\frac{x}{y}$

5 The range of the function $f : f(x) = \frac{x-3}{3-x}$ equal ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\{-1\}$

6 The solution set of the inequality $|2x - 3| \leq 1$ in $\mathbb{R}$ is ....

(a) $[1, 2]$ (b) $]1, 2[$ (c) $\mathbb{R} -]1, 2[$ (d) $\mathbb{R} - [1, 2]$

7 The curve of $g(x) = |x + 3|$ is the same curve of $f(x) = |x|$ by translation of magnitude 3 units in the direction ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{OY}$ (d) $\overrightarrow{OY'}$

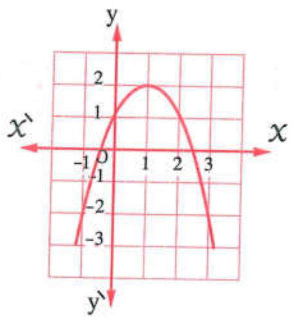
8 In all of the following relation, y is function in x except ....

(a) $y = 3x + 1$ (b) $y = x^2 - 1$ (c) $x = y^2 - 1$ (d) $y = \sin x$

9 If $\log_x (x + 6) = 2$, then $x =$ ....

(a) $\{3, -2\}$ (b) $\{3\}$ (c) $\{3, 1\}$ (d) $\{6, 1\}$

10 The rule of the function represented in the opposite figure is $f(x) =$ ....



(a) $(x-2)^2 + 1$

(b) $-(x-2)^2 + 1$

(c) $-(x-1)^2 + 2$

(d) $(-x+1)^2 + 2$

11 The domain of the function $f : f(x) = \sqrt{x-9}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{9\}$ (c) $]-\infty , 9[$ (d) $[9 , \infty[$

12 The type of the function $f : f(x) = \frac{\cos x}{x}$ is ....

(a) even. (b) odd.

(c) neither even nor odd. (d) both even and odd.

13 The curve of the function $f(x) = x^2 + 4$ is increasing in interval ....

(a) $]0 , \infty[$ (b) $[0 , \infty[$ (c) $]-\infty , 0]$ (d) $]-\infty , 0[$

14 The function $f : f(x) = a^x$ is decreasing if ....

(a) $a = 1$ (b) $a > 1$ (c) $0 < a < 1$ (d) $a = -1$

15 $\lim\limits_{x \to \infty} \frac{3x+4}{7x^2+5} =$ ....

(a) 3 (b) 0 (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

16 $\lim\limits_{x \to 1} (2x - 5) =$ ....

(a) -3 (b) -2 (c) 0 (d) 2

17 $\lim\limits_{x \to 0} \frac{(x+2)^5 - 32}{x} =$ ....

(a) 25 (b) 64 (c) 80 (d) 100

18 $\lim\limits_{x \to 3} \frac{x-3}{x^2-9} =$ ....

(a) 3 (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

19 $\lim\limits_{x \to -3} \frac{\sqrt{x+7}-2}{x+3} =$ ....

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$

20 $\lim\limits_{x \to 3} \frac{x^2-7x+12}{x-3} =$ ....

(a) 1 (b) -1 (c) -2 (d) 3

21 If $\lim\limits_{x \to 1} \frac{x^2-k^2}{x+2} = -1$, then $k =$ ....

(a) 2 (b) -2 (c) 4 (d) ± 2

22 $\lim\limits_{x \to \frac{\pi}{2}} (2x - \cos x) =$ ....

(a) 0 (b) 2 (c) π (d) $\frac{\pi}{2}$

---

23 The measure of the greatest angle of the triangle whose sides length are 3 cm. , 5 cm. and 7 cm. is ....°

(a) 150 (b) 120 (c) 60 (d) 30

---

24 In triangle ABC , $\frac{a}{\sin A} = 6$ cm. , then the length of the radius of the circumcircle of triangle = ... cm.

(a) 2 (b) 3 (c) 5 (d) 6

---

25 In triangle ABC , m (∠ B) = 60° , m (∠ C) = 30° and c = 4 cm. , then b = ... cm.

(a) 4 (b) 8 (c) $2\sqrt{3}$ (d) $4\sqrt{3}$

---

26 If ABC is triangle , a = 4 cm. , $b = 4\sqrt{3}$ cm. and c = 8 cm. , then cos A = ...

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) 0

---

27 In triangle ABC , if 4 sin A = 3 sin B = 6 sin C , then m (∠ C) = ...

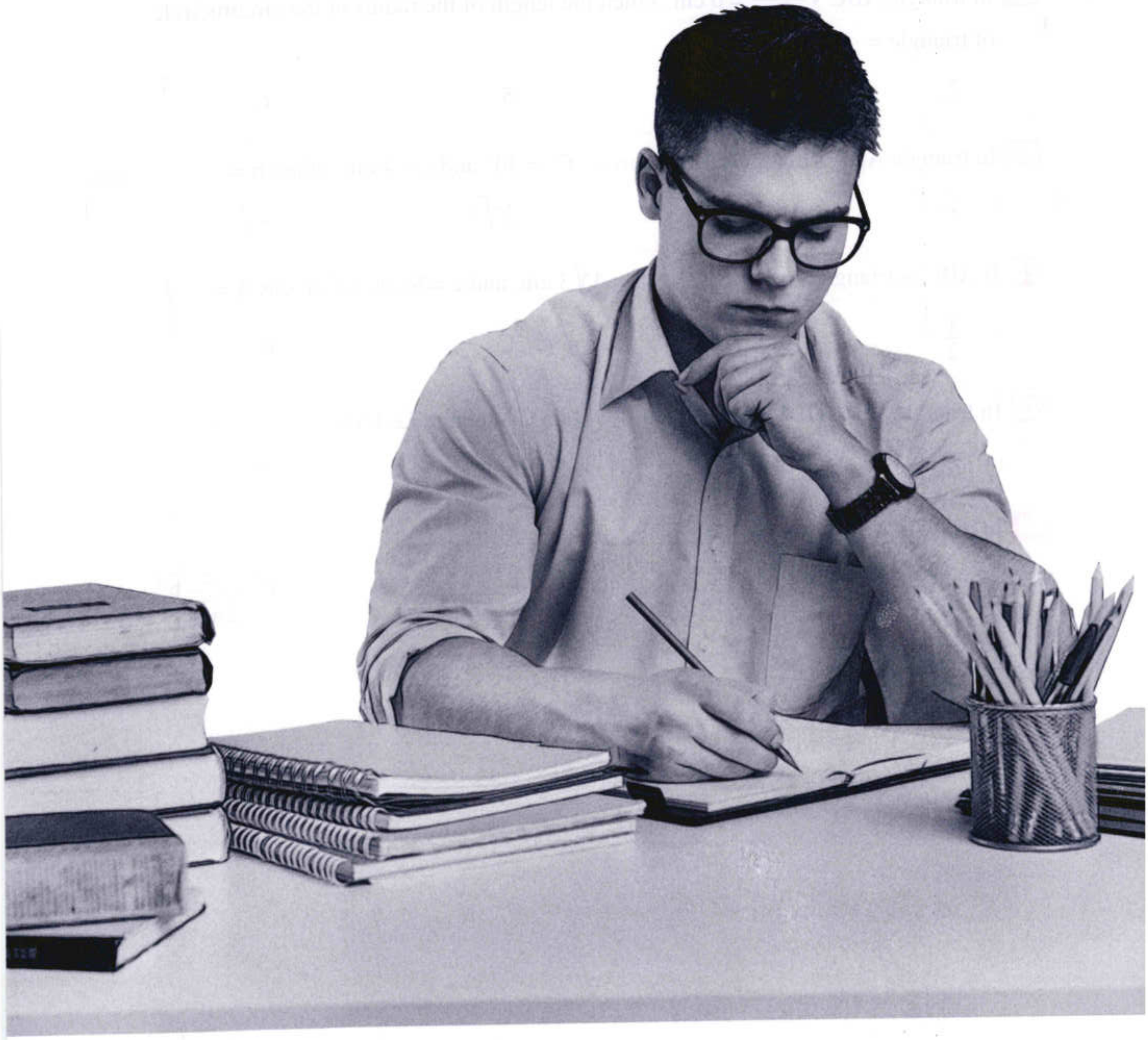
(a) 89° (b) 29° (c) 57° (d) 82°

---

28 In triangle ABC , cos (A + B) = ...

(a) $\frac{a^2 + b^2 - c^2}{2ab}$ (b) $\frac{a^2 + c^2 - b^2}{2ab}$ (c) $\frac{b^2 + c^2 - a^2}{2bc}$ (d) $\frac{c^2 - a^2 - b^2}{2ab}$

Guide answers



Guide answers of accumulative quizzes on algebra

Accumulative quiz 1

1 (1) a (2) d (3) d (4) a

2

(1) increasing on $]0, 2[$ and constant on $]2, \infty[$

(2) increasing on $]-1, 0[$ and decreasing on $]0, 4[$

(3) decreasing on $]-\infty, 1[$ and increasing on $]1, \infty[$

Accumulative quiz 2

1 (1) b (2) c (3) b (4) d

2 (1) Even function (2) Odd function

(3) Neither odd nor even

Accumulative quiz 3

1

Graph by yourself , range = $[0, \infty[$

, its type is neither odd nor even

* Decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$

2

Graph by yourself , range = $[0, 2]$

* Decreasing on $]-2, 0[$ and increasing on $]0, 2[$

3

Graph by yourself , Range = $]1, 3] \cup \{-1\}$

Accumulative quiz 4

1 (1) d (2) d (3) b (4) c

2

Graph by yourself , domain = $\mathbb{R}$, range = $\mathbb{R}$

, increasing on $\mathbb{R}$, type : Neither odd nor even.

Accumulative quiz 5

1 (1) b (2) c (3) c (4) d

2 The solution set = $\{9, -7\}$

3 (1) $f(x) = |x+3| + 2$ (2) $(-3, 2)$

(3) Range = $[2, \infty[$,

monotony : Decreasing on $]-\infty, -3[$

Increasing on $]-3, \infty[$

(4) The symmetric axis equation : $x = -3$

Accumulative quiz 6

1 (1) a (2) a (3) c (4) a

2 (1) The solution set = $\{4, \frac{2}{3}\}$

(2) The solution set = $[-1, \frac{7}{3}]$

3

Graph by yourserlf , range = $[0, \infty[$

Monotony : Decreasing on $]-\infty, 3[$

Increasing on $]3, \infty[$

Type : Neither odd nor even.

Accumulative quiz 7

1 (1) b (2) d (3) c (4) c

(5) d (6) a

2 (1) The solution set = $\{15\}$

(2) The solution set = $\{0, 2\}$

Accumulative quiz 8

1 (1) c (2) b (3) c (4) c

2

Graph by yourself , the area of the triangle

= one square unit

3

The price after three years = 1092.727 pounds.

Accumulative quiz 9

1 (1) c (2) b (3) d (4) c

2

The function is even , the solution set = $\{-1, 1\}$

3

$x = -\frac{1}{2}$

Accumulative quiz 10

1 (1) a (2) c (3) d (4) a

2 (1) The solution set = $\{2\}$

(2) The solution set = $\{1.32\}$

(3) The solution set = $\{1\}$

(4) The solution set = $[-2, 5]$

3

Graph by yourself , Range of the function = $[2, \infty[$

Prove by yourself.

Guide answers of accumulative quizzes on calculus

Accumulative quiz 1

1 (1) zero (2) 2 (3) 3 (4) 2

2 (1) d (2) d (3) c

Accumulative quiz 2

1 (1) c (2) b (3) b (4) d

2

(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$

3

a = 12

Accumulative quiz 3

1 (1) c (2) d (3) d (4) a

2 (1) $\frac{1}{2}$ (2) 20 (3) 5 (4) 4

Accumulative quiz 4

1 (1) b (2) b (3) d (4) c

2 (1) $\frac{2}{3}$ (2) 80 (3) $\frac{5}{4}$ (4) 5

Guide answers of accumulative quizzes on Trigonometry

Accumulative quiz 1

1

(1) d (2) d (3) b (4) a

2

a ≃ 8 cm. , b ≃ 18 cm.

3

AB = $8\sqrt{2}$ cm. , AC ≃ 15.45 cm.

Accumulative quiz 2

1

(1) b (2) c (3) c (4) a

2

m (∠ A) = 120°

3

Perimeter of parallelogram ≃ 25.24 cm.

Accumulative quiz 3

1

(1) c (2) c (3) d (4) d

2

m (∠ A) = 70° , b ≃ 19.68 cm. , c ≃ 14.35 cm.

3

Prove by yourself.

Guide answers of school book examinations in Algebra

Model 1

1

(1) (b) (2) (b) (3) (a) (4) (c)

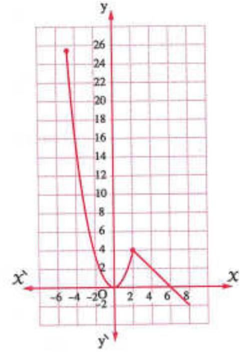
2

(a) * The range = ℝ − {0}

* The symmetry point = (0 , 0)

* The S.S. = {4}

(b)

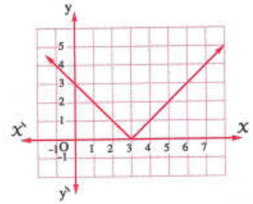


* The range = [−2 , 25]

* f is decreasing on each of]−5 , 0[,]2 , 8[and increasing on]0 , 2[

3

(a)



* The range = [0 , ∞[

* f is decreasing on]−∞ , 3[and increasing on]3 , ∞[

* f is neither even nor odd.

(b) (1) $X - 3 \geq 5$, then $X \geq 8$

or $X - 3 \leq -5$, then $X \leq -2$

∴ The S.S. = ℝ −]−2 , 8[

(2) ∵ $|X - 3| = 0$ ∴ $X - 3 = 0$

∴ $X = 3$ ∴ The S.S. = {3}

4

(a) (1) ∵ $\log X = \log (3 \times 10)$ ∴ $X = 30$

∴ The S.S. = {30}

(2) ∵ $3^{2X} - 3 \times 3^{X} = 0$ ∴ $3^{X}(3^{X} - 3) = 0$

∴ $3^{X} = 0$ (refused) or $3^{X} - 3 = 0$

∴ $3^{X} = 3$ ∴ $X = 1$

∴ The S.S. = {1}

(b) (1) $\frac{4^{2n+1} \times 2^{1-n}}{8^{n+2}} = \frac{(2^2)^{2n+1} \times 2^{1-n}}{(2^3)^{n+2}}$

$= \frac{2^{4n+2} \times 2^{1-n}}{2^{3n+6}}$

$= 2^{4n+2+1-n-3n-6}$

$= 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(2) $\log_6 54 - \log_6 9 = \log_6 \frac{54}{9} = \log_6 6 = 1$

5

(a) $\frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30} = \log_{30} 2 + \log_{30} 3 + \log_{30} 5$

$= \log_{30} (2 \times 3 \times 5)$

$= \log_{30} 30 = 1$

(b) (1) ∵ $f(-X) = (-X) + \sin(-X) = -X - \sin X$

$= -(X + \sin X)$

$= -f(X)$

∴ f is odd.

(2) ∵ $f(-X) = (-X)^3 - 2(-X)^2$

$= -X^3 - 2X^2 \neq f(X) \neq -f(X)$

∴ f is neither even nor odd.

Model 2

1

(1) (a) (2) (c) (3) (b) (4) (a)

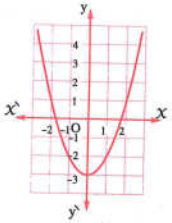
2

(a) ∴ $f(2) = |2 - 3| + |2 + 2| = 1 + 4 = 5$

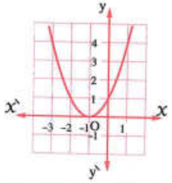
, $f(-1) = |-1 - 3| + |-1 + 2| = 4 + 1 = 5$

∴ $f(2) = f(-1)$

(b) (1)



(2)



3

(a) (1) $\because \text{Log}_2 X(X+1)=1$

$\therefore X^2+X=2 \qquad \therefore X^2+X-2=0$

$\therefore (X+2)(X-1)=0$

$\therefore X=-2$ (refused) or $X=1$

$\therefore$ The S.S. $=\{1\}$

(2) $3^X+3^{1+X}=36 \qquad \therefore 3^X(1+3)=36$

$\therefore 3^X\times 4=36 \qquad \therefore 3^X=9$

$\therefore 3^X=3^2 \qquad \therefore X=2$

$\therefore$ The S.S. $=\{2\}$

(b) (1) $\because 4^X+2^{X+1}=8 \qquad \therefore 2^{2X}+2(2)^X-8=0$

$\therefore (2^X+4)(2^X-2)=0$

$\therefore 2^X=-4$ (refused) or $2^X=2$

$\therefore X=1 \qquad \therefore$ The S.S. $=\{1\}$

(2) $\because$ L.H.S. $=\log_6(8\times 27)=\log_6 216$

$=\log_6 6^3=3\log_6 6=3$

, R.H.S. $=\log_3 3^3=3\log_3 3=3$

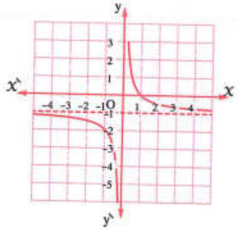
$\therefore$ L.H.S = R.H.S.

4

(a) $\because |X|<1 \qquad \therefore -1<X<1$

$\therefore$ The S.S. $=]-1,1[$

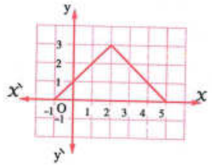
(b)



* The domain $=\mathbb{R}-\{0\}$
* The range $=\mathbb{R}-\{-1\}$
* f is decreasing on each of $]-\infty,0[$ and $]0,\infty[$
* f is neither even nor odd.

5

(a)



* The range $=[0,3]$
* f is increasing on $]-1,2[$ and decreasing on $]2,5[$
* f is neither even nor odd.

(b) (1) $\because 2^{X+1}=32 \qquad \therefore 2^{X+1}=2^5$

$\therefore X+1=5 \qquad \therefore X=4$

$\therefore$ The S.S. $=\{4\}$

(2) $\because 2^{X-2+1}=\frac{1}{8} \qquad \therefore 2^{X-1}=2^{-3}$

$\therefore X-1=-3 \qquad \therefore X=-2$

$\therefore$ The S.S. $=\{-2\}$

Guide answers of school book examinations in calculus & trigonometry

Model 1

1

(1) (c) (2) (c) (3) (d) (4) (a)

2

(a) (1) By dividing both of numerator and denominator by X^4, we get : $\lim_{X\to\infty}\dfrac{5+\frac{3}{X^2}-\frac{6}{X^4}}{\frac{2}{X^3}+1}=5$

(2) $\lim_{X\to-2}\dfrac{X+2}{X-3}=\dfrac{-2+2}{-2-3}=0$

(b) $\because a:b:c=2:3:4$

Let $a=2k$, $b=3k$, $c=4k$

, $\because$ c is the length of the largest side.

$\therefore \angle C$ is the largest angle.

$\therefore \cos C=\dfrac{(2k)^2+(3k)^2-(4k)^2}{2\times 2k\times 3k}$

$\therefore m(\angle C)\simeq 104°\,29'$

3

(a) (1) By dividing both of numerator and denominator by $X^2=\sqrt{X^4}$, we get : $\lim_{X\to\infty}\dfrac{\frac{4}{X^2}-3}{\sqrt{1+\frac{5}{X^4}}}=-3$

(2) $\lim_{X\to3}\dfrac{(\sqrt{X+1}-2)(\sqrt{X+1}+2)}{(X-3)(\sqrt{X+1}+2)}$

$=\lim_{X\to3}\dfrac{X+1-4}{(X-3)(\sqrt{X+1}+2)}$

$=\lim_{X\to3}\dfrac{1}{\sqrt{X+1}+2}=\dfrac{1}{4}$

(b) $\because c^2=8^2+6^2-2\times8\times6\cos 48° \qquad \therefore c\simeq 6$ cm.

$\therefore$ The perimeter of $\Delta ABC=20$ cm.

4

(a) (1) $\lim_{X\to3}\dfrac{(X-3)^2}{X-3}=\lim_{X\to3}(X-3)=0$

(2) $\lim_{X\to2}\dfrac{2(X-2)(X+2)}{X-2}=\lim_{X\to2}[2(X+2)]=8$

(b) (1) $2r=\dfrac{a}{\sin A}=\dfrac{21}{\sin 75°}\simeq 21.7$ cm.

(2) $2r=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}=\dfrac{c-b}{\sin C-\sin B}$

$=\dfrac{6}{\sin 65°-\sin 50°}$

$\simeq 42.8$ cm.

5

(a) (1) $\lim_{X\to3}\dfrac{(X-6+3)(X-6-3)}{(X+3)(X-3)}$

$=\lim_{X\to3}\dfrac{(X-3)(X-9)}{(X+3)(X-3)}=\lim_{X\to3}\dfrac{X-9}{X+3}=-1$

(2) $\lim_{X\to-1}\dfrac{2X^3-2X-X^2+1}{X^3+1}$

$=\lim_{X\to-1}\dfrac{2X(X^2-1)-(X^2-1)}{X^3+1}$

$=\lim_{X\to-1}\dfrac{(X^2-1)(2X-1)}{X^3+1}$

$=\lim_{X\to-1}\dfrac{(X-1)(X+1)(2X-1)}{(X+1)(X^2-X+1)}$

$=\lim_{X\to-1}\dfrac{(X-1)(2X-1)}{X^2-X+1}=2$

(b) $\because m(\angle B)=180°-(36°+45°)=99°$

, $\because \dfrac{b}{\sin B}=2r \qquad \therefore r=\dfrac{b}{2\sin B}=\dfrac{9}{2\sin 99°}$

$\therefore$ The area of the circumcircle of Δ

$=\pi r^2=\pi\left(\dfrac{9}{2\sin 99°}\right)^2\simeq 65.2$ cm².

Model 2

1

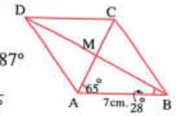
(1) (c) (2) (d) (3) (b) (4) (d)

2

(a) (1) $\lim_{X\to2}\dfrac{X^5-2^5}{X-2}=5\times2^4=80$

(2) $\lim_{(X-2)\to-1}\dfrac{(X-2)^4-(-1)^4}{(X-2)-(-1)}=4(-1)^3=-4$

(b) In Δ ABM :



$\because m(\angle AMB)$

$=180°-(65°+28°)=87°$

$\therefore \dfrac{BM}{\sin 65°}=\dfrac{AM}{\sin 28°}=\dfrac{7}{\sin 87°}$

$\therefore BM=\dfrac{7\sin 65°}{\sin 87°} \qquad \therefore BD=2\,BM\simeq 12.7$ cm.

, $AM=\dfrac{7\sin 28°}{\sin 87°} \qquad \therefore AC=2\,AM\simeq 6.6$ cm.

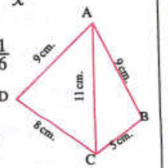
3

(a) (1) $\lim\limits_{x \to 3} \frac{x^3 - 3^3}{x^2 - 3^2} = \frac{3}{2} \times 3^{3-2} = \frac{9}{2}$

(2) By dividing both of numerator and denominator

By x^2, we get : $\lim\limits_{x \to \infty} \frac{4 + \frac{1}{x^2}}{1 - \frac{2}{x^2}} = 4$

(b) In Δ ABC :



$\cos B = \frac{(9)^2 + (5)^2 - (11)^2}{2 \times 9 \times 5} = -\frac{1}{6}$

, in Δ ADC :

$\cos D = \frac{(9)^2 + (8)^2 - (11)^2}{2 \times 9 \times 8} = \frac{1}{6}$

$\therefore \cos B = -\cos D$

$\therefore m(\angle B) + m(\angle D) = 180°$

$\therefore$ ABCD is a cyclic quadrilateral.

4

(a) (1) $\lim\limits_{x \to 1} \frac{(x-1)(x+6)}{(x-1)(x+1)} = \lim\limits_{x \to 1} \frac{x+6}{x+1} = \frac{7}{2}$

(2) $\lim\limits_{(x+1) \to 2} \frac{(x+1)^5 - 2^5}{(x+1) - 2} = 5 \times 2^4 = 80$

(b) $\because a^2 = (2.5)^2 + (2)^2 - 2 \times 2.5 \times 2 \times \frac{2}{5} = 6.25$

$\therefore a = 2.5$ cm. $\therefore \Delta$ ABC is isosceles.

5

(a) (1) $\lim\limits_{x \to 1} \frac{(x-1)(x^2 + x - 1)}{(x-1)(x+1)}$ «By long division»

$= \lim\limits_{x \to 1} \frac{x^2 + x - 1}{x + 1} = \frac{1}{2}$

(2) $\lim\limits_{x \to 1} \left(\frac{1}{x} + 3\right) = 4$

(b) $\because m(\angle A) = 180° - (35° + 70°) = 75°$

$\therefore \frac{a}{\sin 75°} = \frac{b}{\sin 35°} = \frac{c}{\sin 70°} = 32$

$\therefore a = 32 \sin 75°$, $b = 32 \sin 35°$, $c = 32 \sin 70°$

$\therefore$ The area of the triangle

$= \frac{1}{2} \times 32 \sin 75° \times 32 \sin 35° \times 32 \sin 70°$

$\simeq 267$ cm^2.

, the perimeter of the triangle

$= 32 \sin 75° + 32 \sin 35° + 32 \sin 70° \simeq 79$ cm.

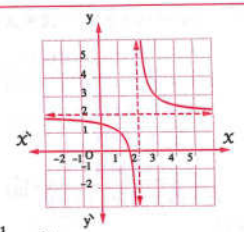
Guide answers of examination models

Model 1

1 (b) **2** (b) **3** (b) **4** (b)

5 (d) **6** (b) **7** (b)

8



$g(x) = \frac{1}{x-2} + 2$

The range of g is $\mathbb{R} - \{2\}$, g is decreasing on $]-\infty, 2[$, $]2, \infty[$

9 (c) **10** (d)

11

$\because \cos B = \frac{3}{5}$ (positive) $\therefore \angle B$ is an acute

$\therefore m(\angle B) \approx 53° 8'$

, $\because \tan C = \frac{5}{12}$ (positive) $\therefore \angle C$ is acute

$\therefore m(\angle C) \approx 22° 37'$ $\therefore m(\angle A) = 104° 15'$

$\therefore \frac{21}{\sin 104° 15'} = \frac{b}{\sin 53° 8'} = \frac{c}{\sin 22° 37'}$

$\therefore b \approx 17.3$ cm., $c \approx 8.3$ cm.

12 (b) **13** (a) **14** (c) **15** (a) **16** (c)

17 (b) **18** (a) **19** (d) **20** (d)

21

By dividing both of numerator and denominator by $(x = \sqrt{x^2})$

$\lim\limits_{x \to \infty} = \frac{\sqrt{4 + \frac{1}{x^2}}}{5 + \frac{2}{x}} = \frac{2}{5}$

22 (a) **23** (c) **24** (b) **25** (a)

26 (c) **27** (c)

28

$\because \sqrt{4x^2 - 12x + 9} \le 9$ $\therefore \sqrt{(2x-3)^2} \le 9$

$\therefore |2x - 3| \le 9$ $\therefore -9 \le 2x - 3 \le 9$

$\therefore -6 \le 2x \le 12$ $\therefore -3 \le x \le 6$

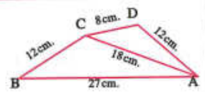
$\therefore$ S.S. $= [-3, 6]$

29 (a) **30** (c) **31** (d) **32** (a)

Model 2

1 (a) **2** (a)

3



In Δ ADC :

$\cos(\angle DAC)$

$= \frac{(12)^2 + (18)^2 - (8)^2}{2 \times 12 \times 18} = \frac{101}{108}$

, in Δ CAB :

$\cos(\angle CAB) = \frac{(27)^2 + (18)^2 - (12)^2}{2 (27)(18)} = \frac{101}{108}$

$\therefore m(\angle DAC) = m(\angle CAB)$

i.e. $\overrightarrow{AC}$ bisects $\angle$ BAD

4 (a) **5** (b) **6** (c)

7

$\frac{1}{x} = \frac{1}{5 + 2\sqrt{6}} \times \frac{5 - 2\sqrt{6}}{5 - 2\sqrt{6}} = \frac{5 - 2\sqrt{6}}{25 - 24} = 5 - 2\sqrt{6}$

$\therefore x + \frac{1}{x} = 5 + 2\sqrt{6} + 5 - 2\sqrt{6} = 10$

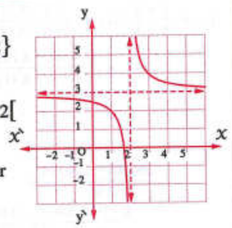
$\therefore \log\left(x + \frac{1}{x}\right) = \log 10 = 1$

8 (c) **9** (c) **10** (c) **11** (a)

12 (b) **13** (d) **14** (b) **15** (b)

16

- Domain of $g = \mathbb{R} - \{2\}$
- Range $= \mathbb{R} - \{3\}$
- Decreasing on $]-\infty, 2[$, $]2, \infty[$
- The function is neither even nor odd



17 (d) **18** (b) **19** (c) **20** (c)
21 (d) **22** (c) **23** (d) **24** (d)
25 (a) **26** (b) **27** (b)

28

$$\lim_{(x+2)\to 3} \frac{(x+2)^4-(3)^4}{(x+2)-3} = \frac{4}{1}(3)^3 = 108$$

29 (b) **30** (c) **31** (a) **32** (a)

Model 3

1 (a) **2** (c) **3** (d) **4** (c)
5 (d) **6** (d) **7** (b) **8** (b)
9 (a) **10** (a) **11** (d) **12** (a)
13 (c) **14** (d) **15** (d) **16** (d)
17 (b) **18** (a) **19** (a) **20** (b)

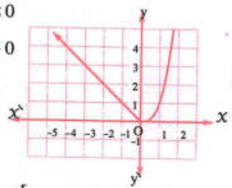
21

$$\text{L.H.S.} = \frac{2^x \times 3^{2x+2}}{3\times 2^x \times 3^{2x}}$$

$$= 3^{2x+2-1-2x} \times 2^{x-x} = 3^1 \times 2^0 = 3$$

22

$$f(x) = \begin{cases} -x &, x \le 0 \\ x^3 &, x > 0 \end{cases}$$



- The range is $[0, \infty[$
- f is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$

23 (a) **24** (b)

25

$\because \frac{a}{\sin A} = \frac{b}{\sin B}$ $\therefore \frac{5}{\sin 40^\circ} = \frac{7}{\sin B}$

$\therefore m(\angle B) = 64^\circ\ 8'\ 43''$ or $115^\circ\ 51'\ 17''$

$\therefore m(\angle C) = 75^\circ\ 51'\ 17''$ or $24^\circ\ 8'\ 43''$

$\therefore$ area of $\Delta ABC = \frac{1}{2}\, a\, b \sin C \approx 17\ cm^2$ or $7\ cm^2$

26 (c) **27** (c)

28

By dividing both of numerator and denominator by x^2

$$\therefore \lim_{x\to\infty} \frac{\left(1+\frac{1}{x}\right)\left(5-\frac{3}{x}\right)}{1+\frac{3}{x^2}} = 5$$

29 (c) **30** (d) **31** (d) **32** (b)

Model 4

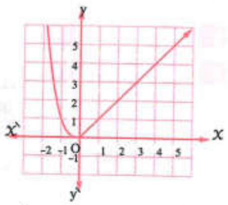
1 (d) **2** (a) **3** (a) **4** (b)
5 (b) **6** (a) **7** (b) **8** (d)
9 (c) **10** (b) **11** (c) **12** (a)

13

$$\because \lim_{x\to -2} \frac{3(x^2-4)}{x+2} = \lim_{x\to -2} \frac{3(x-2)(x+2)}{(x+2)} = -12$$

14 (d) **15** (c)

16



- The range $= [0, \infty[$
- The function is neither even nor odd
- f is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$

17 (c) **18** (a)

19

$f(x+1) - f(x-1) = 24$

$\therefore 2^{x+1} - 2^{x-1} = 24$ $\therefore 2^x(2-2^{-1}) = 24$

$\therefore 2^x = 16 = 2^4$ $\therefore x = 4$

20 (d) **21** (d) **22** (a)

23

$m(\angle A) = 3k$, $m(\angle B) = 4k$

, $m(\angle C) = 3k$ $\therefore 3k + 4k + 3k = 180^\circ$

$\therefore k = 18^\circ$ $\therefore m(\angle A) = 54^\circ$

$\therefore \frac{a}{\sin A} = 2r$ $\therefore \frac{5}{\sin 54^\circ} = 2r$

$\therefore$ circumference of the circumcircle of the triangle

$ABC = 2\pi r = \frac{5\pi}{\sin 54^\circ} \approx 19.4$ cm.

24 (a) **25** (b) **26** (b) **27** (a) **28** (b)
29 (a) **30** (b) **31** (c) **32** (c)

Model 5

1 (c) **2** (d) **3** (a) **4** (c)
5 (c) **6** (d) **7** (c)

8

$\because 3^{2x-1} - 4\times 3^x + 9 = 0$ (Multiplying by 3)

$\therefore 3^{2x} - 12\times 3^x + 27 = 0$ $\therefore (3^x-3)(3^x-9) = 0$

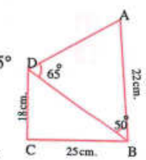
$\therefore 3^x = 3$ $\therefore x = 1$

or $3^x = 9 = 3^2$ $\therefore x = 2$

$\therefore$ The S.S. $= \{1, 2\}$

9 (c) **10** (c) **11** (c) **12** (d)
13 (b) **14** (b) **15** (c)

16



In ΔABD :

$m(\angle A) = 180^\circ - (50^\circ + 65^\circ) = 65^\circ$

$\therefore BD = AB = 22$ cm.

, in ΔBDC :

$\cos(\angle CBD) = \frac{(22)^2 + (25)^2 - (18)^2}{2\times 22\times 25}$

$\therefore m(\angle CBD) = 44^\circ\ 28'\ 6''$

17 (d) **18** (d) **19** (a) **20** (d)

21

* The domain of $f = \mathbb{R} - \{0\}$

* The point of symmetry is $(0, 0)$

* $\because f\left(\frac{1}{x}\right) = 4$ $\therefore x = 4$

$\therefore$ The S.S. $= \{4\}$

22 (b) **23** (b) **24** (b) **25** (a)
26 (c) **27** (c)

28

$$\lim_{x\to 0} \frac{\left(\sqrt{x+4}-2\right)\left(\sqrt{x+4}+2\right)}{x\left(\sqrt{x+4}+2\right)}$$

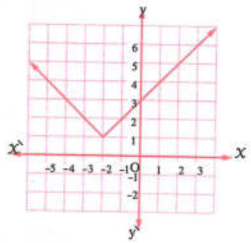
$$= \lim_{x\to 0} \frac{x}{x\left(\sqrt{x+4}+2\right)} = \frac{1}{4}$$

29 (c) **30** (a) **31** (c) **32** (d)

Model 6

1 (a) **2** (b) **3** (b) **4** (d)
5 (c) **6** (b) **7** (c) **8** (b)
9 (b) **10** (d) **11** (b) **12** (a)

13



* The range $= [1, \infty[$

* The function is decreasing at $]-\infty, -2[$ and increasing at $]-2, \infty[$

* The function neither even nor odd.

14 (a) **15** (d)

16

$\lim_{x \to \infty} \frac{(\sqrt{x^2+5x}-x)(\sqrt{x^2+5x}+x)}{\sqrt{x^2+5x}+x}$

$= \lim_{x \to \infty} \frac{x^2+5x-x^2}{\sqrt{x^2+5x}+x} = \lim_{x \to \infty} \frac{5x}{\sqrt{x^2+5x}+x}$

"dividing both numberator and denominator by $x = \sqrt{x^2}$"

$= \lim_{x \to \infty} \frac{5}{\sqrt{1+\frac{5}{x}}+1} = \frac{5}{2}$

17

The expression $= \log_2 \frac{\frac{3}{25} \times 5^5 \times 27}{\frac{125}{12} \times 243} = \log_2 4 = 2$

18 (b) **19** (b) **20** (d)
21 (c) **22** (b) **23** (b)

24

$m(\angle C) = 180° - (40° + 30°) = 110°$

$, \because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\therefore \frac{a}{\sin 40°} = \frac{10}{\sin 30°} = \frac{c}{\sin 110°}$

$\therefore a \simeq 12.9$ cm. , $c = 18.8$ cm.

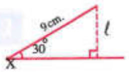
25 (a) **26** (a) **27** (a) **28** (d)
29 (b) **30** (b) **31** (c) **32** (b)

Model 7

1 (a) **2** (b) **3** (b) **4** (d)
5 (b) **6** (c) **7** (b)

8

$\because \angle X$ is acute and length of the perpendicular from Z to $\overline{XY}$



is $\ell = y \sin X = 9 \sin 30° = 4.5$ cm.

$\therefore \ell < x < y$

$\therefore$ There are two solutions

$\because \frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$ $\therefore \frac{6}{\sin 30°} = \frac{9}{\sin Y} = \frac{z}{\sin Z}$

$\therefore \sin Y = \frac{9 \sin 30°}{6} = \frac{3}{4}$ $\therefore m(\angle Y) \simeq 48°\ 35'\ 25''$

or $131°\ 24'\ 35''$

• At $m(\angle Y) = 48°\ 35'\ 25''$

$\therefore m(\angle Z) = 101°\ 24'\ 35''$

$\therefore \frac{6}{\sin 30°} = \frac{z}{\sin 101°\ 24'\ 35''}$ $\therefore z \simeq 11.8$ cm.

• At $m(\angle Y) = 131°\ 24'\ 35''$

$\therefore m(\angle Z) = 18°\ 35'\ 25''$

$\therefore \frac{6}{\sin 30°} = \frac{z}{\sin 18°\ 35'\ 25''}$ $\therefore z \simeq 3.8$ cm.

9 (c) **10** (d) **11** (d) **12** (d)

13

$\log_2 \frac{3}{25} + 5 \log_2 5 + \log_2 27 - \log_2 \frac{125}{12} - \log_2 243$

$= \log_2 \left(\frac{3}{25} \times 5^5 \times 27 \times \frac{12}{125} \times \frac{1}{243}\right)$

$= \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2$

14 (d) **15** (b) **16** (b)
17 (b) **18** (c) **19** (d)

20

By dividing both numerator and denominator by $x = \sqrt[3]{x^3}$

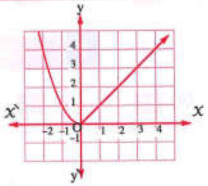
$\therefore \lim_{x \to \infty} \frac{2-\frac{3}{x}}{\sqrt[3]{125+\frac{5}{x^3}}} = \frac{2}{5}$

21 (c) **22** (d) **23** (b)

24

* The range $= [0, \infty[$

* The function is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$



25 (b) **26** (b) **27** (b) **28** (a)
29 (c) **30** (a) **31** (d) **32** (b)

Model 8

1 (d) **2** (c) **3** (c)

4

$\log_3 54 - \log_3 \frac{8}{15} + \log_3 \frac{4}{5}$

$= \log_3 \left(54 \times \frac{15}{8} \times \frac{4}{5}\right) = \log_3 81$

$= \log_3 3^4 = 4 \log_3 3 = 4$

5 (a) **6** (a) **7** (b) **8** (a)
9 (d) **10** (c) **11** (b) **12** (c)
13 (a) **14** (a) **15** (c)

16

$\lim_{x \to 5} \frac{x^2-5x}{\sqrt{x+4}-3} \times \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$

$= \lim_{x \to 5} \frac{x(x-5)(\sqrt{x+4}+3)}{(x-5)} = 30$

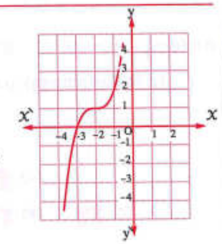
17 (d) **18** (a) **19** (d) **20** (d)

21

• The range $= \mathbb{R}$

• The function is increasing on its domain.

• The function is neither even nor odd.



22 (b) **23** (d) **24** (d)
25 (b) **26** (b) **27** (c)

28

$\because \angle C$ is obtuse , $c > a$

$\therefore$ exists unique solution of the triangle.

$\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\therefore \frac{10}{\sin A} = \frac{b}{\sin B} = \frac{12}{\sin 116°}$ $\therefore \sin A = \frac{10 \sin 116°}{12}$

$\therefore m(\angle A) = 48°\ 30'\ 12''$

$\therefore m(\angle B) = 180° - (116° + 48°\ 30'\ 12'') = 15°\ 29'\ 48''$

$\therefore b = \frac{12 \sin (15°\ 29'\ 48'')}{\sin 116°} \simeq 3.6$ cm.

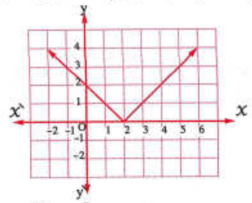
29 (a) **30** (d) **31** (b) **32** (c)

Model 9

1 (d) **2** (a) **3** (c) **4** (b)
5 (c) **6** (b) **7** (b) **8** (b)
9 (a) **10** (d) **11** (c) **12** (a)
13 (c) **14** (b) **15** (d)

16

$f(x) = \sqrt{x^2-4x+4} = \sqrt{(x-2)^2} = |x-2|$



• The range $= [0, \infty[$

• f is decreasing on $]-\infty, 2[$ and increasing on $]2, \infty[$

17 (c) **18** (b) **19** (b) **20** (a)

21

$\because \frac{21}{\sin A} = \frac{25}{\sin B} = \frac{c}{\sin C} = 28$

$\therefore \sin A = \frac{3}{4}$

$, \because$ the triangle is acute angled triangle.

$\therefore m(\angle A) = 48°\ 35'\ 25''$

$, \sin B = \frac{25}{28}$

$, \because$ the triangle is acute angled triangle.

$\therefore m(\angle B) = 63°\ 14'\ 4''$, $m(\angle C) = 68°\ 10'\ 31''$

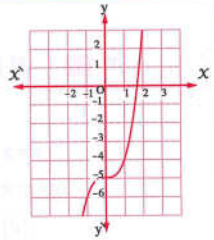
$\therefore c = 28 \sin (68°\ 10'\ 31'') \simeq 26$ cm.

22 (a) **23** (d) **24** (d)

25

* The function is increasing on $\mathbb{R}$

* The function is neither even nor odd.



26 (a) **27** (a)

28

The limit $= 9 \times \lim_{x \to 0} \frac{(9x+16)^{\frac{1}{2}} - (16)^{\frac{1}{2}}}{9x}$

$= 9 \times \lim_{x \to 0} \frac{(9x+16)^{\frac{1}{2}} - (16)^{\frac{1}{2}}}{(9x+16) - (16)}$

$= 9 \times \frac{1}{2} \times (16)^{-\frac{1}{2}} = \frac{9}{8}$

29 (b) **30** (b) **31** (c) **32** (d)

Model 10

1 (a) **2** (c) **3** (b) **4** (a)

5 (d) **6** (c) **7** (b)

8

$\lim_{(x-1) \to 1} \frac{(x-1)^6 - (1)^6}{(x-1) - 1} = 6\,(1)^5 = 6$

9

$\because 26 + b + c = 70$

$\therefore b + c = 44$ $\therefore b = 44 - c$

$, \because a^2 = b^2 + c^2 - 2\,b\,c \cos A$

$\therefore (26)^2 = (44 - c)^2 + c^2 - 2\,(44 - c)\,c \cos 60°$

$\therefore 676 = 1936 - 88\,c + c^2 + c^2 - 44\,c + c^2$

$\therefore 3\,c^3 - 132\,c + 1260 = 0$

$\therefore c^2 - 44\,c + 420 = 0$

$\therefore (c - 30)(c - 14) = 0$

$\therefore c = 30$ cm. , then $b = 14$ cm.

or $c = 14$ cm. , then $b = 30$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2}$ bc sin A

$= \frac{1}{2} \times 420 \times \sin 60°$

$= 105\sqrt{3}$ cm^2

10 (b) **11** (d) **12** (a) **13** (c)

14 (a) **15** (a) **16** (d)

17

$\therefore$ L.H.S. $= \log_5 \left(\frac{15}{7} \times \frac{35}{3} \times 5\right) = \log_5 125$

$= \log_5 5^3 = 3$

, R.H.S. $= \log_2 2^3 = 3$

$\therefore$ L.H.R. = R.H.S.

18 (b) **19** (d) **20** (c) **21** (b)

22

$\because f(-x) = (-x)^2 + \sin(-x) = x^2 - \sin x$

$\therefore f$ is neither even nor odd.

23 (b) **24** (d) **25** (d) **26** (d)

27 (c) **28** (a) **29** (a) **30** (d)

31 (c) **32** (c)

Guide answers of multiple choice examinations

Model 1

1 (b)

2 (c)

Solution :

$2^{x-5} = 3^{5-x}$ $\therefore \left(\frac{1}{2}\right)^{5-x} = 3^{5-x}$

$\therefore 5 - x = 0$ $\therefore x = 5$

3 (b)

Solution :

$x - 5 \geq 0$ $\therefore x \geq 5$

$x \in [5, \infty[$

4 (a) **5** (c)

6 (c)

Solution :

$x = 2^4 = 16$

7 (b)

8 (d)

Solution :

$\log 12 = \log (3 \times 4) = \log 3 + \log 4 = x + y$

9 (d)

10 (a)

Solution :

Let $x^2 - 4 = 0$

$\therefore x = \pm 2$

$\therefore$ The domain is $\mathbb{R} - \{2, -2\}$

11 (c)

Solution :

Let $f(x) = x \sin x$

$\therefore f(-x) = (-x) \sin(-x)$

$= x \sin x$

$= f(x)$

$\therefore f(x)$ is even

12 (b)

13 (a)

Solution :

$\because |x| - 2 > 0$ $\therefore |x| > 2$

$\therefore x \in \mathbb{R} - [-2, 2]$

14 (b)

Solution :

$\because 7^{x+2} = 5$ $\therefore x + 2 = \log_7 5$

$\therefore x = (\log_7 5) - 2$

15 (d)

Solution :

$\lim_{x \to 0} \frac{x^2 + x}{2x} = \lim_{x \to 0} \frac{x(x+1)}{2x} = \lim_{x \to 0} \frac{x+1}{2} = \frac{1}{2}$

16 (a)

Solution :

$\lim_{x \to \infty} \frac{|x|}{x} = \lim_{x \to \infty} \frac{x}{x} = \lim_{x \to \infty} 1 = 1$

17 (a)

Solution :

$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} \times \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1}$

$= \lim_{x \to 0} \frac{x + 1 - 1}{x\left(\sqrt{x+1} + 1\right)}$

$= \lim_{x \to 0} \frac{1}{\sqrt{x+1} + 1} = \frac{1}{2}$

18 (d)

19 (a)

Solution :

$\lim_{x \to 1} \frac{x^5 - 1^5}{x - 1} = \frac{5}{1}\,(1^{5-1}) = 5$

20 (d)

$\because \lim_{x \to 3} \frac{a}{x+1} = 2$ $\therefore \frac{a}{3+1} = 2$

$\therefore a = 8$

21 (b)

Solution :

$\lim_{x \to 3} \frac{x^2 - 3x}{2x - 6} = \lim_{x \to 3} \frac{x(x-3)}{2(x-3)} = \lim_{x \to 3} \frac{x}{2} = \frac{3}{2}$

22 (d)

Solution :

$\lim_{x \to 0} \frac{(x+3)^4 - 81}{x} = \lim_{x+3 \to 3} \frac{(x+3)^4 - 3^4}{x+3-3} = \frac{4}{1}(3)^{4-1} = 108$

23 (c)

24 (d)

Solution :

$\because \frac{a}{\sin A} = \frac{b}{\sin B}$ $\therefore \frac{6}{\sin 30} = \frac{b}{\sin B}$

$\therefore \frac{b}{\sin B} = 12$

25 (b)

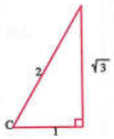
Solution :

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{b^2+c^2-b^2}{2bc} = \frac{c^2}{2bc} = \frac{c}{2b}$

26 (b)

Solution :

$\because \cos C = \frac{1}{2}$ $\therefore \sin C = \frac{\sqrt{3}}{2}$



$\therefore$ area of $(\Delta ABC) = \frac{1}{2} \times 8 \times 7 \times \frac{\sqrt{3}}{2}$

$= 14\sqrt{3}$ cm^2.

27 (a)

28 (c)

Solution :

greatest angle is opposite to longest side (c)

$\therefore \cos c = \frac{a^2+b^2-c^2}{2ab} = \frac{9+16-25}{2 \times 3 \times 4} = $ zero

m (∠ C) = 90°

Model 2

1 (d)

Solution :

$\lim_{x \to 0} \frac{x(x+1)}{2x} = \lim_{x \to 0} \frac{x+1}{2} = \frac{1}{2}$

2 (c)

Solution :

let $x - 5 \geq 0$

$\therefore x \geq 5$ $\therefore$ The domain is $[5, \infty[$

3 (b)

4 (c)

Solution :

$f(x) = \frac{3x-2}{x} = 3 - \frac{2}{x} = \frac{-2}{x} + 3$

$\therefore$ The symmetric point is (0 , 3)

5 (c)

Solution :

$\because |x - 1| = 4$

then $x - 1 = 4$ $\therefore x = 5$

or $x - 1 = -4$ $\therefore x = -3$

$\therefore$ The solution set $= \{5, -3\}$

6 (b)

7 (c)

Solution :

$\because -1 < \cos x < 1$

$\therefore -1 < \frac{y^2+z^2-x^2}{2yz} < 1$ (Multiply by 2)

$\therefore -2 < \frac{y^2+z^2-x^2}{yz} < 2$

$\therefore$ (a , b) = (– 2 , 2)

8 (c)

Solution :

$\lim_{x \to \infty} \frac{\sqrt[3]{8x^3-2x+1}}{5x} = \lim_{x \to \infty} \frac{\sqrt[3]{8 - \frac{2}{x^2} + \frac{1}{x^3}}}{5}$

$= \frac{\sqrt[3]{8}}{5} = \frac{2}{5}$

9 (d)

Solution :

The smallest angle is opposite to the shortest side (∠ C)

$\therefore \cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{8^2+10^2-7^2}{2 \times 8 \times 10} = \frac{23}{32}$

$\therefore$ m (∠ C) = 44° 2̀ 55̀ ≃ 44°

10 (a)

Solution :

$\lim_{x \to 1} \frac{x^5-1}{x-1} = \lim_{x \to 1} \frac{x^5-1^5}{x-1} = \frac{5}{1}(1)^{5-1} = 5$

11 (d)

Solution :

$\because 3^{x-7} = 5^{x-7}$

$\therefore x - 7 = 0$ $\therefore x = 7$

12 (b)

Solution :

$\log_5 2 \times \log_3 5 \times \log_2 3$

$= \frac{\log 2}{\log 5} \times \frac{\log 5}{\log 3} \times \frac{\log 3}{\log 2} = 1$

13 (a)

14 (b)

Solution :

$\because \log_3 x + \log_3 (x+2) = 1$ $\therefore \log_3 x(x+2) = 1$

$\therefore x^2 + 2x = 3^1$ $\therefore x^2 + 2x - 3 = 0$

$\therefore (x+3)(x-1) = 0$

then $x = -3$ (refused) or $x = 1$

$\therefore$ The solution set is {1}

15 (b)

Solution :

m (∠ C) = 180° – (30° + 70°) = 80°

$, \because \frac{a}{\sin A} = \frac{c}{\sin C}$

$\therefore \frac{a}{\sin 30°} = \frac{15}{\sin 80°}$ $\therefore a = \frac{15 \sin 30°}{\sin 80°} \simeq 7.6$ cm.

16 (c)

17 (b)

Solution :

$\lim_{x \to 3} \frac{x^2-7x+12}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x-4)}{\cancel{(x-3)}}$

$= \lim_{x \to 3} (x-4) = -1$

18 (c)

Solution :

$\because \frac{a}{\sin A} = \frac{b}{\sin B}$

$\therefore \frac{a}{\sin A} = \frac{a+b}{\sin A + \sin B}$ $\therefore \frac{a}{a+b} = \frac{\sin A}{\sin A + \sin B}$

19 (d)

Solution :

$\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3} \times \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$= \lim_{x \to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$

$= \lim_{x \to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$

$= \lim_{x \to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$

20 (c)

21 (d)

Solution :

$(f+g)(x) = x^2 + 5 + x + 1 = x^2 + x + 6$

$\therefore (f+g)(3) = 3^2 + 3 + 6 = 18$

22 (b) **23** (c) **24** (c)

25 (b)

Solution :

$\because f(x)$ is odd function

$\therefore f(-x) = -f(x)$

$\therefore \frac{5f(x)+3f(-x)}{4f(x)} = \frac{5f(x)-3f(x)}{4f(x)} = \frac{2f(x)}{4f(x)} = \frac{1}{2}$

26 (c)

Solution :

$\because$ m (∠ A) : m (∠ B) : m (∠ C) = 3 : 5 : 4

$\therefore$ m (∠ A) = 3 m , m (∠ B) = 5 m , m (∠ C) = 4 m.

where m ≠ 0

$\therefore$ 3 m + 5 m + 4 m = 180°

12 m = 180° $\therefore$ m = 15°

$\therefore$ m (∠ A) = 45° , m (∠ C) = 60°

$, \because a : c = \sin A : \sin C = \frac{1}{\sqrt{2}} : \frac{\sqrt{3}}{2}$

$\therefore a^4 : c^4 = \frac{1}{4} : \frac{9}{16} = 4 : 9$

27 (b)

Solution :

$\lim_{x \to 4} \frac{x^3-64}{x-4} = \lim_{x \to 4} \frac{x^3-4^3}{x-4}$

$= \frac{3}{1}(4)^{3-1} = 48$

28 (d)

Solution :

$\because \frac{\sin A}{3} = \frac{\sin B}{2} = \frac{\sin c}{4}$

$\therefore$ a : b : c = 3 : 2 : 4

$\therefore$ a = 3 m , b = 2 m , c = 4 m , where m ≠ 0

$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2\,b\,c} = \frac{(2\text{ m})^2 + (4\text{ m})^2 - (3\text{ m})^2}{2 \times 2\text{ m} \times 4\text{ m}}$

$= \frac{11\text{ m}^2}{16\text{ m}^2} = \frac{11}{16}$

$\therefore m(\angle A) = 46° \, 34'$

Model 3

1 (c)

Solution :

$\because f(X)$ is odd function

$\therefore f(-a) = -f(a)$

$\therefore f(a) + f(-a) = f(a) + (-f(a)) = 0$

2 (d)

Solution :

$\lim_{X \to 2} \frac{X^5 - 32}{X^4 - 16} = \lim_{X \to 2} \frac{X^5 - 2^5}{X^4 - 2^4} = \frac{5}{4}(2)^{5-4} = \frac{5}{2}$

3 (d)

Solution :

$\because \log_{X+3} 125 = 3$

$\therefore (X+3)^3 = 125$

$\therefore X + 3 = 5$ $\qquad \therefore X = 2$

$\therefore$ S.S. = {2}

4 (a)

5 (c)

Solution :

Let $9 - X \geq 0$ $\qquad \therefore -X \geq -9$

$\therefore X \leq 9$

$\therefore$ The domain is $]-\infty , 9]$

6 (a)

Solution :

$f(-X) = \frac{\sin(-X)}{-X} = \frac{-\sin X}{-X} = \frac{\sin X}{X} = f(X)$

$\therefore f(X)$ is even

7 (d)

Solution :

$\lim_{X \to 1} \frac{X^2 + 5X - 6}{X^2 - 1} = \lim_{X \to 1} \frac{(X+6)\cancel{(X-1)}}{\cancel{(X-1)}(X+1)}$

$= \lim_{X \to 1} \frac{X+6}{X+1} = \frac{7}{2}$

8 (a)

Solution :

$\frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30} = \log_{30} 2 + \log_{30} 3 + \log_{30} 5$

$= \log_{30} (2 \times 3 \times 5)$

$= \log_{30} 30 = 1$

9 (a)

Solution :

m (∠ F) = 180° − (80° + 60°) = 40°

$, \because \frac{d}{\sin D} = \frac{f}{\sin F}$

$\therefore \frac{d}{\sin 80°} = \frac{12}{\sin 40°}$ $\qquad \therefore d = \frac{12 \sin 80°}{\sin 40°}$

10 (b)

11 (a)

Solution :

$\lim_{X \to 2} |f(X)| = 1$



12 (b)

Solution :

$\because$ 4 sin A = 3 sin B = 6 sin C

(divide by 12)

$\therefore \frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{2}$

$\therefore$ a : b : c = sin A : sin B : sin C = 3 : 4 : 2

$\therefore$ a = 3 m , b = 4 m , c = 2 m where m ≠ 0

$\therefore \cos C = \frac{(3\text{ m})^2 + (4\text{ m})^2 - (2\text{ m})^2}{2 \times 3\text{ m} \times 4\text{ m}} = \frac{21\text{ m}^2}{24\text{ m}^2} = \frac{7}{8}$

$\therefore m(\angle C) = 28° 57' 18'' \simeq 29°$

13 (c)

14 (a)

Solution :

$\because X^{\frac{4}{3}} = 81$ $\qquad \therefore X = \pm (81)^{\frac{3}{4}} = \pm 27$

$\therefore$ The solution set = {27 , −27}

15 (c)

Solution :

$2\,r = \frac{a}{\sin A} = \frac{5\sqrt{3}}{\sin 60} = 10$ cm

16 (d)

Solution :

$\because \cos C = \frac{a^2 + b^2 - c^2}{2\,ab}$

$\therefore a^2 + b^2 - c^2 = 2\,ab \cos C$

17 (b)

18 (c)

Solution :

$\lim_{X \to 1} \frac{2X + a}{X + 1} = \frac{2 + a}{2}$

$\therefore \frac{2 + a}{2} = 5$ $\qquad \therefore 2 + a = 10$

$\therefore a = 8$

19 (b)

20 (a)

Solution :

a is the shortest side.

$\therefore \angle A$ is the smallest angle

$\cos A = \frac{(4\sqrt{3})^2 + (8)^2 - (4)^2}{2 \times 4\sqrt{3} \times 8} = \frac{\sqrt{3}}{2}$

$\therefore m(\angle A) = 30°$ $\qquad \therefore \sin A = \frac{1}{2}$

21 (a)

Solution :

$|3 - 2X| \leq 1$

$\therefore |2X - 3| \leq 1$ $\qquad \therefore -1 \leq 2X - 3 \leq 1$

$\therefore 2 \leq 2X \leq 4$ $\qquad \therefore 1 \leq X \leq 2$

$\therefore$ The solution set = [1 , 2]

22 (d)

Solution :

$\lim_{X \to \infty} \frac{12^{\frac{1}{X}}}{X + 7} = \lim_{X \to \infty} 12^{\frac{1}{X}} \times \lim_{X \to \infty} \frac{1}{X + 7} = 1 \times 0 = 0$

23 (d)

Solution :

Let $|X| - 3 = 0$

$\therefore |X| = 3$ $\qquad \therefore X = \pm 3$

$\therefore$ The domain is $\mathbb{R} - \{3 , -3\}$

24 (b)

Solution :

$\lim_{X \to \infty} \frac{5X^2 - 3X + 6}{1 + 4X - 7X^2} = \lim_{X \to \infty} \frac{5 - \frac{3}{X} + \frac{6}{X^2}}{\frac{1}{X^2} + \frac{4}{X} - 7} = \frac{-5}{7}$

25 (b)

26 (a)

Solution :

$\lim_{X \to 0} \frac{(2X+1)^2 - 1}{X} = \lim_{X \to 0} \frac{4X^2 + 4X + 1 - 1}{X}$

$= \lim_{X \to 0} \frac{4X(X+1)}{X}$

$= \lim_{X \to 0} 4(X+1) = 4$

27 (b)

Solution :

$\log_a (X+2) - \log_a (X-1) = \log_a 4$

$\therefore \log_a \frac{(X+2)}{(X-1)} = \log_a 4$

$\therefore \frac{X+2}{X-1} = 4$ $\qquad \therefore X + 2 = 4X - 4$

$\therefore 3X = 6$ $\qquad \therefore X = 2$

28 (b)

Solution :

$\lim_{X \to 0} \frac{X^2 + X}{X^3 + X} = \lim_{X \to 0} \frac{X(X+1)}{X(X^2+1)} = \lim_{X \to 0} \frac{X+1}{X^2+1} = 1$

Model 4

1 (d)

Solution :

$\because 4^{x+2} = 5^{x+2}$

$\therefore x + 2 = 0 \qquad \therefore x = -2$

$\therefore 6^{x+1} = 6^{-2+1} = \frac{1}{6}$

2 (c)

3 (c)

4 (a)

Solution :

$\frac{1}{3} \sin X = \frac{1}{6} \sin Y = \frac{1}{7} \sin Z$

$\therefore \frac{\sin X}{3} = \frac{\sin Y}{6} = \frac{\sin Z}{7}$

$\therefore x : y : z = \sin X : \sin Y : \sin Z = 3 : 6 : 7$

5 (c)

6 (b)

Solution :

$\therefore \lim_{x \to 3} \frac{kx}{x-4} = \frac{3k}{-1} = -3k$

$\therefore -3k = 6 \qquad \therefore k = -2$

7 (d)

Solution :

$\because \cos H = \frac{d^2 + e^2 - h^2}{2de}$

$\therefore \cos H = 0 \qquad \therefore m(\angle H) = 90^\circ$

8 (c)

Solution :

$\because \sqrt[3]{x^2} = 4 \qquad \therefore x^{\frac{2}{3}} = 4$

$\therefore x = \pm (4)^{\frac{3}{2}} = \pm 8 \qquad \therefore x \in \{8 , -8\}$

9 (b)

Solution :

$\because \log_3 27^x - \log_9 9^x = 8$

$\therefore x \log_3 27 - x \log_9 9 = 8 \qquad \therefore 3x - x = 8$

$\therefore 2x = 8 \qquad \therefore x = 4$

10 (c)

Solution :

$f(-x) = f(x) = 7 \qquad \therefore f(x)$ is even.

11 (c)

12 (d)

Solution :

$x^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos 120^\circ = 196$

$\therefore x = 14$

$\therefore$ The perimeter $= 14 + 6 + 10 = 30$ cm.

13 (b)

14 (c)

Solution :

$\lim_{x \to -2} \frac{x^2 - 4}{x^2 - 3x - 10} = \lim_{x \to -2} \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x-5)}$

$= \lim_{x \to -2} \frac{(x-2)}{(x-5)} = \frac{4}{7}$

15 (a)

Solution :

$f(27) = \log_3 27 = c$

$\therefore c = 3$

16 (d)

17 (d)

Solution :

$\because |2x - 1| > 1 \qquad \therefore 2x - 1 > 1$

$\therefore 2x > 2$

$x > 1$

and $2x - 1 < -1$

$2x < 0$

$x < 0$

$\therefore x \in \mathbb{R} - [0 , 1]$

18 (b)

19 (b)

Solution :

$\lim_{x \to \infty} \frac{kx^2}{3x^2 + 1} = \frac{k}{3 + \frac{1}{x^2}} = \frac{k}{3}$

$\therefore \frac{k}{3} = 4 \qquad \therefore k = 12$

20 (c)

Solution :

$\because h = 10 \sin H \qquad \therefore \frac{h}{\sin H} = 10 \qquad \therefore 2r = 10$

$\therefore r = 5$ cm.

the area of its circumcircle $= \pi (5)^2 = 25\pi$

21 (c)

Solution :

$\because 3^{x+1} - 3^x = 54 \qquad \therefore 3^x (3 - 1) = 54$

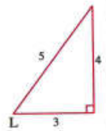
$\therefore 3^x = 27 = 3^3 \qquad \therefore x = 3$

22 (c)

Solution :

$\cos L = \frac{14^2 + 15^2 - 13^2}{2 \times 14 \times 15} = \frac{3}{5}$

$\therefore \sin L = \frac{4}{5}$



23 (d)

Solution :

x	-2	-1	0	1	2
$f(x)$	6	3	2	3	6

$\therefore$ The range is $\{6 , 3 , 2\}$

24 (c)

Solution :

$\lim_{x \to \infty} \left(\frac{1}{3x^2} + \frac{4x}{5 + x}\right) = \lim_{x \to \infty} \left(\frac{1}{3x^2} + \frac{4}{\frac{5}{x} + 1}\right)$

$=$ zero $+ 4 = 4$

25 (c)

Solution :

$f(x+1) \times f(-x) = 3^{(x+1)+2} \times 3^{(-x)+2}$

$= 3^{x+3} \times 3^{-x+2} = 3^5 = 243$

26 (d)

Solution :

$\cos (X + Y) = \cos (180 - Z) = -\cos Z$

27 (b)

28 (d)

Solution :

$\lim_{h \to 0} \frac{(x+h)^7 - x^7}{7h} = \frac{1}{7} \lim_{h \to 0} \frac{(x+h)^7 - x^7}{(x+h) - x}$

$= \frac{1}{7} \times 7x^6 = x^6$

Model 5

1 (a)

2 (d)

Solution :

$\because 3^{x-2} = 2^{x-2}$

$\therefore x - 2 = 0 \qquad \therefore x = 2$

3 (d)

4 (a)

Solution :

$\log 15 = \log (3 \times 5) = \log 3 + \log 5 = x + y$

5 (d)

Solution :

$f(x) = \frac{x-3}{3-x} = -1$ when $x \in \mathbb{R} - \{3\}$

$\therefore$ The range $= \{-1\}$

6 (a)

Solution :

$\because |2x - 3| \le 1$

$\therefore -1 \le 2x - 3 \le 1$ (add 3)

$2 \le 2x \le 4$ (divide by 2)

$1 \le x \le 2 \qquad$ S.S. $= [1 , 2]$

7 (b) **8** (c)

9 (b)

Solution :

$\because \log_x (x + 6) = 2 \qquad \therefore x^2 = x + 6$

$\therefore x^2 - x - 6 = 0 \qquad (x + 2)(x - 3) = 0$

$x = -2$ (refused) or $x = 3$

S.S. $= \{3\}$

10 (c)

11 (d)

Solution :

Let $x - 9 \ge 0 \qquad \therefore x \ge 9$

The domain is $[9 , \infty[$

12 (b)

Solution :

$\because f(-x) = \frac{\cos(-x)}{(-x)} = \frac{\cos x}{-x} = -\frac{\cos x}{x} = -f(x)$

$\therefore f(x)$ is odd

13 (a) **14** (c)

15 (b)

Solution :

$$\lim_{x \to \infty} \frac{3x+4}{7x^2+5} = \lim_{x \to \infty} \frac{\frac{3}{x}+\frac{4}{x^2}}{7+\frac{5}{x^2}} = \frac{\text{zero}}{7} = \text{zero}$$

16 (a)

17 (c)

Solution :

$$\lim_{x \to 0} \frac{(x+2)^5-32}{x} = \lim_{(x+2) \to 2} \frac{(x+2)^5-2^5}{(x+2)-2} = \frac{5}{1}(2)^{5-1} = 80$$

18 (d)

Solution :

$$\lim_{x \to 3} \frac{x-3}{x^2-9} = \lim_{x \to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \lim_{x \to 3} \frac{1}{x+3} = \frac{1}{6}$$

19 (b)

Solution :

$$\lim_{x \to -3} \frac{\sqrt{x+7}-2}{x+3} \times \frac{\sqrt{x+7}+2}{\sqrt{x+7}+2}$$

$$= \lim_{x \to -3} \frac{x+7-4}{(x+3)\left(\sqrt{x+7}+2\right)}$$

$$= \lim_{x \to -3} \frac{1}{\sqrt{x+7}+2} = \frac{1}{4}$$

20 (b)

$$\lim_{x \to 3} \frac{x^2-7x+12}{x-3} = \lim_{x \to 3} \frac{(x-4)\cancel{(x-3)}}{\cancel{(x-3)}} = \lim_{x \to 3} (x-4) = -1$$

21 (d)

Solution :

$\because \lim_{x \to 1} \frac{x^2-k^2}{x+2} = \frac{1-k^2}{3}$

$\therefore \frac{1-k^2}{3} = -1$ $\therefore 1-k^2 = -3$

$\therefore k^2 = 4$ $\therefore k = \pm 2$

22 (c)

Solution :

$$\lim_{x \to \frac{\pi}{2}} (2x - \cos x) = 2\left(\frac{\pi}{2}\right) - \cos\frac{\pi}{2} = \pi$$

23 (b)

Solution :

the greatest angle is the opposite to longest side (let it is called $\angle C$)

$\cos C = \frac{3^2+5^2-7^2}{2 \times 3 \times 5} = \frac{-1}{2}$

$\angle C = 120°$

24 (b)

25 (d)

Solution :

$\because \frac{b}{\sin B} = \frac{c}{\sin C}$ $\therefore \frac{b}{\sin 60} = \frac{4}{\sin 30}$

$\therefore b = \frac{4 \sin 60°}{\sin 30°} = 4\sqrt{3}$ cm.

26 (b)

Solution :

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{\left(4\sqrt{3}\right)^2+8^2-4^2}{2 \times 4\sqrt{3} \times 8} = \frac{\sqrt{3}}{2}$

27 (b)

Solution :

$\because 4 \sin A = 3 \sin B = 6 \sin C$ (divide by 12)

$\therefore \frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{2}$

$\therefore a : b : c = \sin A : \sin B : \sin C$

$= 3 : 4 : 2$

$\therefore a = 3m , b = 4m , c = 2m , m \neq 0$

$\therefore \cos C = \frac{(3m)^2+(4m)^2-(2m)^2}{2 \times 3m \times 4m} = \frac{7}{8}$

$\therefore m(\angle C) \simeq 29°$

28 (d)

Solution :

$\cos(A+B) = \cos(180 - C)$

$= -\cos C = -\frac{a^2+b^2-c^2}{2ab} = \frac{c^2-a^2-b^2}{2ab}$

ARTS SECTION

General Mathematics

By a group of supervisors



FIRST TERM

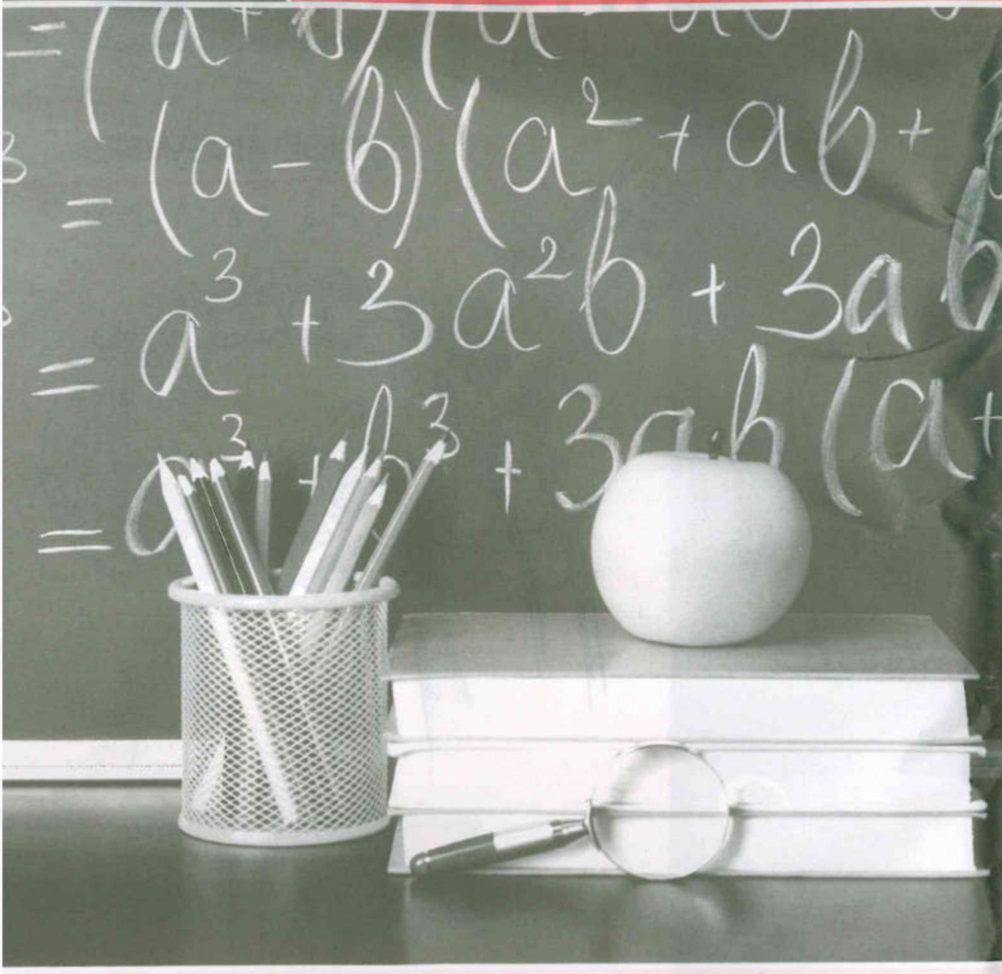
2 SEC. 2022

GUIDE ANSWERS

EL-MOASSER

FIRST

Answers of Algebra



Answers of "Unit One"

Answers of Pre-requirements

(1) (d) (2) (b) (3) (d) (4) (c)
(5) (c) (6) (a) (7) (c)

Exercise 1

First Multiple choice questions

(1) c (2) d (3) a (4) d (5) d (6) b
(7) a (8) d (9) c (10) b (11) c (12) d
(13) c (14) b (15) b (16) a (17) b (18) a
(19) d (20) a (21) b (22) c (23) c (24) d
(25) b (26) d (27) c (28) d (29) b (30) c
(31) d (32) d (33) a (34) b (35) c
(36) First : b Second : c (37) d (38) d
(39) c

Second Essay questions

1

(1) Function. (2) Not function.
(3) Function. (4) Not function.
(5) Function. (6) Not function.

2

(1) Function. (2) Not function.
(3) Not function. (4) Function.
(5) Not function. (6) Function.

3

(1) $\mathbb{R}-\{-3 , 3\}$ (2) $\mathbb{R}-\{1 , 2\}$
(3) $\mathbb{R}-\{3\}$ (4) $\mathbb{R}-\{2 , 3\}$
(5) $\mathbb{R}-\{1 , \frac{-2}{3}\}$ (6) $\mathbb{R}-\{-1\}$

4

(1) $[0 , \infty[$ (2) $\mathbb{R}-\{\frac{5}{2}\}$
(3) $]-4 , \infty[$ (4) $]-\infty , 3[$

5

(1) $\mathbb{R}$ (2) $]-\infty , 4[$
(3) $[0 , 6]$

6 The range $= \{2 , 7 , -13 , -18\}$

7

(1) The range $= \{1 , 5 , 9 , 13 , 17\}$
(2) $\because 4k - 3 = 17$ $\therefore k = 5$

8

(1) The domain $= \mathbb{R}$, the range $= \{2\}$
, the function is constant on its domain $\mathbb{R}$

(2) The domain $= \mathbb{R}$, the range $= \{-2 , 3\}$
, the function is constant on $]-\infty , 0[$, $]0 , \infty[$

(3) The domain $= \mathbb{R}-\{0\}$, the range $= \mathbb{R}-[-2 , 2]$
, the function is decreasing on $]-\infty , 0[$, $]0 , \infty[$

(4) The domain $= \mathbb{R}-\{1\}$, the range $=]-2 , \infty[$
, the function is decreasing on $]-\infty , 1[$
, increasing on $]1 , \infty[$

(5) The domain $= \mathbb{R}-\{2\}$, the range $= \mathbb{R}-\{2\}$
, the function is increasing on $]-\infty , 2[$, $]2 , \infty[$

(6) The domain $= [0 , \infty[$, the range $= [0 , 4]$
, the function is increasing on $]0 , 2[$, constant on $]2 , \infty[$

(7) The domain $= \mathbb{R}-\{-1 , 2\}$, the range $= \{3\}$
, the function is constant on its domain.

(8) The domain $= \mathbb{R}-]0 , 1]$, the range $=]3 , \infty[\cup \{2\}$, constant on $]-\infty , 0[$
and increasing on $]1 , \infty[$

(9) The domain $= \mathbb{R}$, the range $=]-\infty , 4]$
, the function is increasing on $]-\infty , -2[$, $]0 , 2[$
and decreasing on $]-2 , 0[$, $]2 , \infty[$

(10) The domain $= \mathbb{R}$, the range $= \mathbb{R}-]1 , 2]$
, the function is decreasing on $]-\infty , 0[$, $]1 , \infty[$
and increasing on $]0 , 1[$

(11) The domain $= \mathbb{R}-\{0\}$, the range $= \mathbb{R}-[-2 , 2]$
, the function is increasing on $]-\infty , 0[$, $]0 , \infty[$

(12) The domain $= [-2 , \infty[$, the range $= [0 , \infty[$
, the function is increasing on $]-2 , 0[$ and decreasing on $]0 , 2[$, $]2 , \infty[$

Third Higher skills

(1) (d) (2) (d) (3) (d) (4) (c)

Instructions to solve :

(1) $\because x$ is the number of sides

$\therefore x$ is an integer more than 2

$\therefore$ The domain $= \mathbb{Z}^+ - \{1, 2\}$

(2) Put $\sqrt[3]{x} - 2 = 0 \quad \therefore \sqrt[3]{x} = 2 \quad \therefore x = 8$

$\therefore$ The domain $= \mathbb{R} - \{8\}$

(3) $\because 3x \geq 0 \quad \therefore x \geq 0 \quad \therefore x \in [0, \infty[$

, put $\sqrt{3x} - x = 0 \quad \therefore \sqrt{3x} = x$

$\therefore 3x = x^2 \quad \therefore x^2 - 3x = 0$

$\therefore x(x-3) = 0 \quad \therefore x = 0$ or $x = 3$

$\therefore$ The domain $=]0, \infty[- \{3\}$

(4) $\because x - 1 \geq 0 \quad \therefore x \geq 1 \quad \therefore x \in [1, \infty[$

, put $\sqrt{x-1} = 3 \quad \therefore x - 1 = 9 \quad \therefore x = 10$

$\therefore$ The domain $= [1, \infty[- \{10\}$

Exercise 2

First Multiple choice questions

(1) d	(2) a	(3) a	(4) a	(5) b	(6) a
(7) c	(8) b	(9) b	(10) a	(11) c	(12) a
(13) d	(14) b	(15) b	(16) d	(17) c	(18) c
(19) a	(20) c	(21) d	(22) c	(23) c	(24) a
(25) a	(26) d	(27) d	(28) b		

Second Essay questions

1

Figure (1) : symmetric about x-axis , y-axis and the origin point.

Figure (2) : symmetric about x-axis.

Figure (3) : symmetric about the origin point.

Figure (4) : symmetric about the origin point.

Figure (5) : symmetric about the y-axis.

Figure (6) : symmetric about the origin point.

2

(1) Odd (2) Neither even nor odd

(3) Even (4) Neither even nor odd

(5) Odd (6) Odd

3

Figure (1) :

* The range $= \{-2, 2\}$ * Neither even nor odd

Figure (2) :

* The range $= \mathbb{R} - [-2, 2]$ * odd

Figure (3) :

* The range $=]-\infty, 3]$ * even

Figure (4) :

* The range $= [0, 4]$ * even

Figure (5) :

* The range $= \mathbb{R} - \{0\}$ * odd

Figure (6) :

* The range $= \mathbb{R}$ * odd

4

Figure (1) :

$f(x) = x^3 + x$

$\because$ The domain $= \mathbb{R}$

, the curve is symmetric about the origin point.

$\therefore$ The function f is odd.

algebraically verifying :

for every $x, -x \in \mathbb{R}$

$\because f(-x) = (-x)^3 + (-x) = -x^3 - x$

$= -(x^3 + x) = -f(x)$

$\therefore$ the function f is odd.

Figure (2) :

$f(x) = x^3 - 2$

$\because$ The domain $= \mathbb{R}$

, the curve is neither symmetric about y-axis nor symmetric about the origin point.

$\therefore$ The function f is neither even nor odd.

algebraically verifying :

for every $x, -x \in \mathbb{R}$

$\because f(-x) = (-x)^3 - 2 = -x^3 - 2 = -(x^3 + 2)$

$\therefore f(-x) \neq f(x) \neq -f(x)$

$\therefore$ The function f is neither even nor odd.

Figure (3) :

$f(x) = 2 - x^2$

$\because$ The domain $= [-2, 2]$,

the curve is symmetric about y-axis

∴ The function f is even.

algebraically verifying : For every $x , -x \in [-2 , 2]$

$\because f(-x) = 2 - (-x)^2 = 2 - x^2 = f(x)$

∴ The function is even.

5

First :

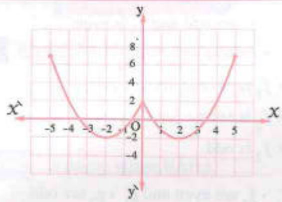


Fig. (1)

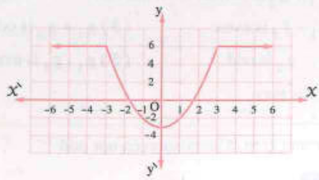


Fig. (3)

Second :

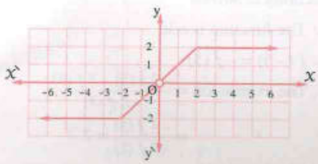


Fig. (2)

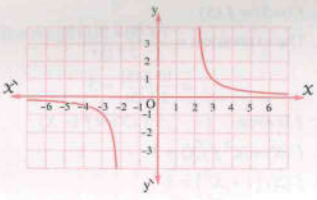


Fig. (4)

Third :

Figure (1) :

The domain = $[-5 , 5]$, the range = $[-2 , 7]$, the function is decreasing on each of $]-5 , -2[$, $]0 , 2[$ and increasing on each of $]-2 , 0[$, $]2 , 5[$

Figure (2) :

The domain = $\mathbb{R} - \{0\}$, the range = $[-2 , 2] - \{0\}$, the function is constant on each of $]-\infty , -2[$, $]2 , \infty[$ and increasing on each of $]-2 , 0[$, $]0 , 2[$

Figure (3) :

The domain = $\mathbb{R}$, the range = $[-3 , 6]$, the function is constant on each of $]-\infty , -3[$, $]3 , \infty[$, decreasing on $]-3 , 0[$ and increasing on $]0 , 3[$

Figure (4) :

The domain = $\mathbb{R} - [-2 , 2]$, the range = $\mathbb{R} - \{0\}$, the function is decreasing on each of $]-\infty , -2[$, $]2 , \infty[$

6

(1) $f(-x) = 5 = f(x)$ ∴ f is even.

(2) $f(-x) = (-x)^4 + (-x)^2 - 1$
$= x^4 + x^2 - 1 = f(x)$

∴ f is even.

(3) $f(-x) = 3(-x) - 4(-x)^3 = -3x + 4x^3$
$= -(3x - 4x^3) = -f(x)$

∴ f is odd.

(4) $f(-x) = (-x)^2 - 3(-x) + 4$
$= x^2 + 3x + 4 \neq f(x) \neq -f(x)$

∴ f is neither even nor odd.

(5) $f(-x) = (-x)^3\left((-x)^2 - 1\right)$
$= -x^3(x^2 - 1) = -f(x)$

∴ f is odd.

(6) $f(-x) = (-x - 3)^2 - 7 = (x + 3)^2 - 7$
$\neq f(x) \neq -f(x)$

∴ f is neither even nor odd.

(7) $f(-x) = \frac{(-x)^3 + 2}{-x - 3}$
$= \frac{-x^3 + 2}{-x - 3} = \frac{x^3 - 2}{x + 3} \neq f(x) \neq -f(x)$

∴ f is neither even nor odd.

(8) $f(-x) = \frac{2(-x)^3 - (-x)^5}{-x} = \frac{-2x^3 + x^5}{-x}$
$= \frac{2x^3 - x^5}{x} = f(x)$

∴ f is even.

(9) $f(-x) = \frac{(-x)^2}{1-x} = \frac{x^2}{1-x} \neq f(x) \neq -f(x)$

$\therefore$ f is neither even nor odd.

(10) The domain of $f = [-3, \infty[$

$\therefore$ For each $x \in [-3, \infty[$,

it is not necessary to find $-x \in [-3, \infty[$

$\therefore$ The function f is neither even nor odd.

(11) $f(-x) = \sqrt[3]{(-x)^3 + (-x)} = \sqrt[3]{-(x^3+x)}$

$= -\sqrt[3]{x^3+x} = -f(x)$

$\therefore$ f is odd.

(12) $f(-x) = (-x)^3 - \frac{1}{-x} = -x^3 + \frac{1}{x} = -\left(x^3 - \frac{1}{x}\right)$

$= -f(x)$

$\therefore$ f is odd.

(13) $f(-x) = \left(-x - \frac{2}{-x}\right)^3 = -\left(x - \frac{2}{x}\right)^3$

$= -f(x)$

$\therefore$ f is odd.

(14) $f(-x) = -x\cos(-x) = -x\cos x = -f(x)$

$\therefore$ f is odd.

(15) $f(-x) = \frac{-3x}{\tan(-x)} = \frac{-3x}{-\tan x} = \frac{3x}{\tan x} = f(x)$

$\therefore$ f is even.

(16) $f(-x) = \frac{(-x)^3 \times \sin(-3x)}{1+(-x)^4}$

$= \frac{x^3 \sin 3x}{1+x^4} = f(x)$

$\therefore$ f is even.

(17) $f(-x) = (-x)^2 \left(\sin(-x)\right)^3$

$= x^2(-\sin x)^3 = -x^2 \sin^3 x = -f(x)$

$\therefore$ f is odd.

(18) $f(-x) = -x\sin(-x)^3 = -x(-\sin x^3)$

$= x\sin x^3 = f(x)$

$\therefore$ f is even.

(19) $f(-x) = \frac{(-x)^2 + \tan(-x)}{(-x)^4 + \sin(-x)}$

$= \frac{x^2 - \tan x}{x^4 - \sin x} \neq f(x) \neq -f(x)$

$\therefore$ f is neither even nor odd.

(20) $f(-x) = \frac{\sin(-3x)\cos(-2x)}{\sec(-x)}$

$= \frac{-\sin 3x \cos 2x}{\sec x} = -f(x)$

$\therefore$ f is odd.

7 $\because$ f_1, f_2 are odd and f_3 is even

(1) $f_1 + f_2$ is odd

(2) $f_1 + f_3$ is neither even or odd

(3) $f_1 \times f_2$ is even

(4) $f_3 \times f_2$ is odd

8 $\because$ f_1, f_2 are even and g_1, g_2 are odd

(1) $f_1 + g_2$ is neither even nor odd.

(2) $f_1 - f_2$ is even.

(3) $g_1 + g_2$ is odd.

(4) $f_1 \cdot g_2$ is odd.

(5) $g_1 \cdot g_2$ is even.

(6) $\frac{f_2}{f_1}$ is even.

9 From (1) to (5) neither even nor odd

Third Higher skills

(1) (c) (2) (a) (3) (c) (4) (b)

Instructions to solve :

(1) $\because$ The function is odd

$\therefore f(-5) = -f(5)$

$\therefore$ The expression $= \frac{-7f(5) + 3f(5)}{-2f(5)}$

$= \frac{-4f(5)}{-2f(5)} = 2$

(2) $\because$ The function is even

$\therefore f(-5) = f(5)$

$\therefore$ The expression $= \frac{7f(5) + 3f(5)}{2f(5)}$

$= \frac{10f(5)}{2f(5)} = 5$

(3) $\because$ f is even $\therefore f(x) = f(-x)$

$\therefore f(x) + x^2 f(x) = 3$

$\therefore f(x)[1 + x^2] = 3$

$\therefore f(x) = \frac{3}{1+x^2}$ $\therefore f(1) = 1\frac{1}{2}$

(4) $\because$ f is odd, $f(1) = k$ $\therefore f(-1) = -k$

, $\because f(x+2) = f(x) + f(2)$

Put $x = -1$

$\therefore f(1) = f(-1) + f(2)$ $\therefore k = -k + f(2)$

$\therefore f(2) = 2\,k$

Put $x = 1$

$\therefore f(3) = f(1) + f(2) = k + 2\,k = 3\,k$

Exercise 3

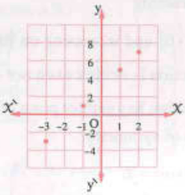
First Multiple choice questions

(1) a (2) c (3) d (4) b (5) c (6) c
(7) d (8) b (9) b (10) b

Second Essay questions

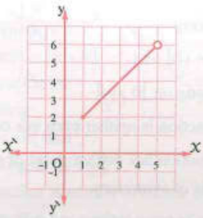
1

(1)



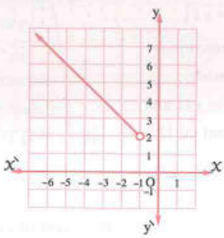
The range $= \{-3 , 1 , 5 , 7\}$

(2)



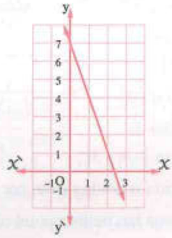
The range $= [2 , 6[$

(3)



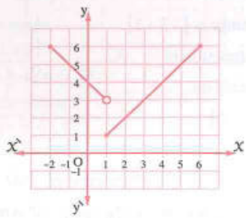
The range $=]2 , \infty[$

(4)



The range $= \mathbb{R}$

2

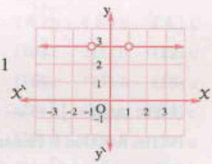


* The range $= [1 , 6]$
* The function is decreasing on $]-2 , 1[$ and increasing on $]1 , 6[$

3

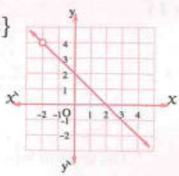
(1) $\because f(x) = \dfrac{3(x^2 - 1)}{(x^2 - 1)}$

$\therefore f(x) = 3 , x \neq \pm 1$



* The domain $= \mathbb{R} - \{1 , -1\}$
* The range $= \{3\}$
* The function is constant on its domain.
* The function is even.
* The axis of symmetry is $x = 0$

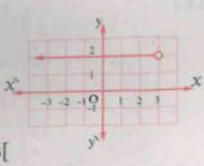
(2) $g(x) = \dfrac{(2 - x)(2 + x)}{x + 2} = 2 - x , x \neq -2$



* The domain $= \mathbb{R} - \{-2\}$
* The range $= \mathbb{R} - \{4\}$
* The function is decreasing on its domain.
* The function is neither even nor odd.

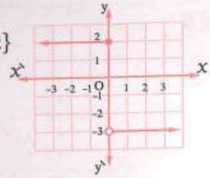
4

(1) * The domain $=]-\infty , 3[$



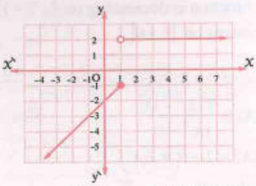
* The range $= \{2\}$
* The function is constant on $]-\infty , 3[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(2) * The domain $= \mathbb{R}$



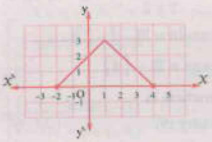
* The range $= \{-3 , 2\}$
* The function is constant on $]-\infty , 0[$, $]0 , \infty[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(3)



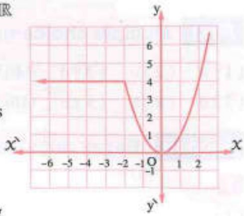
* The range $=]-\infty , -1] \cup \{2\}$
* The function is constant on $]1 , \infty[$ and increasing on $]-\infty , 1[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(4)



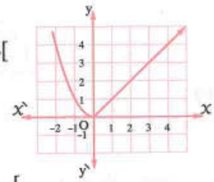
* The domain $= [-2 , 4]$
* The range $= [0 , 3]$
* The function is increasing on $]-2 , 1[$ and decreasing on $]1 , 4[$
* The function is neither odd nor even.
* The axis of symmetry is the straight line $x = 1$

(5) * The domain $= \mathbb{R}$



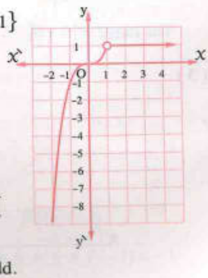
* The range $= [0 , \infty[$
* The function is constant on $]-\infty , -2[$ and decreasing on $]-2 , 0[$ and increasing on $]0 , \infty[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(6) * The domain $= \mathbb{R}$



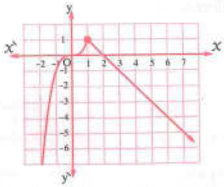
* The range $= [0 , \infty[$
* The function is decreasing on $]-\infty , 0[$ and increasing on $]0 , \infty[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(7) * The domain $= \mathbb{R} - \{1\}$



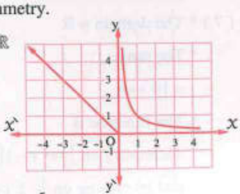
* The range $=]-\infty , 1]$
* The function is increasing on $]-\infty , 1[$ and constant on $]1 , \infty[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(8)



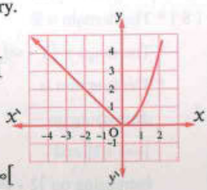
* The domain = $\mathbb{R}$ * The range = $]-\infty , 1]$
* The function is increasing on $]-\infty , 1[$ and decreasing on $]1 , \infty[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(9) * The domain = $\mathbb{R}$



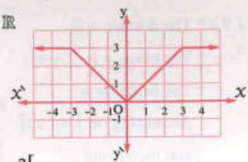
* The range $= [0 , \infty[$
* The function is decreasing on each of $]-\infty , 0[,]0 , \infty[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(10) * The domain = $\mathbb{R}$



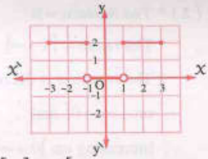
* The range = $[0 , \infty[$
* The function is decreasing on $]-\infty , 0[$ and increasing on $]0 , \infty[$
* The function is neither even nor odd.
* The function has neither point of symmetry nor axis of symmetry.

(11) * The domain = $\mathbb{R}$



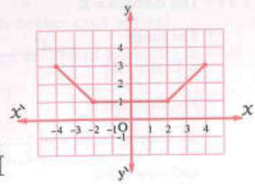
* The range $= [0 , 3]$
* The function is constant on each of $]-\infty , -3[$, $]3 , \infty[$ and decreasing on $]-3 , 0[$ and increasing on $]0 , 3[$
* The function is even.
* The axis of symmetry is $x = 0$

(12) * The domain $= [-3 , 3]$



* The range $= \{0 , 2\}$
* The function is constant on each $]-3 , -1[,]-1 , 1[,]1 , 3[$
* The function is even.
* The axis of symmetry is the straight line $x = 0$

(13) * The domain $= [-4 , 4]$



* The range $= [1 , 3]$
* The function is decreasing on $]-4 , -2[$, constant on $]-2 , 2[$ and increasing on $]2 , 4[$
* The function is even.
* The axis of symmetry is the straight line $x = 0$

Exercise 4

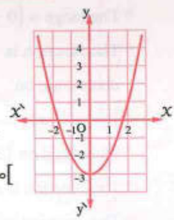
First Multiple choice questions

(1) a	(2) b	(3) c	(4) b	(5) d	(6) c
(7) c	(8) a	(9) d	(10) b	(11) c	(12) c
(13) a	(14) c	(15) b	(16) c	(17) b	(18) a
(19) b	(20) b	(21) b	(22) b	(23) c	(24) c
(25) c	(26) c	(27) a	(28) d	(29) b	(30) c
(31) d	(32) c	(33) d	(34) a	(35) b	(36) b
(37) c	(38) a	(39) d	(40) b	(41) d	(42) c

Second Essay questions

1

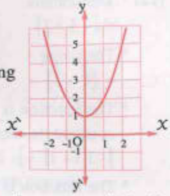
(1) * The domain = $\mathbb{R}$



* The range $= [-3 , \infty[$
* The function is decreasing on $]-\infty , 0[$ and increasing on $]0 , \infty[$
* The function is even.
* The equation of the axis of symmetry is $x = 0$

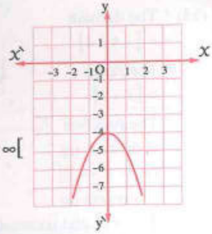
(2) * The domain = ℝ

* The range = $[1 , \infty[$
* The function is decreasing on $]-\infty , 0[$ and increasing on $]0 , \infty[$
* The function is even
* The equation of the axis of symmetry is $x = 0$



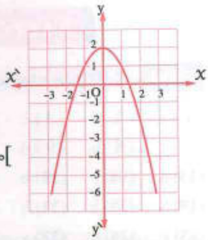
(3) * The domain = ℝ

* The range $=]-\infty , -4]$
* The function is decreasing on $]0 , \infty[$ and increasing on $]-\infty , 0[$
* The function is even.
* The equation of the axis of symmetry is $x = 0$



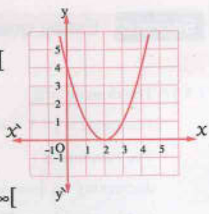
(4) * The domain = ℝ

* The range $=]-\infty , 2]$
* The function is decreasing on $]0 , \infty[$ and increasing on $]-\infty , 0[$
* The function is even
* The equation of the axis of symmetry is $x = 0$



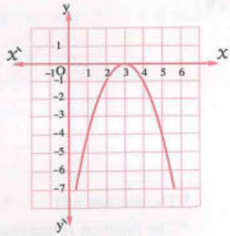
(5) * The domain = ℝ

* The range = $[0 , \infty[$
* The function is decreasing on $]-\infty , 2[$ and increasing on $]2 , \infty[$
* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = 2$



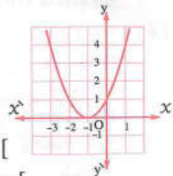
(6) * The domain = ℝ

* The range $=]-\infty , 0]$
* The function is decreasing on $]3 , \infty[$ and increasing on $]-\infty , 3[$
* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = 3$



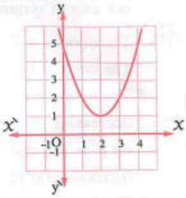
(7) * The domain = ℝ

* The range $= [0 , \infty[$
* The function is decreasing on $]-\infty , -1[$ and increasing on $]-1 , \infty[$
* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = -1$



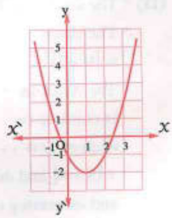
(8) * The domain = ℝ

* The range = $[1 , \infty[$
* The function is decreasing on $]-\infty , 2[$ and increasing on $]2 , \infty[$
* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = 2$



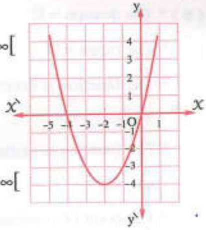
(9) * The domain = ℝ

* The range = $[-2 , \infty[$
* The function is decreasing on $]-\infty , 1[$ and increasing on $]1 , \infty[$
* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = 1$



(10) * The domain = $\mathbb{R}$

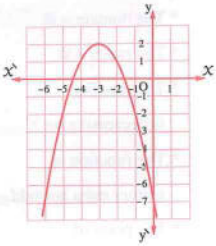
* The range = $[-4 , \infty[$
* The function is decreasing on $]-\infty , -2[$ and increasing on $]-2 , \infty[$
* The function is neither even nor odd



* The equation of the axis of symmetry is $x = -2$

(11) * The domain = $\mathbb{R}$

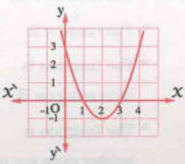
* The range $=]-\infty , 2]$
* The function is decreasing on $]-3 , \infty[$ and increasing on $]-\infty , -3[$



* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = -3$

(12) * The domain = $\mathbb{R}$

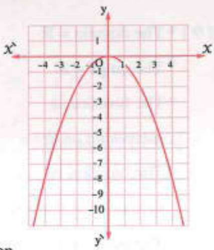
* The range = $]-1 , \infty[$
* The function is decreasing on $]-\infty , 2[$ and increasing on $]2 , \infty[$



* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = 2$

(13) * The domain = $\mathbb{R}$

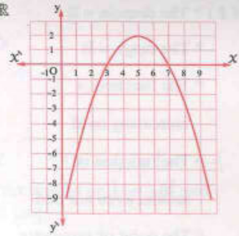
* The range $=]-\infty , 0]$
* The function is decreasing on $]0 , \infty[$ and increasing on $]-\infty , 0[$



* The function is even.
* The equation of the axis of symmetry is $x = 0$

(14) * The domain = $\mathbb{R}$

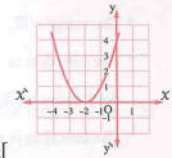
* The range $=]-\infty , 2]$
* The function is decreasing on $]5 , \infty[$ and increasing on $]-\infty , 5[$



* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = 5$

(15) * $g(x) = (x + 2)^2$

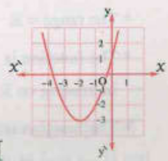
* The domain = $\mathbb{R}$
* The range = $[0 , \infty[$
* The function is decreasing on $]-\infty , -2[$ and increasing on $]-2 , \infty[$



* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = -2$

(16) * $g(x) = (x + 2)^2 - 3$

* The domain = $\mathbb{R}$
* The range = $[-3 , \infty[$
* The function is decreasing on $]-\infty , -2[$ and increasing on $]-2 , \infty[$

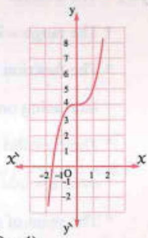


* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = -2$

2

(1) * The domain = $\mathbb{R}$

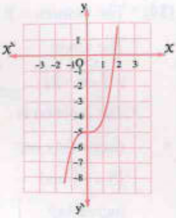
* The range = $\mathbb{R}$
* The function is increasing on $\mathbb{R}$
* The function is neither even nor odd



* The point of symmetry is $(0 , 4)$

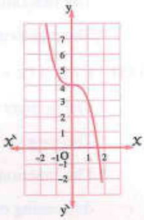
(2) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is increasing on $\mathbb{R}$
* The function is neither even nor odd.
* The point of symmetry is $(0 , -5)$



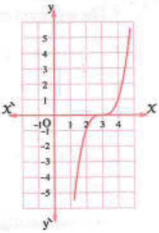
(3) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is decreasing on $\mathbb{R}$
* The function is neither even nor odd.
* The point of symmetry is $(0 , 4)$



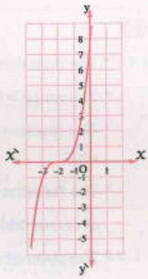
(4) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is increasing on $\mathbb{R}$
* The function is neither even nor odd
* The point of symmetry is $(3 , 0)$



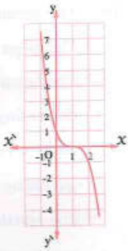
(5) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is increasing on $\mathbb{R}$
* The function is neither even nor odd
* The point of symmetry is $(-2 , 0)$



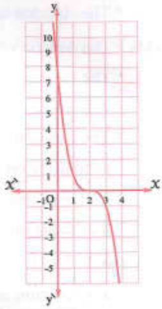
(6) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is decreasing on $\mathbb{R}$
* The function is neither even nor odd
* The point of symmetry is $(1 , 0)$



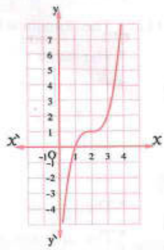
(7) $g(x) = -(x-2)^3$

* The domain = $\mathbb{R}$
* The range = $\mathbb{R}$
* The function is decreasing on $\mathbb{R}$
* The function is neither even nor odd.
* The point of symmetry is $(2 , 0)$



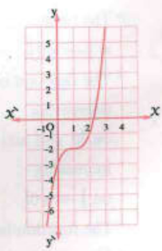
(8) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is increasing on $\mathbb{R}$
* The function is neither even nor odd.
* The point of symmetry is $(2 , 1)$



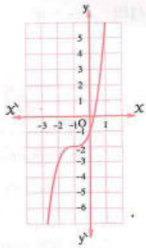
(9) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is increasing on $\mathbb{R}$
* The function is neither even nor odd.
* The point of symmetry is $(1 , -2)$



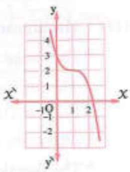
(10) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is increasing on $\mathbb{R}$
* The function is neither even nor odd.
* The point of symmetry is $(-1, -2)$



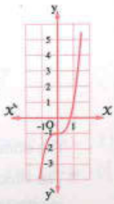
(11) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is decreasing on $\mathbb{R}$
* The function is neither even nor odd.
* The point of symmetry is $(1, 2)$



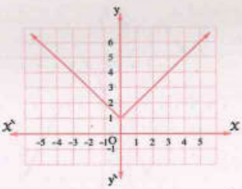
(12) * The domain = $\mathbb{R}$

* The range = $\mathbb{R}$
* The function is increasing on $\mathbb{R}$
* The function is neither even nor odd.
* The point of symmetry is $(0, -1)$



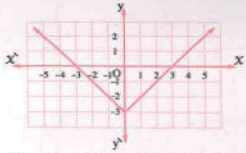
3

(1)



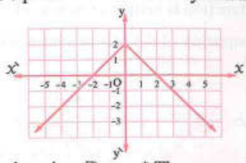
* The domain = $\mathbb{R}$ * The range = $[1, \infty[$
* The function is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$
* The function is even.
* The equation of the axis of symmetry is $x = 0$

(2)



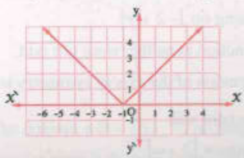
* The domain = $\mathbb{R}$ * The range = $[-3, \infty[$
* The function is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$
* The function is even.
* The equation of the axis of symmetry is $x = 0$

(3)



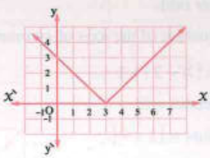
* The domain = $\mathbb{R}$ * The range = $]-\infty, 2]$
* The function is decreasing on $]0, \infty[$ and increasing on $]-\infty, 0[$
* The function is even.
* The equation of the axis of symmetry is $x = 0$

(4)

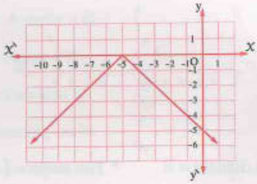


* The domain = $\mathbb{R}$ * The range = $[0, \infty[$
* The function is decreasing on $]-\infty, -1[$ and increasing on $]-1, \infty[$
* The function is neither even nor odd.
* The equation of the axis of symmetry $x = -1$

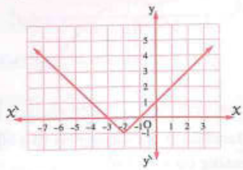
(5)



* The domain = $\mathbb{R}$ * The range = $[0, \infty[$
* The function is decreasing on $]-\infty, 3[$ and increasing on $]3, \infty[$
* The function is neither even nor odd
* The equation of the axis of symmetry $x = 3$

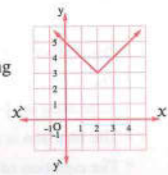
(6) 

* The domain = ℝ   * The range =]−∞ , 0]
* The function is decreasing on]− 5 , ∞[and increasing on]−∞ , − 5[
* The function is neither even nor odd.
* The equation of the axis of symmetry $X = -5$

(7) 

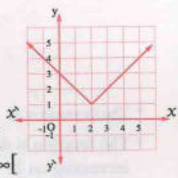
* The domain = ℝ   * The range = [− 1 , ∞[
* The function is decreasing on]−∞ , − 2[and increasing on]− 2 , ∞[
* The function is neither even nor odd.
* The equation of the axis of symmetry is $X = -2$

(8) * The domain = ℝ

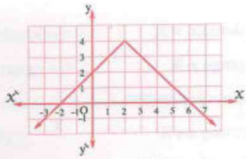


* The range = [3 , ∞[
* The function is decreasing on]−∞ , 2[and increasing on]2 , ∞[
* The function is neither even nor odd.
* The equation of the axis of symmetry is $X = 2$

(9) * $g(X) = |X - 2| + 1$

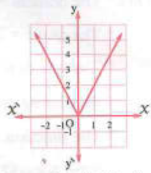


* The domain = ℝ
* The range = [1 , ∞[
* The function is decreasing on]−∞ , 2[and increasing on]2 , ∞[
* The function is neither even nor odd.
* The equation of the axis of symmetry is $X = 2$

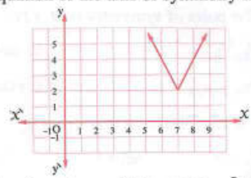
(10) 

* The domain = ℝ   * The range = [−∞ , 4]
* The function is decreasing on]2 , ∞[and increasing on]−∞ , 2[
* The function is neither even nor odd.
* The equation of the axis of symmetry is $X = 2$

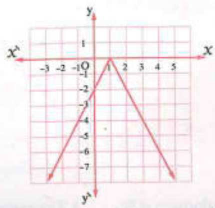
(11) * The domain = ℝ



* The range = [0 , ∞[
* The function is decreasing on]−∞ , 0[and increasing on]0 , ∞[
* The function is even.
* The equation of the axis of symmetry is $X = 0$

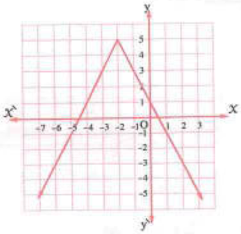
(12) 

* The domain = ℝ   * The range = [2 , ∞[
* The function is decreasing on]−∞ , 7[and increasing on]7 , ∞[
* The function is neither even nor odd.
* The equation of the axis of symmetry $X = 7$

(13) 

* The domain = ℝ   * The range = [−∞ , 0]
* The function is decreasing on]1 , ∞[and increasing on]−∞ , 1[
* The function is neither even nor odd.
* The equation of the axis of symmetry is $X = 1$

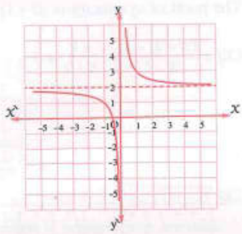
(14)



* The domain = $\mathbb{R}$ * The range = $]-\infty , 5]$
* The function is decreasing on $]-2 , \infty[$ and increasing on $]-\infty , -2[$
* The function is neither even nor odd.
* The equation of the axis of symmetry is $x = -2$

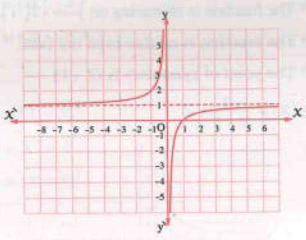
4

(1)



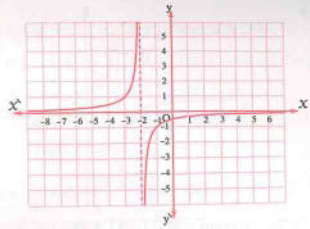
* The domain = $\mathbb{R} - \{0\}$ * The range = $\mathbb{R} - \{2\}$
* The function is decreasing on $]0 , \infty[,]-\infty , 0[$
* The function is neither even nor odd.
* The point of symmetry is $(0 , 2)$

(2)



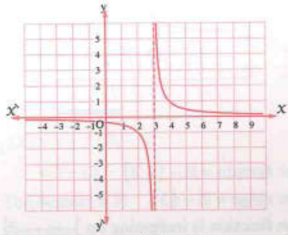
* The domain = $\mathbb{R} - \{0\}$ * The range = $\mathbb{R} - \{1\}$
* The function is increasing on $]0 , \infty[,]-\infty , 0[$
* The function is neither even nor odd.
* The point of symmetry is $(0 , 1)$

(3)



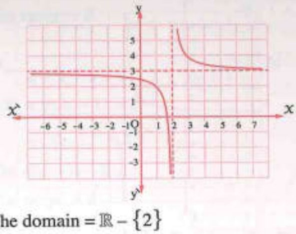
* The domain = $\mathbb{R} - \{-2\}$
* The range = $\mathbb{R} - \{0\}$
* The function is increasing on $]-\infty , -2]$ $,]-2 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(-2 , 0)$

(4)



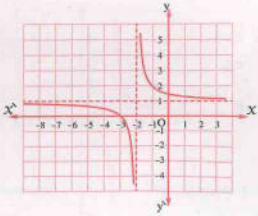
* The domain = $\mathbb{R} - \{3\}$ * The range = $\mathbb{R} - \{0\}$
* The function is decreasing on $]-\infty , 3[,]3 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(3 , 0)$

(5)



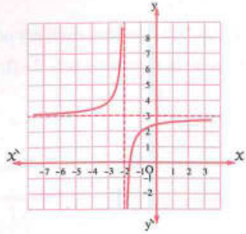
* The domain = $\mathbb{R} - \{2\}$
* The range = $\mathbb{R} - \{3\}$
* The function is decreasing on $]-\infty , 2[,]2 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(2 , 3)$

(6)



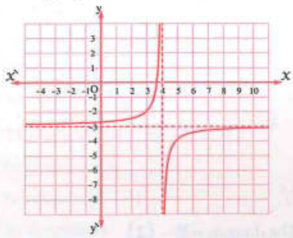
* The domain = $\mathbb{R} - \{-2\}$ * The range = $\mathbb{R} - \{1\}$
* The function is decreasing on $]-\infty , -2[$, $]-2 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(-2 , 1)$

(7)



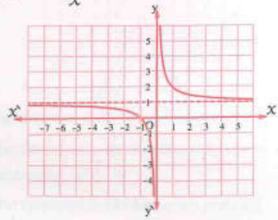
* The domain = $\mathbb{R} - \{-2\}$
* The range = $\mathbb{R} - \{3\}$
* The function is increasing on $]-\infty , -2[$, $]-2 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(-2 , 3)$

(8) $g(x) = \frac{-1}{x-4} - 3$



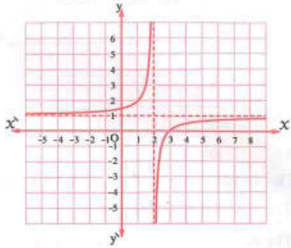
* The domain = $\mathbb{R} - \{4\}$
* The range = $\mathbb{R} - \{-3\}$
* The function is increasing on $]-\infty , 4[$, $]4 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(4 , -3)$

(9) $g(x) = 1 + \frac{1}{x}$



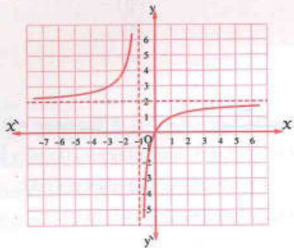
* The domain = $\mathbb{R} - \{0\}$
* The range = $\mathbb{R} - \{1\}$
* The function is decreasing on $]-\infty , 0[$, $]0 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(0 , 1)$

(10) $g(x) = \frac{(x-2)-1}{x-2} = \frac{-1}{x-2} + 1$



* The domain = $\mathbb{R} - \{2\}$ * The range = $\mathbb{R} - \{1\}$
* The function is increasing on $]-\infty , 2[$, $]2 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(2 , 1)$

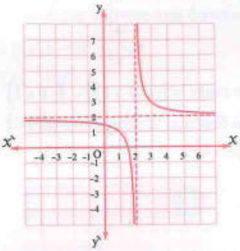
(11) $g(x) = \frac{-2}{x+1} + 2$



* The domain = $\mathbb{R} - \{-1\}$ * The range = $\mathbb{R} - \{2\}$
* The function is increasing on $]-\infty , -1[$, $]-1 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(-1 , 2)$

(12) $g(x) = \frac{(2x-4)+1}{x-2} = \frac{2(x-2)}{(x-2)} + \frac{1}{x-2}$

$= \frac{1}{x-2} + 2$



* The domain = $\mathbb{R} - \{2\}$
* The range = $\mathbb{R} - \{2\}$
* The function is decreasing on $]-\infty , 2[$, $]2 , \infty[$
* The function is neither even nor odd.
* The point of symmetry is $(2 , 2)$

5

* $g(x) = |x-3|$
* $h(x) = |x| - 2$
* $z(x) = |x+3| + 2$

6

* $g(x) = (x-3)^3$
* $h(x) = x^3 - 4$
* $z(x) = (x+4)^3 - 1$

7

(1) $f(x) = (x-2)^2$ (2) $g(x) = x^3 + 2$

(3) $h(x) = \frac{1}{x-1} + 2$ (4) $[0 , \infty[$

(5) (2) (6) $(1 , 2)$

(7) $x = 2$

8

(1) $f(x) = -1$ (2) $f(x) = x - 2$

(3) $f(x) = (x+2)^2 - 3$ (4) $f(x) = -(x-1)^2 + 2$

(5) $f(x) = x^3$ (6) $f(x) = (x-2)^3$

(7) $f(x) = (x+1)^3 - 3$ (8) $f(x) = |x-2| - 3$

(9) $f(x) = -|x-1| + 3$ (10) $f(x) = \frac{-1}{x}$

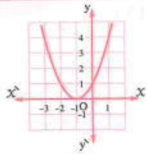
(11) $f(x) = \frac{1}{x} + 2$ (12) $f(x) = \frac{-1}{x+2} - 2$

9

(1) $f_1(x) = f(x+1)$

$= (x+1)^2$

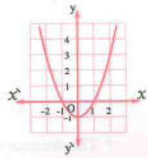
* The domain = $\mathbb{R}$
* The range = $[0 , \infty[$



(2) $f_2(x) = f(x) - 1$

$= x^2 - 1$

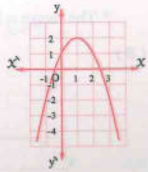
* The domain = $\mathbb{R}$
* The range = $[-1 , \infty[$



(3) $f_3(x) = 2 - f(x-1)$

$= 2 - (x-1)^2$

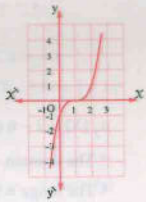
* The domain = $\mathbb{R}$
* The range = $]-\infty , 2]$



(4) $g_1(x) = g(x-1)$

$= (x-1)^3$

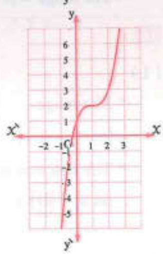
* The domain = $\mathbb{R}$
* The range = $\mathbb{R}$



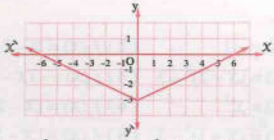
(5) $g_2(x) = g(x-1) + 2$

$= (x-1)^3 + 2$

* The domain = $\mathbb{R}$
* The range = $\mathbb{R}$



(6)

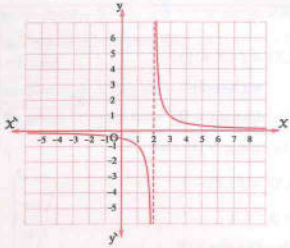


$k_1(x) = \frac{1}{2}k(x) - 3 = \frac{1}{2}|x| - 3$

* The domain = ℝ

* The range =]– 3 , ∞[

(7)

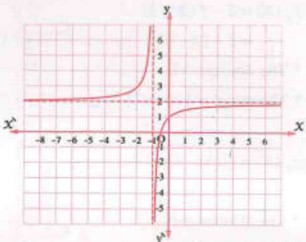


$n_1(x) = n(x-2) = \frac{1}{x-2}$

* The domain = ℝ – {2}

* The range = ℝ – {0}

(8)



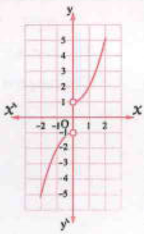
$n_2(x) = 2 - n(x+1) = 2 - \frac{1}{x+1}$

* The domain = ℝ – {– 1}

* The range = ℝ – {2}

10

(1)

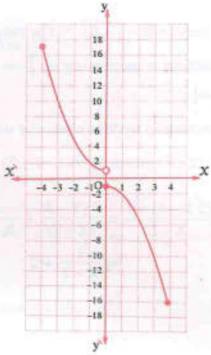


* The range

= ℝ – [– 1 , 1]

* The function is increasing on ℝ – {0}

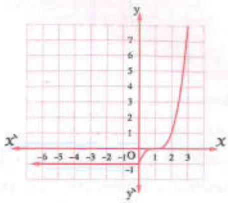
(2)



* The range = [– 17 , 17] –]–1 , 1]

* The function is decreasing on]– 4 , 0[,]0 , 4[

(3)



* The range = [– 1 , ∞[

* The function is constant on]–∞ , 0[and increasing on]0 , ∞[

Third Higher skills

(1) (c) (2) (b) (3) (a) (4) (c) (5) (c)

Instructions to solve:

(1) ∵ The curve g (x) is the same as the curve f (x) by translation 3 units to the right

∴ Each point of the intersection points of the curve with the x-axis move 3 units to the right too

∴ $x \in \{-3+3 , 1+3 , 0+3\}$

i.e. $x \in \{0 , 4 , 3\}$

(2) ∵ The range of the quadratic functions = [1 , ∞[

∴ b – 2 = 1 ∴ b = 3

, ∵ the curve passes through the point (3 , 2)

∴ $f(3) = 2$ ∴ $(3 - a + 1)^2 + 1 = 2$

∴ $(4 - a)^2 = 1$ ∴ $4 - a = \pm 1$

∴ $a = 4 \pm 1$ ∴ $a = 3$ or $a = 5$

(3) ∵ The curve of $f(x)$ is the same as the curve of g (x) by translation one unit to the left

∴ The function is increasing on $]-1 , \infty[$

(4) The curve $y = 3(x - 5)^2 + 7$ by translation 3 units to the right and one unit downwards

∴ $y = 3(x - 5 - 3)^2 + 7 - 1$

∴ $y = 3(x - 8)^2 + 6$

(5) ∵ The function is symmetric about y-axis

∴ $g(x) = -x^3 + 2$

Exercise 5

First Multiple choice questions

(1) b (2) c (3) a (4) c (5) a (6) c
(7) c (8) d (9) a (10) b (11) c (12) c
(13) b (14) d

Second Essay questions

1

(1) $|x| = 7$ ∴ $x = \pm 7$

∴ The S.S. = $\{7 , -7\}$

(2) $|x| = -3$ (refused) ∴ The S.S. = $\varnothing$

(3) $4|x| = 20$ ∴ $|x| = 5$

∴ $x = \pm 5$ ∴ The S.S. = $\{5 , -5\}$

(4) $|x - 2| = 2$ ∴ $x - 2 = \pm 2$

∴ $x = 4$ or $x = 0$

∴ The S.S. = $\{4 , 0\}$

(5) $|x - 3| = 0$ ∴ $x - 3 = 0$

∴ $x = 3$ ∴ The S.S. = $\{3\}$

(6) $|x + 3| = 6$ ∴ $x + 3 = \pm 6$

∴ $x = 3$ or $x = -9$

∴ The S.S. = $\{3 , -9\}$

(7) $|2x - 7| = 5$ ∴ $2x - 7 = \pm 5$

∴ $2x - 7 = 5$ ∴ $x = 6$

or $2x - 7 = -5$ ∴ $x = 1$

∴ The S.S. = $\{6 , 1\}$

(8) $|2x - 3| = 7$ ∴ $2x - 3 = \pm 7$

∴ $2x - 3 = 7$ ∴ $2x = 10$

∴ $x = 5$ or $2x - 3 = -7$

∴ $2x = -4$ ∴ $x = -2$

∴ The S.S. = $\{5 , -2\}$

(9) $|x + 2| = 1$ ∴ $x + 2 = \pm 1$

∴ $x = -1$ or $x = -3$

∴ The S.S. = $\{-1 , -3\}$

(10) $3|x| = 3$ ∴ $|x| = 1$

∴ $x = \pm 1$ ∴ The S.S. = $\{1 , -1\}$

(11) $|x - 3| = |x + 1|$

∴ $x - 3 = \pm (x + 1)$

∴ $x - 3 = x + 1$ (refused)

or $x - 3 = -x - 1$ ∴ $2x = 2$

∴ $x = 1$ (satisfy)

∴ The S.S. = $\{1\}$

(12) $|x + 5| = |x - 3|$

∴ $x + 5 = \pm (x - 3)$

∴ $x + 5 = x - 3$ (refused)

or $x + 5 = -x + 3$ ∴ $2x = -2$

∴ $x = -1$ (satisfy) ∴ The S.S. = $\{-1\}$

(13) $|x - 1| = |2x + 3|$

∴ $x - 1 = \pm (2x + 3)$

∴ $x - 1 = 2x + 3$ ∴ $x = -4$ (satisfy)

or $x - 1 = -2x - 3$ ∴ $3x = -2$

∴ $x = \frac{-2}{3}$ (satisfy)

∴ The S.S. = $\left\{-4 , \frac{-2}{3}\right\}$

(14) ∵ $|2(x - 3)| = |x - 3|$

∴ $2|x - 3| = |x - 3|$ ∴ $|x - 3| = 0$

∴ $x - 3 = 0$ ∴ $x = 3$

∴ The S.S. = $\{3\}$

(15) $|2x+1|=|x-3|$ $\quad \therefore 2x+1=\pm(x-3)$

$\therefore 2x+1=x-3$ $\quad \therefore x=-4$ (satisfy)

or $2x+1=-x+3$ $\quad \therefore x=\frac{2}{3}$ (satisfy)

$\therefore$ The S.S. $=\left\{-4, \frac{2}{3}\right\}$

(16) $|x-1|=2|x-2|$

$\therefore x-1=\pm 2(x-2)$ $\quad \therefore x-1=2x-4$

$\therefore x=3$ (satisfy)

or $x-1=-2x+4$ $\quad \therefore 3x=5$

$\therefore x=\frac{5}{3}$ (satisfy) $\quad \therefore$ The S.S. $=\left\{3, \frac{5}{3}\right\}$

(17) $\because \sqrt{x^2-4x+4}=4$

$\therefore \sqrt{(x-2)^2}=4$ $\quad \therefore |x-2|=4$

$\therefore x-2=\pm 4$

$\therefore x=6$ or $x=-2$

$\therefore$ The S.S. $=\{6, -2\}$

(18) $|x-3|\left(|x-3|-1\right)=0$

$\therefore |x-3|=0$ and hence $x=3$

or $|x-3|-1=0$ $\quad \therefore x-3=\pm 1$

and hence $x=2$ or $x=4$

$\therefore$ The S.S. $=\{3, 2, 4\}$

(19) $\sqrt{4x^2-12x+9}=|x+1|$

$\therefore \sqrt{(2x-3)^2}=|x+1|$

$\therefore |2x-3|=|x+1|$

$\therefore 2x-3=\pm(x+1)$

$\therefore 2x-3=x+1$ $\quad \therefore x=4$ (satisfy)

or $2x-3=-x-1$ $\quad \therefore x=\frac{2}{3}$ (satisfy)

$\therefore$ The S.S. $=\left\{4, \frac{2}{3}\right\}$

(20) $|x-1||x+1|=|x-1|$

$\therefore |x-1|\left(|x+1|-1\right)=0$

$\therefore |x-1|=0$ $\quad \therefore x=1$

or $|x+1|-1=0$ $\quad \therefore x+1=\pm 1$

$\therefore x=0$ or $x=-2$

$\therefore$ The S.S. $=\{1, 0, -2\}$

(21) $|x^2-1|=26$ $\quad \therefore x^2-1=\pm 26$

$\therefore x^2=-25$ (refused)

or $x^2=27$ and hence $x=\pm 3\sqrt{3}$

$\therefore$ The S.S. $=\left\{3\sqrt{3}, -3\sqrt{3}\right\}$

(22) $\left(|x+1|+2\right)\left(|x+1|-5\right)=0$

$\therefore |x+1|+2=0$ (refused)

or $|x+1|-5=0$

$\therefore x+1=\pm 5$ and hence $x=4$ or $x=-6$

$\therefore$ The S.S. $=\{4, -6\}$

(23) $|x-5|^2=2|x-5|$

$\therefore |x-5|\left(|x-5|-2\right)=0$

$\therefore |x-5|=0$ and hence $x=5$

or $|x-5|-2=0$ and hence $x-5=\pm 2$

$\therefore x=7$ or $x=3$

$\therefore$ The S.S. $=\{5, 7, 3\}$

(24) $x^2+x-10=\pm 10$ $\quad \therefore x^2+x-10=-10$

$\therefore x^2+x=0$ $\quad \therefore x(x+1)=0$

$\therefore x=0$ or $x=-1$

or $x^2+x-20=0$ $\quad \therefore (x+5)(x-4)=0$

$\therefore x=-5$ or $x=4$

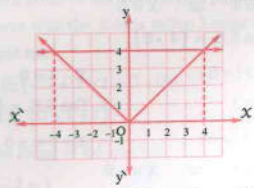
$\therefore$ The S.S. $=\{0, -1, -5, 4\}$

2

* We shall give the solution graphically and you can verify it algebraically.

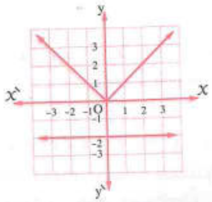
* Draw the curves of the two functions $f(x)$, $g(x)$, the x - coordinate of the intersection point of the two curves represents the S.S.

(1) $\because |x|=4$ $\quad \therefore f(x)=|x|$, $g(x)=4$



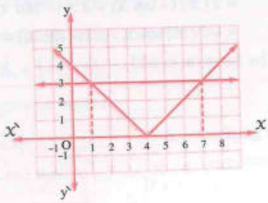
From the graph : The S.S. $=\{4, -4\}$

(2) $\because |x| = -2$ $\therefore f(x) = |x|$, $g(x) = -2$



From the graph : The S.S. = $\varnothing$

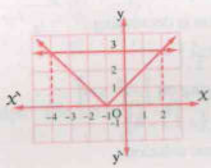
(3) $\because |x-4| = 3$ $\therefore f(x) = |x-4|$, $g(x) = 3$



From the graph : The S.S. = $\{1 , 7\}$

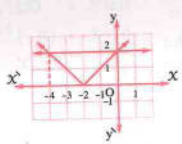
(4) $\because |x+1| = 3$

$\therefore f(x) = |x+1|$, $g(x) = 3$



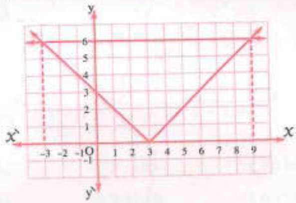
From the graph : The S.S. = $\{2 , -4\}$

(5) $\because |x+2| = 2$ $\therefore f(x) = |x+2|$, $g(x) = 2$



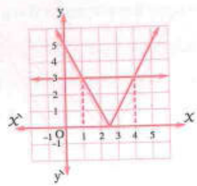
From the graph : The S.S. = $\{0 , -4\}$

(6) $\because |x-3| = 6$ $\therefore f(x) = |x-3|$, $g(x) = 6$



From the graph : The S.S. = $\{9 , -3\}$

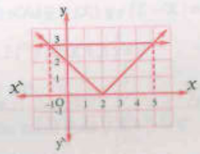
(7) $f(x) = |2x-5|$, $g(x) = 3$



From the graph : The S.S. = $\{1 , 4\}$

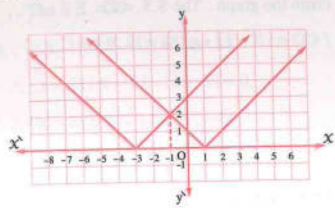
(8) $\because \sqrt{(x-2)^2} = 3$ $\therefore |x-2| = 3$

$\therefore f(x) = |x-2|$, $g(x) = 3$



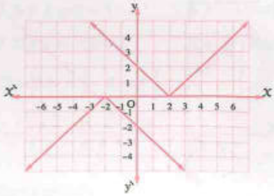
From the graph : The S.S. = $\{5 , -1\}$

(9) $f(x) = |x-1|$, $g(x) = |x+3|$



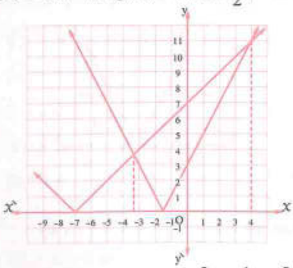
From the graph : The S.S. = $\{-1\}$

(10) $f(x) = |x - 2|$, $g(x) = -|x + 2|$



From the graph : The S.S. = $\varnothing$

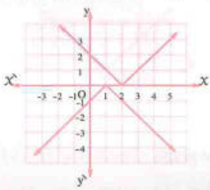
(11) $f(x) = |x + 7|$, $g(x) = 2\left|x + \frac{3}{2}\right|$



From the graph : The S.S. = $\left\{-3\frac{1}{3} , 4\right\}$

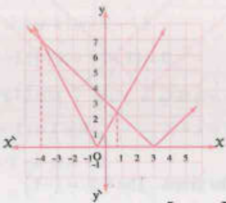
(12) $\because |x - 2| = -|x - 1|$

$\therefore f(x) = |x - 2|$, $g(x) = -|x - 1|$



From the graph : The S.S. = $\varnothing$

(13) $f(x) = |x - 3|$, $g(x) = |2x + 1|$



From the graph : The S.S. = $\left\{-4 , \frac{2}{3}\right\}$

3

(1) $f(-x) = -x|-x| = -x|x| = -f(x)$

$\therefore$ The function is odd.

(2) $f(-x) = (-x)^2|-x| - 1 = x^2|x| - 1 = f(x)$

$\therefore$ The function is even.

(3) $f(-x) = -x|-x-2| + 4$

$= -x|x + 2| + 4 \neq f(x) \neq -f(x)$

$\therefore$ The function is neither even nor odd.

(4) $f(-x) = \frac{(-x)^2 \cos(-2x)}{5 + |-2x|} = \frac{x^2 \cos 2x}{5 + |2x|} = f(x)$

$\therefore$ The function is even.

(5) $f(-x) = 2|-x| \tan(-x) + 2(-x)|\tan(-x)|$

$= 2|x|(-\tan x) - 2x|-\tan x|$

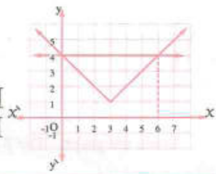
$= -2|x| \tan x - 2x|\tan x| = -f(x)$

$\therefore$ The function is odd.

4

From the graph :

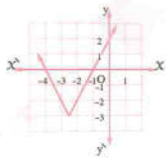
* The function is decreasing on $]-\infty , 3[$ and increasing on $]3 , \infty[$
* The S.S. = $\{0 , 6\}$



5

From the graph :

* The range = $[-3 , \infty[$
* The function is decreasing on $\left]-\infty , \frac{-5}{2}\right[$ and increasing on $\left]\frac{-5}{2} , \infty\right[$
* The S.S. = $\{-1 , -4\}$
* The algebraic solution :



$\therefore |2x + 5| = 3$ $\quad\therefore 2x + 5 = \pm 3$

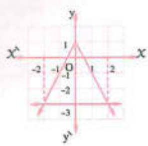
$\therefore 2x + 5 = 3$ then $x = -1$

or $2x + 5 = -3$ then $x = -4$

6

From the graph :

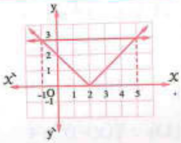
* The range = $]-\infty , 1]$
* The function is increasing on $]-\infty , 0[$ and decreasing on $]0 , \infty[$



* The function is even because it is symmetric about the y-axis
* The S.S. = $\{-2 , 2\}$

, you can verify algebraically.

7



The domain of the function f is $\mathbb{R}$

From the graph :

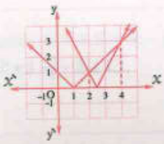
* The range = $[0 , \infty[$
* The function is decreasing on $]-\infty , 2[$ and increasing on $]2 , \infty[$
* The function is neither even nor odd.
* The S.S. = $\{-1 , 5\}$
* The algebraic solution :

$\because |x-2| = 3$ $\quad\therefore x-2 = \pm 3$

$\therefore x-2 = 3$ then $x = 5$ or $x-2 = -3$ then $x = -1$

8

$f_1(x) = |x-1| , f_2(x) = 2|x - \frac{5}{2}|$



From the graph : The S.S. = $\{2 , 4\}$

9

$\because f(-x) = (-x)^2|-x| = x^2|x| = f(x)$

$\therefore f$ is even

$$\therefore f(x) = \begin{cases} x^2(x) & , x \geq 0 \\ x^2(-x) & , x < 0 \end{cases}$$

$$= \begin{cases} x^3 & , x \geq 0 \\ -x^3 & , x < 0 \end{cases}$$

at $x \geq 0$ $\quad\therefore x^3 = 1$

$\therefore x = 1 \in [0 , \infty[$

at $x < 0$ $\quad\therefore -x^3 = 1$

$\therefore x^3 = -1$ $\quad\therefore x = -1 \in]-\infty , 0[$

$\therefore$ The S.S. = $\{1 , -1\}$

Third Higher skills

(1) (b) (2) (b) (3) (c)

(4) (c) (5) (d) (6) (d)

Instructions to solve :

(1) $\because$ The domain = $\mathbb{R} - \{-2 , 2\}$

$\because |x| + a = 0$ when $x = \pm 2$ $\therefore a = -2$

(2) $\because f(x+2) = |x+2-2| + 4 = |x| + 4$

$, \because f(x+2) = 6$ $\quad\therefore |x| + 4 = 6$

$\therefore |x| = 2$ $\quad\therefore x = \pm 2$

$\therefore$ The S.S. = $\{2 , -2\}$

(3) $\because f(x+2) = |x+2-2| + 4 = |x| + 4$

$, \because f(x+2) = 3$ $\quad\therefore |x| + 4 = 3$

$\therefore |x| = -1$ $\quad\therefore$ The S.S. = $\varnothing$

(4) $\because |x-3| = |3-x|$ for all values of x

$\therefore$ The S.S. = $\mathbb{R}$

(5) $\because |x+1|^2 + |2x+3| = 0$

$\therefore x + 1 = 0$ $\quad\therefore x = -1$

and in the same time $2x + 3 = 0$

$\therefore x = \frac{-3}{2}$

and this is a contradiction

$\therefore$ The S.S. = $\varnothing$

(6) $\because |(x-1)(x-3)| = |x-3|$

$\therefore |x-3||x-1| - |x-3| = 0$

$\therefore |x-3|(|x-1| - 1) = 0$

$\therefore |x-3| = 0$ $\quad\therefore x = 3$

or $|x-1| = 1$ $\quad\therefore x-1 = \pm 1$

$x = 0$ or 2

$\therefore$ The S.S. = $\{0 , 2 , 3\}$

Exercise 6

First Multiple choice questions

(1) c	(2) c	(3) b	(4) c	(5) c	(6) c
(7) a	(8) b	(9) d	(10) d	(11) d	(12) b
(13) c	(14) b	(15) b	(16) a	(17) c	(18) d

Second Essay questions

1

(1) $-5 \le x-3 \le 5$ $\quad \therefore -2 \le x \le 8$

$\therefore$ The S.S. $= [-2, 8]$

(2) $x-3 \ge 5$, then $x \ge 8$

or $x-3 \le -5$, then $x \le -2$

$\therefore$ The S.S. $= \mathbb{R} -]-2, 8[$

(3) $-5 < 2x-3 < 5$ $\quad \therefore -2 < 2x < 8$

$\therefore -1 < x < 4$ $\quad \therefore$ The S.S. $=]-1, 4[$

(4) $2x+5 > 3$, then $x > -1$

or $2x+5 < -3$, then $x < -4$

$\therefore$ The S.S. $= \mathbb{R} - [-4, -1]$

(5) $-10 \le 3x-5 \le 10$ $\quad \therefore -5 \le 3x \le 15$

$, -\frac{5}{3} \le x \le 5$ $\quad \therefore$ The S.S. $= [-\frac{5}{3}, 5]$

(6) $|x-3| < 7$ $\quad \therefore -7 < x-3 < 7$

$\therefore -4 < x < 10$ $\quad \therefore$ The S.S. $=]-4, 10[$

(7) $\because -4 \le 2x+6 \le 4$ $\quad \therefore -10 \le 2x \le -2$

$\therefore -5 \le x \le -1$

$\therefore$ The S.S. $= [-5, -1]$

(8) $|5-x| > 3$

$\therefore x-5 > 3$ $\quad \therefore x > 8$

or $x-5 < -3$ $\quad \therefore x < 2$

$\therefore$ The S.S. $= \mathbb{R} - [2, 8]$

(9) $-7 \le 2x+3 \le 7$ $\quad \therefore -10 \le 2x \le 4$

$\therefore -5 \le x \le 2$

$\therefore$ The S.S. $= [-5, 2]$

(10) $|2x+3| \le -1$ $\quad \therefore$ The S.S. $= \varnothing$

(11) $|2x-3| > 4$ $\quad \therefore 2x-3 > 4$

, then $x > \frac{7}{2}$ or $2x-3 < -4$, then $x < -\frac{1}{2}$

$\therefore$ The S.S. $= \mathbb{R} - [-\frac{1}{2}, \frac{7}{2}]$

(12) $\sqrt{(x-3)^2} \le 3$ $\quad \therefore |x-3| \le 3$

$\therefore -3 \le x-3 \le 3$ $\quad \therefore 0 \le x \le 6$

$\therefore$ The S.S. $= [0, 6]$

(13) $\because \sqrt{(x-1)^2} \ge 4$ $\quad \therefore |x-1| \ge 4$

$\therefore x-1 \ge 4$, then $x \ge 5$

or $x-1 \le -4$, then $x \le -3$

$\therefore$ The S.S. $= \mathbb{R} -]-3, 5[$

(14) $2|x-2| < 6$ $\quad \therefore |x-2| < 3$

$\therefore -3 < x-2 < 3$ $\quad \therefore -1 < x < 5$

$\therefore$ The S.S. $=]-1, 5[$

(15) $|3x-2| + 2 \times 2|3x-2| \ge 20$

$\therefore 5|3x-2| \ge 20$ $\quad \therefore |3x-2| \ge 4$

$\therefore 3x-2 \ge 4$

, then $x \ge 2$ or $3x-2 \le -4$, then $x \le -\frac{2}{3}$

$\therefore$ The S.S. $= \mathbb{R} -]\frac{-2}{3}, 2[$

2

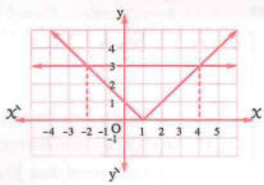
(1) $]-4, 0[$ $\qquad$ (2) $\mathbb{R} - [-1, 5]$

(3) $[-5, -1]$

3

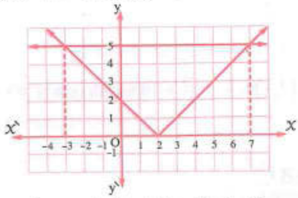
The following is the graphically solution, verify algebraically by yourself :

(1) $f(x) = |x-1|$, $g(x) = 3$



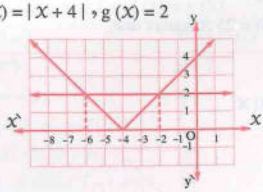
From the graph : The S.S. $=]-2, 4[$

(2) $f(x) = |x - 2|$, $g(x) = 5$



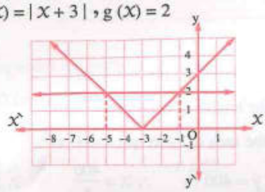
From the graph : The S.S. $= [-3 , 7]$

(3) $f(x) = |x + 4|$, $g(x) = 2$



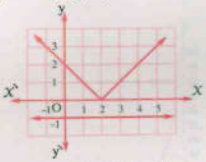
From the graph : The S.S. $= [-6 , -2]$

(4) $f(x) = |x + 3|$, $g(x) = 2$



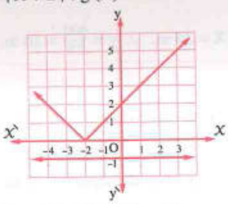
From the graph : The S.S. $= \mathbb{R} -]-5 , -1[$

(5) $f(x) = |2 - x|$, $g(x) = -1$



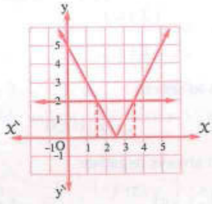
From the graph : The S.S. $= \varnothing$

(6) $f(x) = |x + 2|$, $g(x) = -1$



From the graph : The S.S. $= \mathbb{R}$

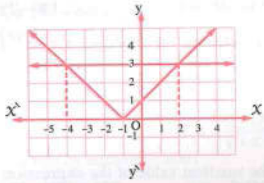
(7) $f(x) = 2\left|x - \frac{5}{2}\right|$, $g(x) = 2$



From the graph : The S.S. $= \mathbb{R} - \left]\frac{3}{2} , \frac{7}{2}\right[$

(8) $\because \sqrt{x^2 + 2x + 1} = \sqrt{(x+1)^2} = |x + 1|$

$\therefore f(x) = |x + 1|$, $g(x) = 3$



From the graph : The S.S. $= \mathbb{R} - [-4 , 2]$

4

(1) $\because -4 \le x \le 4$ $\qquad \therefore |x| \le 4$

(2) $\because 0 < x < 6$, adding (-3) to the terms of the inequality $\qquad \therefore -3 < x - 3 < 3$

$\therefore |x - 3| < 3$

(3) $\because x \le -2$ or $x \ge 2$ $\qquad \therefore |x| \ge 2$

5

(1) Let the mark of the student be x

$\therefore 60 < x < 100$, adding (-80) to the terms of the inequality

$\therefore -20 < x - 80 < 20$ $\qquad \therefore |x - 80| < 20$

(2) Let the temperature $= x$ degree

$\therefore 35 < x < 42$, adding (-38.5) to the terms of the inequality

$\therefore -3.5 < x - 38.5 < 3.5$ $\therefore |x - 38.5| < 3.5$

Third Higher skills

(1) (c) (2) (c) (3) (a)

(4) (c) (5) (c) (6) (d)

Instructions to solve :

(1) $\because b<0$ $\therefore |a|b<0$

i.e. $|a|b$ always negative

(2) $\because x\in[-1,4]$ $\therefore -1\le x\le 4$

$\therefore -2\le 2x\le 8$ $\therefore -5\le 2x-3\le 5$

$\therefore |2x-3|\le 5$

(3) $\because \sqrt{x^2-4x+4}>0$

$\therefore \sqrt{(x-2)^2}>0$ $\therefore |x-2|>0$

$\therefore$ it is satisfied for all values $x\in\mathbb{R}-\{2\}$

(4) $\because |x|+|y|\ge|x+y|$

$\therefore \frac{|x|+|y|}{|x+y|}\ge 1$

$\therefore$ The smallest value of the expression $\frac{|x|+|y|}{|x+y|}$ is 1

(5) $\because 32<61<64$ $\therefore 2^5<2^x<2^6$

$\therefore 5<x<6$

$\therefore |x-6|=-x+6$, $|x-5|=x-5$

$\therefore |x-6|+|x-5|=-x+6+x-5=1$

(6) $\because a^2b>0$ $\therefore$ b is positive

, $\because \frac{a}{b}<0$ $\therefore$ a is negative

$\therefore \sqrt{a^2}=|a|=-a$, $\sqrt{b^2}=|b|=b$

$\therefore \sqrt{a^2}+\sqrt{b^2}-(b-a)=-a+b-b+a=$ zero

Answers of Life Applications on Unit One

1

(1) $f(5000)=\frac{5}{2}\times 5000=12500$ L.E.

(2) $f(10000)=2\times 10000+2500=22500$ L.E.

(3) $f(50000)=\frac{3}{2}\times 50000+10000=85000$ L.E.

2

$P(\ell)=4\ell$

(1) $P(3)=4\times 3=12$ length unit.

(2) $P\left(\frac{15}{4}\right)=4\times\frac{15}{4}=15$ length unit.

3

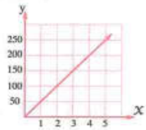
$A(r)=\pi r^2$

(1) $A\left(\frac{1}{2}\right)=\frac{1}{4}\pi$ square unit.

(2) $A(5)=25\pi$ square unit.

4

$f(x)=50x$



5

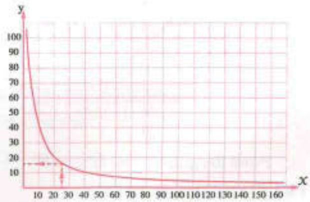
Let the width of each piece = y m.

(1) $\because$ The length of each one = x m.

and the area of each one = 400 m^2.

$\therefore xy=400$ $\therefore x=\frac{400}{y}$ $\therefore x\propto\frac{1}{y}$

(2) $y=\frac{400}{x}$



(3) When $x=25$ m. $\therefore y=\frac{400}{25}=16$ m.

6

(1) $\because$ The point (0 , 3) belongs to the curve of the function

$\therefore$ (0 , 3) satisfies the equation of the function

$f(x) = a(x-2)^2 + 4$

$\therefore 3 = a(0-2)^2 + 4$ $\quad \therefore 3 = 4a + 4$

$\therefore 4a = -1$ $\quad \therefore a = -\frac{1}{4}$

(2) $\because$ The point of the vertex of the curve is (2 , 4)

$\therefore$ The maximum height of the gate = 4 m.

(3) The width of the gate = 2 + 2 = 4 m.

7

The two roads intersect at $f(x) = g(x)$

$\therefore |x-4| = 3$ $\quad \therefore x-4 = \pm 3$

$\therefore x-4 = 3$ $\quad \therefore x = 7$

or $x-4 = -3$ $\quad \therefore x = 1$

, $\because g(7) = 3$, $g(1) = 3$

$\therefore$ A (1 , 3) , B (7 , 3)

$\therefore$ Length of $\overline{AB} = 7 - 1 = 6$

$\therefore$ The distance between A and B = 6 km.

8

Let the expected temperature on that day = x°

$\therefore |x-32| = 7$ $\quad \therefore x-32 = \pm 7$

$\therefore x-32 = 7$ $\quad \therefore x = 39$

or $x-32 = -7$ $\quad \therefore x = 25$

$\therefore$ The expected temperature recorded is 25° or 39°

9

Let Bassem's weight = x kg.

$\therefore |x-60| = 5$ $\quad \therefore x-60 = \pm 5$

$\therefore x-60 = 5$ $\quad \therefore x = 65$

or $x-60 = -5$ $\quad \therefore x = 55$

$\therefore$ The probable weight of Bassem = 55 kg. or 65 kg.

10

$\because$ B = (8 , 4) $\quad \because f(8) = \frac{4}{3}|8-5| = 4$

$\therefore$ B lies on the curve of the function f

$\therefore$ The black ball will be fall in the pocket B

11

Let the height of the applicant = x cm.

$\therefore 178 \le x \le 192$, adding (– 185) to the terms of the inequality

$\therefore -7 \le x - 185 \le 7$ $\quad \therefore |x-185| \le 7$

12

Let the aviation speed of the plane = x km/hr.

$\therefore 700 \le x \le 900$ by adding (– 800)

$\therefore -100 \le x - 800 \le 100$ $\quad \therefore |x-800| \le 100$

13

$\because |x-1600| \le 15$

$\therefore -15 \le x - 1600 \le 15$ $\quad \therefore 1585 \le x \le 1615$

$\therefore$ The greatest weight of the can = 1615 gm. and the least weight = 1585 gm.

Answers of "Unit Two"

Exercise 1

First Multiple choice questions

(1) b	(2) d	(3) b	(4) c	(5) b	(6) c
(7) b	(8) d	(9) d	(10) c	(11) d	(12) c
(13) b	(14) a	(15) c	(16) d	(17) b	(18) d
(19) b	(20) c	(21) c	(22) c	(23) c	(24) a
(25) c	(26) a	(27) b	(28) b	(29) b	(30) d
(31) c	(32) a	(33) a	(34) c	(35) a	(36) b
(37) c	(38) a	(39) d	(40) b	(41) b	(42) d
(43) a	(44) a	(45) c			

Second Essay questions

1

(1) $x^{\frac{1}{3}}$ (2) $a^{\frac{3}{4}}$ (3) $2n^{\frac{1}{3}}$ (4) $3y^{\frac{4}{5}}$

(5) $a^{\frac{3}{2}}b^3$ (6) $a^{\frac{1}{2}}b^{\frac{3}{4}}$ (7) $x^{\frac{4}{3}}$

(8) $x^{\frac{1}{3}-\frac{3}{5}}=x^{-\frac{4}{15}}$ (9) $a^{3-\frac{1}{2}}=a^{\frac{5}{2}}$

2

(1) $\sqrt{a}$ (2) $\sqrt[3]{b^2}$ (3) $5\sqrt[5]{a^4}$

(4) $8\sqrt[9]{b^4}$ (5) $\frac{1}{\sqrt[3]{9x^2}}$ (6) $\sqrt[6]{x^5}$

3

(1) $x^5=0$ $\quad\therefore x=0$

(2) $x^4=81$ $\quad\therefore x=\pm\sqrt[4]{81}=\pm 3$

(3) $x^2=-4$ $\quad\therefore x\notin\mathbb{R}$

(4) $x^3=-8$ $\quad\therefore x=\sqrt[3]{-8}=-2$

4

(1) $(16)^{\frac{3}{2}}=\sqrt{16^3}=64$

(2) $\left(\frac{16}{625}\right)^{-\frac{3}{4}}=\left(\frac{625}{16}\right)^{\frac{3}{4}}=\left(\frac{5}{2}\right)^{4\times\frac{3}{4}}=\frac{125}{8}$

(3) $\sqrt[3]{(-8)^2}=\sqrt[3]{64}=4$

(4) $\left(\sqrt[3]{10^2}\right)^{-\frac{3}{2}}=\left(10^{\frac{2}{3}}\right)^{-\frac{3}{2}}=10^{-1}=0.1$

(5) $\sqrt[4]{\left(2-\sqrt{3}\right)^4}=|2-\sqrt{3}|=2-\sqrt{3}$

(6) $\sqrt[6]{\left(1-\sqrt{7}\right)^6}=|1-\sqrt{7}|=\sqrt{7}-1$

(7) $\sqrt[5]{\left(2-\sqrt{5}\right)^5}=2-\sqrt{5}$

(8) $\sqrt[5]{243}+\sqrt[9]{512}=3+2=5$

(9) $(27)^{\frac{2}{3}}-(64)^{\frac{5}{6}}=\left(3^3\right)^{\frac{2}{3}}-\left(2^6\right)^{\frac{5}{6}}=3^2-2^5=9-32$
$=-23$

(10) $(16)^{\frac{3}{2}}\div(8)^{\frac{2}{3}}=\left(2^4\right)^{\frac{3}{2}}\div\left(2^3\right)^{\frac{2}{3}}=2^6\div 2^2=2^4=16$

(11) $\sqrt{16x^2}=4|x|$ (12) $\sqrt[5]{-32x^5}=-2x$

(13) $-\sqrt[3]{8a^6b^9}=-2a^2b^3$

(14) $\pm\sqrt{64(a^2+3)^6}=\pm 8(a^2+3)^3$

(15) $\sqrt[4]{81a^{12}}=3|a^3|$ (16) $\sqrt[7]{128(a+b)^7}=2(a+b)$

5

(1) $\left(a^{-\frac{2}{3}}\right)^{-3}=a^{-\frac{2}{3}\times(-3)}=a^2$

(2) $\sqrt[3]{x}\times x^{\frac{1}{2}}=x^{\frac{1}{3}}\times x^{\frac{1}{2}}=x^{\frac{5}{6}}=\sqrt[6]{x^5}$

(3) $x^{\frac{1}{2}}\div x^{\frac{1}{3}}=x^{\frac{1}{2}-\frac{1}{3}}=x^{\frac{1}{6}}=\sqrt[6]{x}$

(4) $\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)=x-x^{-1}$

(5) $\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}\right)\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)=a-b$

(6) $\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)^2=x+2+x^{-1}$

(7) $\left(x^{\frac{3}{2}}-x^{-\frac{3}{2}}\right)^2=x^3-2+x^{-3}$

(8) $\frac{\sqrt{a}}{a\sqrt[3]{a}}=\frac{a^{\frac{1}{2}}}{a\times a^{\frac{1}{3}}}=a^{\frac{1}{2}-1-\frac{1}{3}}=a^{-\frac{5}{6}}=\frac{1}{\sqrt[6]{a^5}}$

6

(1) $\frac{3^4\times 2^6}{6^3}=\frac{3^4\times 2^6}{2^3\times 3^3}=3^{4-3}\times 2^{6-3}$
$=3\times 2^3=3\times 8=24$

(2) $\frac{6^2\times 9^2\times 8}{(12)^2\times 3^5}=\frac{(2\times 3)^2\times(3^2)^2\times 2^3}{(2^2\times 3)^2\times 3^5}$
$=\frac{2^2\times 3^2\times 3^4\times 2^3}{2^4\times 3^2\times 3^5}$
$=2^{2+3-4}\times 3^{2+4-2-5}$
$=2\times 3^{-1}=\frac{2}{3}$

(3) $\frac{4^{2n+1}\times 2^{1-n}}{8^{n+2}}=\frac{(2^2)^{2n+1}\times 2^{1-n}}{(2^3)^{n+2}}$
$=\frac{2^{4n+2}\times 2^{1-n}}{2^{3n+6}}$
$=2^{4n+2+1-n-3n-6}$
$=2^{-3}=\frac{1}{2^3}=\frac{1}{8}$

(4) $\frac{6^{4n} \times (30)^{-2n} \times 2^{2n}}{(18)^{2n} \times (15)^{-2n}}$

$= \frac{(2 \times 3)^{4n} \times (2 \times 3 \times 5)^{-2n} \times 2^{2n}}{(2 \times 3^2)^{2n} \times (5 \times 3)^{-2n}}$

$= \frac{2^{4n} \times 3^{4n} \times 2^{-2n} \times 3^{-2n} \times 5^{-2n} \times 2^{2n}}{2^{2n} \times 3^{4n} \times 5^{-2n} \times 3^{-2n}}$

$= 2^{4n-2n+2n-2n} \times 3^{4n-2n-4n+2n} \times 5^{-2n+2n}$

$= 2^{2n} \times 3^0 \times 5^0 = 2^{2n} \times 1 \times 1 = 2^{2n}$

(5) $\frac{(27)^{-3} \times (12)^2}{16 \times (81)^{-2}} = \frac{(3^3)^{-3} \times (3)^2 \times (2^2)^2}{2^4 \times (3^4)^{-2}}$

$= \frac{3^{-9} \times 3^2 \times 2^4}{2^4 \times 3^{-8}} = 3^{-9+2+8} = 3$

(6) $\frac{9^{4n+1} \times 4^{2-2n}}{3^{9n+1} \times 48^{1-n}} = \frac{(3^2)^{4n+1} \times (2^2)^{2-2n}}{3^{9n+1} \times 3^{1-n} \times (2^4)^{1-n}}$

$= \frac{3^{8n+2} \times 2^{4-4n}}{3^{9n+1} \times 3^{1-n} \times 2^{4-4n}}$

$= 3^{8n+2-9n-1-1+n} \times 2^{4-4n-4+4n}$

$= 3^0 \times 2^0 = 1 \times 1 = 1$

(7) $\frac{16^{x-\frac{1}{4}} \times 9^{x+\frac{1}{2}}}{8^{x-1} \times 18^{x+2}}$

$= \frac{(2^4)^{x-\frac{1}{4}} \times (3^2)^{x+\frac{1}{2}}}{(2^3)^{x-1} \times (2 \times 3^2)^{x+2}} = \frac{2^{4x-1} \times 3^{2x+1}}{2^{3x-3} \times 2^{x+2} \times 3^{2x+4}}$

$= 2^{4x-1-3x+3-x-2} \times 3^{2x+1-2x-4} = 2^0 \times 3^{-3} = \frac{1}{27}$

(8) $\frac{25}{27} \times \left(\frac{1}{25}\right)^{\frac{1}{2}} \times (81)^{\frac{3}{4}} = \frac{5^2}{3^3} \times (5^{-2})^{\frac{1}{2}} \times (3^4)^{\frac{3}{4}}$

$= 5^2 \times 3^{-3} \times 5^{-1} \times 3^3$

$= 5^1 \times 3^0 = 5$

(9) $(125)^{\frac{2}{3}} \times (81)^{\frac{1}{4}} \times (15)^{-1}$

$= (5^3)^{\frac{2}{3}} \times (3^4)^{\frac{1}{4}} \times (3 \times 5)^{-1}$

$= 5^2 \times 3 \times 3^{-1} \times 5^{-1}$

$= 5^{2-1} \times 3^{1-1} = 5 \times 1 = 5$

(10) $\frac{8^{\frac{3}{8}} \times 4^{-\frac{3}{16}}}{2^{-\frac{5}{4}}} = \frac{(2^3)^{\frac{3}{8}} \times (2^2)^{-\frac{3}{16}}}{2^{-\frac{5}{4}}} = \frac{2^{\frac{9}{8}} \times 2^{-\frac{3}{8}}}{2^{-\frac{5}{4}}}$

$= 2^{\frac{9}{8}-\frac{3}{8}+\frac{5}{4}} = 2^2 = 4$

7

(1) L.H.S. $= \frac{2^x \times (3^2)^{x+1}}{3 \times (3^2 \times 2)^x} = \frac{2^x \times 3^{2x+2}}{3 \times 3^{2x} \times 2^x}$

$= 3^{2x+2-1-2x} \times 2^{x-x} = 3 \times 2^0 = 3 \times 1 = 3$

(2) L.H.S. $= \frac{(7^3)^{2x-\frac{1}{3}} \times (2^2)^{3x+1}}{(7^2 \times 2^2)^{3x} \times 2^2}$

$= \frac{7^{6x-1} \times 2^{6x+2}}{7^{6x} \times 2^{6x} \times 2^2}$

$= 7^{6x-1-6x} \times 2^{6x+2-6x-2} = 7^{-1} \times 2^0$

$= \frac{1}{7} \times 1 = \frac{1}{7}$

8

(1) Error because $\sqrt[4]{x^4} = |x|$

(2) Error because $x = \pm (4)^{\frac{3}{2}} = \pm 8$

9

(1) $\because x^2 = 36 \quad \therefore x = \pm 6$

$\therefore$ The S.S. $= \{6, -6\}$

(2) $\because x^2 = -49 \quad \therefore$ The S.S. $= \varnothing$

(3) $\because x^3 = 125 \quad \therefore x = (5^3)^{\frac{1}{3}}$

$\therefore x = 5 \quad \therefore$ The S.S. $= \{5\}$

(4) $\because x^5 = -32 \quad \therefore x = \left((-2)^5\right)^{\frac{1}{5}}$

$\therefore x = -2 \quad \therefore$ The S.S. $= \{-2\}$

(5) $\because x^7 = -128 \quad \therefore x = \left((-2)^7\right)^{\frac{1}{7}}$

$\therefore x = -2 \quad \therefore$ The S.S. $= \{-2\}$

(6) $\because x^4 = 1296 \quad \therefore x = \pm (6^4)^{\frac{1}{4}}$

$\therefore x = \pm 6 \quad \therefore$ The S.S. $= \{6, -6\}$

(7) $\because x^{-4} = \frac{1}{16} \quad \therefore x^4 = 16 \quad \therefore x = \pm (2^4)^{\frac{1}{4}}$

$\therefore x = \pm 2 \quad \therefore$ The S.S. $= \{2, -2\}$

(8) $\because x^{\frac{7}{2}} = 2^7 \quad \therefore x = (2^7)^{\frac{2}{7}} = 4$

$\therefore$ The S.S. $= \{4\}$

(9) $\because x^{\frac{5}{2}} = \frac{1}{2^5} = 2^{-5} \therefore x = (2^{-5})^{\frac{2}{5}}$

$\therefore x = 2^{-2} \quad \therefore$ The S.S. $= \left\{\frac{1}{4}\right\}$

(10) $\because x^{-\frac{5}{3}} = 2^{\frac{5}{3}} \quad \therefore x = (2^{\frac{5}{3}})^{-\frac{3}{5}}$

$\therefore x = 2^{-1} = \frac{1}{2} \quad \therefore$ The S.S. $= \left\{\frac{1}{2}\right\}$

(11) $\because 3x^{-\frac{3}{4}} = \frac{3}{8}$ $\therefore x^{-\frac{3}{4}} = \frac{3}{8} \times \frac{1}{3}$

$\therefore x^{-\frac{3}{4}} = \frac{1}{8}$ $\therefore x^{\frac{3}{4}} = 8$ $\therefore x = (2^3)^{\frac{4}{3}}$

$\therefore x = 2^4 = 16$ $\therefore$ The S.S. $= \{16\}$

(12) $\because x^{-\frac{5}{2}} = 3^5$ $\therefore x = (3^5)^{-\frac{2}{5}} = 3^{-2} = \frac{1}{9}$

$\therefore$ The S.S. $= \left\{\frac{1}{9}\right\}$

(13) $\because x^{\frac{2}{3}} = 5^{-2}$ $\therefore x = \pm(5^{-2})^{\frac{3}{2}}$

$\therefore x = \pm 5^{-3} = \pm\frac{1}{125}$

$\therefore$ The S.S. $= \left\{\frac{1}{125}, \frac{-1}{125}\right\}$

(14) $\because (x+1)^{\frac{3}{4}} = 8$ $\therefore x+1 = (2^3)^{\frac{4}{3}}$

$\therefore x+1 = 16$ $\therefore x = 15$

$\therefore$ The S.S. $= \{15\}$

(15) $\because (x-5)^{\frac{5}{2}} = 32$ $\therefore x-5 = (2^5)^{\frac{2}{5}}$

$\therefore x-5 = 4$ $\therefore x = 9$

$\therefore$ The S.S. $= \{9\}$

(16) $\because x^{\frac{4}{3}} = 3^4$ $\therefore x = \pm(3^4)^{\frac{3}{4}} = \pm 3^3 = \pm 27$

$\therefore$ The S.S. $= \{27, -27\}$

(17) $\because (x-1)^{\frac{5}{3}} = 2^5$ $\therefore x-1 = (2^5)^{\frac{3}{5}} = 2^3 = 8$

$\therefore x = 9$ $\therefore$ The S.S. $= \{9\}$

(18) $\because 2x+3 = \pm(3^4)^{\frac{3}{4}}$ $\therefore 2x+3 = \pm 27$

$\therefore 2x = 24$ or $2x = -30$

$\therefore x = 12$ or $x = -15$

$\therefore$ The S.S. $= \{12, -15\}$

(19) $\because \sqrt{x} + 2 = 9$ $\therefore \sqrt{x} = 7$

$\therefore x = 49$ $\therefore$ The S.S. $= \{49\}$

(20) $\because (x^{\frac{2}{5}} - 1)(x^{\frac{2}{5}} - 4) = 0$

$\therefore x^{\frac{2}{5}} = 1$, then $x = \pm 1$

or $x^{\frac{2}{5}} = 4$, then $x = \pm(2^2)^{\frac{5}{2}} = \pm 32$

$\therefore$ The S.S. $= \{1, -1, 32, -32\}$

10

(1) $3^{x+4} = 9$ $\therefore 3^{x+4} = 3^2$

$\therefore x+4 = 2$ $\therefore x = -2$ $\therefore$ The S.S. $= \{-2\}$

(2) $2^{x-5} = \frac{1}{32}$ $\therefore 2^{x-5} = 2^{-5}$

$\therefore x-5 = -5$ $\therefore x = 0$ $\therefore$ The S.S. $= \{0\}$

(3) $7^{x-2} = 1$ $\therefore x-2 = 0$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

(4) $3^{x+5} = 1$ $\therefore x+5 = 0$

$\therefore x = -5$ $\therefore$ The S.S. $= \{-5\}$

(5) $\because 4^{1-x} = \frac{1}{4}$ $\therefore 4^{1-x} = 4^{-1}$

$\therefore 1-x = -1$ $\therefore x = 2$

$\therefore$ The S.S. $= \{2\}$

(6) $5^{x+3} = 4^{x+3}$ $\therefore x+3 = 0$

$\therefore x = -3$ $\therefore$ The S.S. $= \{-3\}$

(7) $5^{x+2} = x^{x+2}$

$\therefore$ Either $x = 5$ or $x+2 = 0$

$\therefore x = -2$ $\therefore$ The S.S. $= \{-2, 5\}$

(8) $\because 2 \times 3^{x-2} = 54$ $\therefore 3^{x-2} = 27$

$\therefore 3^{x-2} = 3^3$ $\therefore x-2 = 3$

$\therefore x = 5$ $\therefore$ The S.S. $= \{5\}$

(9) $\because 2^{3x-6} = 5^{x-2}$ $\therefore 2^{3(x-2)} = 5^{x-2}$

$\therefore 8^{x-2} = 5^{x-2}$ $\therefore x-2 = 0$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

(10) $2^{x^2-9} = 1$ $\therefore x^2 - 9 = 0$

$\therefore x^2 = 9$ $\therefore x = \pm 3$

$\therefore$ The S.S. $= \{3, -3\}$

(11) $2 \times 4^{x+3} = \frac{1}{32}$ $\therefore 4^{x+3} = \frac{1}{64}$

$\therefore 4^{x+3} = 4^{-3}$ $\therefore x+3 = -3$

$\therefore x = -6$ $\therefore$ The S.S. $= \{-6\}$

(12) $\left(\frac{3}{5}\right)^{2x-1} = \frac{27}{125}$ $\therefore \left(\frac{3}{5}\right)^{2x-1} = \left(\frac{3}{5}\right)^3$

$\therefore 2x-1 = 3$ $\therefore 2x = 4$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

(13) $\because \left(\frac{3}{2}\right)^{x-2} = \frac{8}{27}$ $\therefore \left(\frac{3}{2}\right)^{x-2} = \left(\frac{3}{2}\right)^{-3}$

$\therefore x-2 = -3$ $\therefore x = -1$

$\therefore$ The S.S. $= \{-1\}$

(14) $\because 2^x \times 5^{-x} = \frac{4}{25}$

$\therefore \left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^2$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

(15) $\left(\sqrt{7}\right)^{|x+2|}=49$ $\quad\therefore \left(\sqrt{7}\right)^{|x+2|}=\left(\sqrt{7}\right)^4$

$\therefore |x+2|=4$ $\quad\therefore x+2=\pm 4$

$\therefore$ Either $x+2=4$, then $x=2$

or $x+2=-4$, then $x=-6$, then

$\therefore$ The S.S. $=\{2 , -6\}$

(16) $5^{x^2}=25^{x+4}$ $\quad\therefore 5^{x^2}=5^{2x+8}$

$\therefore x^2=2x+8$ $\quad\therefore x^2-2x-8=0$

$\therefore (x+2)(x-4)=0$

$\therefore x=-2$ or $x=4$ $\quad\therefore$ The S.S. $=\{-2 , 4\}$

11

(1) $3^x+3^{1+x}=36$ $\quad\therefore 3^x(1+3)=36$

$\therefore 3^x\times 4=36$ $\quad\therefore 3^x=9$

$\therefore 3^x=3^2$ $\quad\therefore x=2$

$\therefore$ The S.S. $=\{2\}$

(2) $5^{x+1}+5^{x-1}=26$ $\quad\therefore 5^x(5+5^{-1})=26$

$\therefore 5^x\times\frac{26}{5}=26$ $\quad\therefore 5^x=5$

$\therefore x=1$ $\quad\therefore$ The S.S. $=\{1\}$

(3) $3^{x+3}-3^{x+2}=162$ $\quad\therefore 3^x(3^3-3^2)=162$

$\therefore 3^x=9$ $\quad\therefore 3^x=3^2$

$\therefore x=2$ $\quad\therefore$ The S.S. $=\{2\}$

(4) $2^{3x+1}-2^{3x-2}=56$ $\quad\therefore 2^{3x-2}(8-1)=56$

$\therefore 2^{3x-2}\times 7=56$ $\quad\therefore 2^{3x-2}=8$

$\therefore 2^{3x-2}=2^3$ $\quad\therefore 3x-2=3$

$\therefore 3x=5$ $\quad\therefore x=\frac{5}{3}$

$\therefore$ The S.S. $=\left\{\frac{5}{3}\right\}$

(5) $\because 9^x-3\times 3^x=0$ $\quad\therefore 3^{2x}-3\times 3^x=0$

$\therefore 3^x(3^x-3)=0$ $\quad\therefore 3^x=0$ (refused)

or $3^x-3=0$ $\quad\therefore 3^x=3$ $\quad\therefore x=1$

$\therefore$ The S.S. $=\{1\}$

(6) $2^x+2^{5-x}=12$

$\therefore 2^x+\frac{2^5}{2^x}=12$ (multiplying by 2^x)

$\therefore 2^{2x}+32=12\times 2^x$

$\therefore 2^{2x}-12\times 2^x+32=0$

$\therefore (2^x-4)(2^x-8)=0$

$\therefore 2^x=4$, then $x=2$ or $2^x=8$, then $x=3$

$\therefore$ The S.S. $=\{2 , 3\}$

(7) Multiplying by 5^x

$\therefore 5^{2x}-30\times 5^x+125=0$

$\therefore (5^x-5)(5^x-25)=0$

$\therefore 5^x=5$ $\quad\therefore x=1$

or $5^x=25$ $\quad\therefore 5^x=5^2$ $\quad\therefore x=2$

$\therefore$ The S.S. $=\{1 , 2\}$

(8) $4^x+2^{x+1}=8$ $\quad\therefore 2^{2x}+2(2)^x-8=0$

$\therefore (2^x+4)(2^x-2)=0$

$\therefore 2^x=-4$ (refused) or $2^x=2$

$\therefore x=1$ $\quad\therefore$ The S.S. $=\{1\}$

Third Higher skills

(1) (c) (2) (b) (3) (d)

(4) (a) (5) (a) (6) (d)

Instructions to solve :

(1) $\because \sqrt{2}\times\sqrt[3]{3}=\sqrt[6]{x}$

$\therefore \left(\sqrt[6]{x}\right)^6=\left(\sqrt{2}\right)^6\times\left(\sqrt[3]{3}\right)^6$

$\therefore x=8\times 9=72$

(2) $5^{x+1}+5^x=5^x(5+1)=5^x\times 6$

$\therefore$ The number $(5^{x+1}+5^x)$ is divisible by 6

(3) $\because 3^a=4^b$ $\quad\therefore 3^{\frac{a}{b}}=4$ $\quad\therefore 9^{\frac{a}{b}}=16$

, $\because 3^a=4^b$ $\quad\therefore 4^{\frac{b}{a}}=3$ $\quad\therefore (16)^{\frac{b}{a}}=9$

$\therefore 9^{\frac{a}{b}}+(16)^{\frac{b}{a}}=16+9=25$

(4) $\because 2^a=3$ $\quad\therefore 2^{ab}=3^b=7$

$\therefore 2^{abc}=7^c=11$

(5) $\because$ n is an even integer

$\therefore x^n>0$ for all $x\in\mathbb{R}^*$

(6) $\because x^{\frac{2}{3}}=a$ $\quad\therefore x=a^{\frac{3}{2}}=\left(\sqrt{a}\right)^3$

$\therefore$ it must be $a\geq 0$

i.e. : $a\in\mathbb{R}^+\cup\{0\}$

Activity

(1) 458.69 (2) 1.9

UNIT 2

Exercise 8

First Multiple choice questions

(1) d	(2) d	(3) a	(4) b	(5) c	(6) c
(7) c	(8) c	(9) a	(10) c	(11) a	(12) c
(13) b	(14) c	(15) c	(16) a	(17) d	(18) c
(19) d	(20) c	(21) a	(22) d	(23) a	(24) a
(25) b	(26) b	(27) c	(28) c	(29) b	(30) d
(31) c	(32) d	(33) a	(34) c	(35) d	(36) b
(37) b	(38) b				

Second Essay questions

1

(1) Not exponential.

(2) Exponential function , its base = 5 , its power = x

(3) Not exponential.

(4) Not exponential.

(5) Exponential function , its base = $\frac{2}{3}$, its power = $x - 1$

(6) Not exponential.

2

$$\frac{f(x+4) - f(x+3)}{f(x+5) - f(x+4)} = \frac{5^{x+4} - 5^{x+3}}{5^{x+5} - 5^{x+4}} = \frac{5^{x+3}(5-1)}{5^{x+4}(5-1)} = \frac{1}{5}$$

3

L.H.S. $= f(a) \times f(b) = 3^a \times 3^b = 3^{a+b} = f(a+b) =$ R.H.S.

4

L.H.S. $= \frac{5^{x+1} \times 5^{x-1+1}}{5^{x-2+1} \times 5^{x+1+1}} = \frac{5^{x+1} \times 5^x}{5^{x-1} \times 5^{x+2}} = 5^{x+1+x-x+1-x-2} = 5^0 = 1$

5

L.H.S. $= \frac{2^{x+1}}{2^{x-1}} + \frac{2^{x-1}}{2^{x+1}} = 2^2 + \frac{1}{2^2} = \frac{17}{4}$

6

(1) $\because f(x) = 8 \quad \therefore 2^x = 8$

$\therefore 2^x = 2^3 \quad \therefore x = 3$

$\therefore$ The S.S. $= \{3\}$

(2) $\because f(x+1) = \frac{1}{32} \quad \therefore 2^{x+1} = \frac{1}{32}$

$\therefore 2^{x+1} = 2^{-5} \quad \therefore x + 1 = -5$

$\therefore x = -6 \quad \therefore$ The S.S. $= \{-6\}$

7

(1) $\because f(x) = 27 \quad \therefore 3^{x+1} = 27$

$\therefore 3^{x+1} = 3^3 \quad \therefore x + 1 = 3$

$\therefore x = 2 \quad \therefore$ The S.S. $= \{2\}$

(2) $\because f(x-1) = \frac{1}{9} \quad \therefore 3^{x-1+1} = \frac{1}{9}$

$\therefore 3^x = 3^{-2} \quad \therefore x = -2$

$\therefore$ The S.S. $= \{-2\}$

8

(1) $\because f(x) = 343$

$\therefore 7^{x-2} = 343 \quad \therefore 7^{x-2} = 7^3$

$\therefore x - 2 = 3 \quad \therefore x = 5$

$\therefore$ The S.S. $= \{5\}$

(2) $\because f(2x) = \frac{1}{49} \quad \therefore 7^{2x-2} = \frac{1}{49}$

$\therefore 7^{2x-2} = 7^{-2}$

$\therefore 2x - 2 = -2 \quad \therefore 2x = 0$

$\therefore x = 0 \quad \therefore$ The S.S. $= \{0\}$

9

$\because f(2x-1) + f(x-2) = 50$

$\therefore 7^{(2x-1)+1} + 7^{(x-2)+1} = 50$

$\therefore 7^{2x} + 7^{x-1} = 50$ (multiplying by 7)

$\therefore 7 \times 7^{2x} + 7^x - 350 = 0$

$\therefore (7 \times 7^x + 50)(7^x - 7) = 0$

$\therefore 7 \times 7^x + 50 = 0$, then $7^x = \frac{-50}{7}$ (refused)

or $7^x - 7 = 0 \quad \therefore 7^x = 7 \quad \therefore x = 1$

10

$\because 3^{x+1} + 3^{x-1} = 90 \quad \therefore 3^{x-1}(9+1) = 90$

$\therefore 3^{x-1} = 3^2 \quad \therefore x - 1 = 2$

$\therefore x = 3$

11

$\because 4^{x+1} + 4^{x-1} = 68$ $\therefore 4^{x-1}(16+1) = 68$

$\therefore 4^{x-1} = 4$ $\therefore x - 1 = 1$

$\therefore x = 2$

12

$\because f_1(2x-1) + f_2(x+1) = 756$

$\therefore 3^{2x-1} + 9^{x+1} = 756$ $\therefore 3^{2x-1} + 3^{2x+2} = 756$

$\therefore 3^{2x}(3^{-1} + 3^2) = 756$ $\therefore 3^{2x} \times \frac{28}{3} = 756$

$\therefore 3^{2x} = 756 \times \frac{3}{28}$ $\therefore 3^{2x} = 81$

$\therefore 3^{2x} = 3^4$ $\therefore 2x = 4$

$\therefore x = 2$

13

$\because 7^{2x-1} + 7^{2x+1} = \frac{50}{49}$ $\therefore 7^{2x-1}(1+49) = \frac{50}{49}$

$\therefore 7^{2x-1} = 7^{-2}$ $\therefore 2x - 1 = -2$

$\therefore 2x = -1$ $\therefore x = \frac{-1}{2}$

14

$\because 3^{x+1} + 3^{3-x} = 30$ (multiplying by 3^{x-1})

$\therefore 3^{2x} + 3^2 - 10 \times 3 \times 3^{x-1} = 0$

$\therefore 3^{2x} - 10 \times 3^x + 9 = 0$ $\therefore (3^x - 1)(3^x - 9) = 0$

$\therefore 3^x = 1$ $\therefore x = 0$

or $3^x = 9$ $\therefore x = 2$

15

$\because 2^{2x} - 6 \times 2^x + 8 = 0$ $\therefore (2^x - 2)(2^x - 4) = 0$

$\therefore 2^x = 2$ $\therefore x = 1$ or $2^x = 4$ $\therefore x = 2$

$\therefore$ The S.S. $= \{1, 2\}$

16

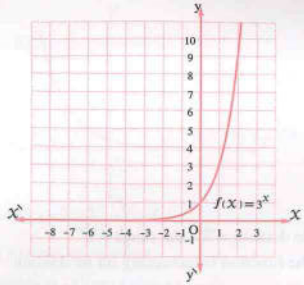
$$\text{L.H.S.} = \frac{3^{2x+2} + 3^{2x-1}}{5 \times 3^{2x} - 7 \times 3^{2x-1}} = \frac{3^{2x-1}(3^3+1)}{3^{2x-1}(5 \times 3 - 7)} = \frac{28}{8} = \frac{7}{2}$$

17

$$\text{L.H.S.} = \frac{3^{3(x+1)-1} \times 3^{3(x+2)-1}}{3^{3(x+3)-1}} = \frac{3^{3x+2} \times 3^{3x+5}}{3^{3x+8}} = 3^{3x+2+3x+5-3x-8} = 3^{3x-1} = f(x)$$

18

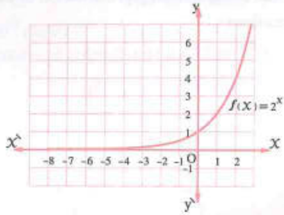
(1)



The domain $= \mathbb{R}$, the range $= \mathbb{R}^+$

, the function is increasing on its domain.

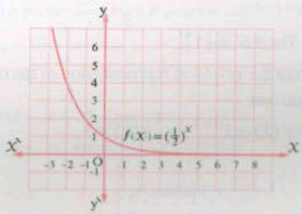
(2)



The domain $= \mathbb{R}$, the range $= \mathbb{R}^+$

, the function is increasing on its domain.

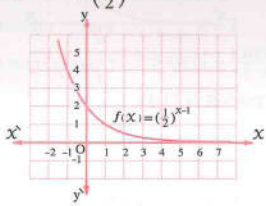
(3)



The domain $= \mathbb{R}$, the range $= \mathbb{R}^+$

, the function is decreasing on its domain.

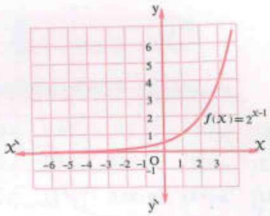
(4) $f(x) = 2^{-(x-1)} = \left(\frac{1}{2}\right)^{x-1}$



The domain $= \mathbb{R}$, the range $= \mathbb{R}^+$

, the function is decreasing on its domain.

(5)



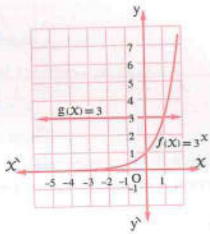
The domain $= \mathbb{R}$, the range $= \mathbb{R}^+$

, the function is increasing on its domain.

19

(1) From the graphical representation of the two functions :

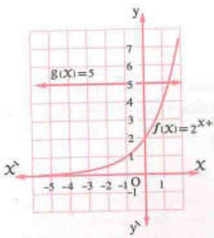
$f : f(x) = 3^x$, $g : g(x) = 3$



$\therefore$ The S.S. $= \{1\}$

(2) From the graphical representation of the two functions :

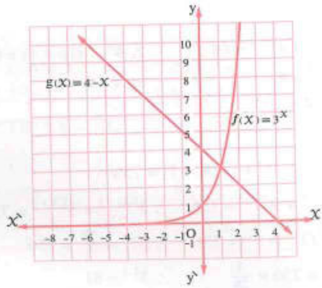
$f : f(x) = 2^{x+1}$, $g : g(x) = 5$



$\therefore$ The S.S. $= \{1.3\}$

(3) From the graphical representation of the two functions :

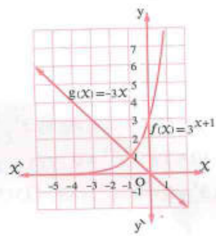
$f : f(x) = 3^x$, $g : g(x) = 4 - x$



$\therefore$ The S.S. $= \{1\}$

(4) From the graphical representation of the two functions :

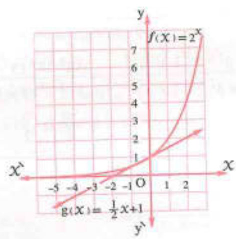
$f : f(x) = 3^{x+1}$, $g : g(x) = -x$



$\therefore$ The S.S. $= \{-1\}$

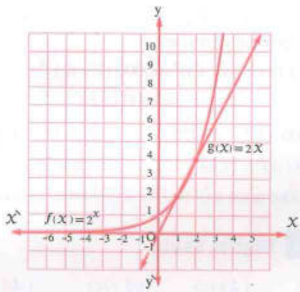
(5) From the graphical representation of the two functions :

$f : f(x) = 2^x$, $g : g(x) = \frac{1}{2} x + 1$



$\therefore$ The S.S. $= \{0 , -1\}$

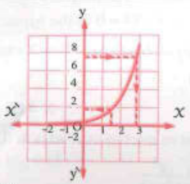
(6) From the graphical representation of the two functions : $f : f(X) = 2^X$, $g : g(X) = 2X$



$\therefore$ The S.S. = $\{1 , 2\}$

20

X	−2	−1	0	1	2	3
$f(X)$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



From the graph :

(1) $f(\frac{3}{2}) \simeq 1.7$

(2) When $3^{X-1} = 7\frac{1}{2}$, then $X \simeq 2.8$

21

The sum of money $c = a\left(1 + \frac{r}{X}\right)^{nX}$

$= 8000\left(1 + \frac{0.05}{1}\right)^{7 \times 1}$

$=$ L.E. 11256.8

22

(1) The number of population after n years since 2000 $= a(1 + r)^n$

$= 43.3\left(1 + \frac{1.5}{100}\right)^n$

$= 43.3\,(1.015)^n$

(2) In 2020 , the number of years will be $= 2020 - 2000 = 20$ years

$\therefore$ The number of population

$= 43.3\,(1.015)^{20} \simeq 58.3$ million people.

23

$a = 36400$, $r = \frac{4}{100} = 0.04$

t = the number of matches

, the numbers of fans $(y) = a(1 - r)^t$

$= 36400\,(1 - 0.04)^t$

$= 36400\,(0.96)^t$

, in the 10th match :

The numbers of fans $(y) = 36400\,(0.96)^{10}$

$\simeq 24200$ fans.

24

$a = 1850$, $r = \frac{9}{100} = 0.09$

(1) The production after n years :

$f(n) = a(1 - r)^n = 1850\,(1 - 0.09)^n$

$= 1850\,(0.91)^n$

(2) The production after 8 years $= 1850\,(0.91)^8$

$\simeq 870$ kg.

25

$a = 2000$, $r = 0.07$, $n = 10$

$\therefore$ The sum of money $c = a\left(1 + \frac{r}{X}\right)^{nX}$

$= 2000\left(1 + \frac{0.07}{X}\right)^{10X}$

(1) At yearly interest $\therefore X = 1$

$\therefore$ The sum of money $c = 2000\,(1 + 0.07)^{10}$

$\simeq$ L.E. 3934.3

(2) At 6 month's interest $\therefore X = 2$

$\therefore$ The sum of money $c = 2000\left(1 + \frac{0.07}{2}\right)^{10 \times 2}$

$\simeq$ L.E. 3979.58

(3) Monthly interest $\therefore X = 12$

$\therefore$ The sum of money $c = 2000\left(1 + \frac{0.07}{12}\right)^{10 \times 12}$

$\simeq$ L.E. 4019.32

26

$\because X = 160000\ (0.95)^n$

(1) $\because$ The car is brand new $\therefore n = 0$

$\therefore X = 160000\ (0.95)^0 = \text{L.E. } 160000$

(2) After 5 years $\therefore n = 5$

$\therefore X = 160000\ (0.95)^5 = 123804.95$

$\therefore$ car price after 5 years of its buying date $= \text{L.E. } 123804.95$

27

After 8 weeks $\therefore n = 8$

$\therefore$ The number of salmons in this lake :

$f(8) = 200\ (1.03)^8$

$\simeq 253$ salmons

28

$a = 4.6 , r = \frac{4}{100} = 0.04$

(1) The exponential growth function after t yeras

$= a(1+r)^t = 4.6\ (1 + 0.04)^t$

$= 4.6\ (1.04)^t$ million people.

(2) After 5 years :

The number of population $= 4.6\ (1.04)^5$

$\simeq 5.6$ million people.

29

$a = 4000 , r = \frac{8}{100} = 0.08$

(1) The price of an article after n year

$f : f(n) = a(1+r)^n$

$= 4000\ (1 + 0.08)^n = 4000\ (1.08)^n$

(2) The price of an article after 4 years

$f(4) = 4000\ (1.08)^4 \simeq \text{L.E. } 5442$

30

$a = 1000000 , r = \frac{6}{100} = 0.06$

(1) The growth function $f : f(n) = a(1+r)^n$

$= 1000000\ (1 + 0.06)^n$

$= 1000000\ (1.06)^n$

(2) The money after 10 years :

$f(10) = 1000000\ (1.06)^{10}$

$= \text{L.E. } 1790847.697$

31

$a = 2000 , r = \frac{10}{100} = 0.1$

(1) The exponential growth function representing the price after n years is

$f : f(n) = a(1+r)^n = 2000\ (1 + 0.1)^n$

$= 2000\ (1.1)^n$

(2) $2000\ (1.1)^n = 2420$ $\therefore (1.1)^n = 1.21$

$\therefore (1.1)^n = (1.1)^2$ $\therefore n = 2$

$\therefore$ The price will be L.E. 2420 after 2 years.

Third Higher skills

(1) (d) (2) (c) (3) (c) (4) (d)

Instructions to solve :

(1) $\because$ The function is decreasing if $0 < 2a < 1$

$\therefore 0 < a < \frac{1}{2}$ $\therefore a \in]0 , \frac{1}{2}[$

(2) $\because$ The function is increasing

$\therefore \frac{a}{3} > 1$ $\therefore a > 3$

(3) The curve intersects the X-axis at $y = 0$

by substitute $f(X) = 0$ in the given functions

, no values can be obtained for X except in case (c)

$3^X - 1 = 0$

$\therefore 3^X = 1$ $\therefore X = 0$

i.e. The curve intersects X-axis at the point $(0 , 0)$

(4) $\because f(X) = \frac{9^X}{9^X + 3}$

$\therefore f(1 - X) = \frac{9^{1-X}}{9^{1-X} + 3}$ (multiply by $\frac{9^X}{9^X}$)

$\therefore f(1 - X) = \frac{9}{9 + 3 \times 9^X} = \frac{9}{3(3 + 9^X)} = \frac{3}{3 + 9^X}$

$\therefore f(X) + f(1 - X) = \frac{9^X}{9^X + 3} + \frac{3}{3 + 9^X} = \frac{9^X + 3}{9^X + 3} = 1$

Exercise 9

First Multiple choice questions

(1) b	(2) b	(3) b	(4) d	(5) d	(6) d
(7) a	(8) a	(9) c	(10) a	(11) b	(12) d
(13) c	(14) a	(15) d	(16) c	(17) c	(18) b
(19) b	(20) b	(21) a	(22) c	(23) a	(24) b
(25) b	(26) c	(27) b	(28) c	(29) a	(30) b
(31) a	(32) c	(33) c			

Second Essay questions

1

(1) $2^7 = 128$

(2) $(49)^{\frac{1}{2}} = 7$

(3) $\left(\frac{2}{5}\right)^2 = \frac{4}{25}$

(4) $3^{-4} = \frac{1}{81}$

(5) $(10)^{-3} = 0.001$

(6) $2^{\frac{5}{2}} = 4\sqrt{2}$

2

(1) $\log_5 125 = 3$

(2) $\log_9 81 = 2$

(3) $\log_5 1 =$ zero

(4) $\log_{\sqrt{2}} 4 = 4$

(5) $\log_5 \frac{1}{125} = -3$

(6) $\log_2 c = n$

3

(1) Let $\log_2 16 = x$ $\quad \therefore 2^x = 16$

$\therefore 2^x = 2^4$ $\quad \therefore x = 4$

$\therefore \log_2 16 = 4$

(2) Let $\log_7 7 = x$ $\quad \therefore 7^x = 7$

$\therefore x = 1$ $\quad \therefore \log_7 7 = 1$

(3) Let $\log_5 1 = x$ $\quad \therefore 5^x = 1 = 5^0$

$\therefore x = 0$ $\quad \therefore \log_5 1 = 0$

(4) Let $\log_3 9 = x$ $\quad \therefore 3^x = 9$

$\therefore 3^x = 3^2$ $\quad \therefore x = 2$

$\therefore \log_3 9 = 2$

(5) Let $\log 0.00001 = x$ $\quad \therefore 10^x = 0.00001$

$\therefore 10^x = \frac{1}{100000}$ $\quad \therefore 10^x = 10^{-5}$

$\therefore x = -5$ $\quad \therefore \log 0.00001 = -5$

(6) Let $\log_4 2\sqrt{2} = x$ $\quad \therefore 4^x = 2\sqrt{2}$

$\therefore 2^{2x} = 2^{\frac{3}{2}}$ $\quad \therefore 2x = \frac{3}{2}$

$\therefore x = \frac{3}{4}$ $\quad \therefore \log_4 2\sqrt{2} = \frac{3}{4}$

(7) Let $\log_{\frac{1}{2}} 128 = x$ $\quad \therefore \left(\frac{1}{2}\right)^x = 128 = 2^7$

$\therefore \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-7}$ $\quad \therefore x = -7$

$\therefore \log_{\frac{1}{2}} 128 = -7$

(8) Let $\log_9 \frac{1}{27} = x$ $\quad \therefore 9^x = \frac{1}{27}$

$\therefore 3^{2x} = 3^{-3}$ $\quad \therefore 2x = -3$

$\therefore x = \frac{-3}{2}$ $\quad \therefore \log_9 \frac{1}{27} = \frac{-3}{2}$

(9) Let $\log_{0.2} 125 = x$ $\quad \therefore (0.2)^x = 125 = 5^3$

$\therefore \left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{-3}$ $\quad \therefore x = -3$

$\therefore \log_{0.2} 125 = -3$

(10) Let $\log_{\sqrt{2}} 8\sqrt{2} = x$

$\therefore (\sqrt{2})^x = 8\sqrt{2} = (\sqrt{2})^7$

$\therefore x = 7$ $\quad \therefore \log_{\sqrt{2}} 8\sqrt{2} = 7$

4

(1) $2^7 = x$ $\quad \therefore x = 128$

(2) $\left(\frac{1}{3}\right)^{-1} = x$ $\quad \therefore x = 3$

(3) $(\sqrt{3})^4 = x$ $\quad \therefore x = 9$

(4) $3^4 = x^2$ $\quad \therefore 81 = x^2$ $\quad \therefore x = \pm 9$

(5) $(81)^{\frac{3}{4}} = x$ $\quad \therefore x = 27$

(6) $(81)^{\frac{1}{4}} = 3x$ $\quad \therefore 3 = 3x$ $\quad \therefore x = 1$

(7) $(0.2)^{-2} = x$ $\quad \therefore x = \left(\frac{10}{2}\right)^2 = 25$

(8) $5^{-2} = \frac{1}{x}$ $\quad \therefore \frac{1}{25} = \frac{1}{x}$ $\quad \therefore x = 25$

(9) $\left(\frac{1}{2}\right)^{-4} = 2^x$ $\quad \therefore 2^x = 2^4$ $\quad \therefore x = 4$

(10) $3^0 = 2x - 5$ $\quad \therefore 1 = 2x - 5$ $\quad \therefore x = 3$

(11) $x + 2 = 5^3 = 125$ $\quad \therefore x = 123$

(12) $x + 5 = 2^3 = 8$ $\quad \therefore x = 3$

(13) $x - 1 = 3^2 = 9$ $\quad \therefore x = 10$

(14) $3x - 1 = 5$ $\quad \therefore 3x = 6$ $\quad \therefore x = 2$

(15) $6^{\frac{1}{2}} = (x+4)^{\frac{1}{2}}$ $\quad \therefore x + 4 = 6$ $\quad \therefore x = 2$

(16) $|x| = 3$ $\quad \therefore x = \pm 3$

(17) $5 = |2x + 1|$

$\therefore 2x + 1 = 5$ or $2x + 1 = -5$

$\therefore x = 2$ or $x = -3$

(18) $2^4 = x(x+6)$ $\quad \therefore x^2 + 6x - 16 = 0$

$\therefore (x-2)(x+8) = 0$ $\quad \therefore x = 2$ or $x = -8$

(19) $2^2 = (x-1)^2$ $\quad \therefore x - 1 = \pm 2$

$\therefore x = 3$ or $x = -1$

(20) $3 = x^2 - 2x$ $\therefore x^2 - 2x - 3 = 0$

$\therefore (x-3)(x+1) = 0$ $\therefore x = 3$ or $x = -1$

(21) $(\log_3 x - 4)(\log_3 x - 5) = 0$

$\therefore \log_3 x = 4$ $\therefore x = 3^4 = 81$

or $\log_3 x = 5$ $\therefore x = 3^5 = 243$

(22) $\log_{10} x - 2 = \pm 2$ $\therefore \log_{10} x - 2 = 2$

$\therefore \log_{10} x = 4$ $\therefore x = 10^4$

or $\log_{10} x - 2 = -2$ $\therefore \log_{10} x = 0$

$\therefore x = 1$

5

(1) $x^2 = 9$ $\therefore x = 3$

(and negative solution is refused)

$\therefore$ The S.S. $= \{3\}$

(2) $x^{\frac{2}{3}} = 9$ $\therefore x = (3^2)^{\frac{3}{2}} = 3^3 = 27$

$\therefore$ The S.S. $= \{27\}$

(3) $x^{\frac{-3}{4}} = \frac{1}{1000}$ $\therefore x = (10^{-3})^{\frac{-4}{3}} = 10000$

$\therefore$ The S.S. $= \{10000\}$

(4) $(-x)^4 = 81 = 3^4$

$\therefore x = -3$ (and the positive solution is refused)

$\therefore$ The S.S. $= \{-3\}$

(5) $(x-1)^2 = 9$ $\therefore x - 1 = \pm 3$

$\therefore x = 4$ or -2 (refused)

$\therefore$ The S.S. $= \{4\}$

(6) $(x-1)^3 = 27 = 3^3$ $\therefore x - 1 = 3$

$\therefore x = 4$ $\therefore$ The S.S. $= \{4\}$

(7) $(x-1)^2 = 7 - x$ $\therefore x^2 - 2x + 1 = 7 - x$

$\therefore x^2 - x - 6 = 0$ $\therefore (x+2)(x-3) = 0$

$\therefore x = -2$ (refused) or $x = 3$

$\therefore$ The S.S. $= \{3\}$

(8) $(x+1)^{\frac{3}{4}} = 8$

$\therefore x + 1 = (2^3)^{\frac{4}{3}} = 2^4 = 16$

$\therefore x = 15$ $\therefore$ The S.S. $= \{15\}$

(9) $x^2 = 5x$ $\therefore x^2 - 5x = 0$

$\therefore x(x-5) = 0$ $\therefore x = 0$ (refused)

or $x = 5$ $\therefore$ The S.S. $= \{5\}$

(10) $x^2 = x + 2$ $\therefore x^2 - x - 2 = 0$

$\therefore (x+1)(x-2) = 0$

$\therefore x = -1$ (refused) or $x = 2$

$\therefore$ The S.S. $= \{2\}$

(11) $x^2 = 2x + 8$ $\therefore x^2 - 2x - 8 = 0$

$\therefore (x-4)(x+2) = 0$

$\therefore x = 4$ or $x = -2$ (refused)

$\therefore$ The S.S. $= \{4\}$

(12) $x = \sqrt{x-2} + 2$

$\therefore \sqrt{x-2} = x - 2$ (by squaring both sides)

$\therefore x - 2 = x^2 - 4x + 4$ $\therefore x^2 - 5x + 6 = 0$

$\therefore (x-2)(x-3) = 0$ $\therefore x = 2$ or $x = 3$

$\therefore$ The S.S. $= \{2, 3\}$

6

(1) $3^x = 81 = 3^4$ $\therefore x = 4$

(2) $3^x = (27)^{-1} = 3^{-3}$ $\therefore x = -3$

(3) $5^{x-1} = 625 = 5^4$ $\therefore x - 1 = 4$ $\therefore x = 5$

(4) $3^{x+2} = 27 = 3^3$ $\therefore x + 2 = 3$ $\therefore x = 1$

(5) $(27)^{x^3} = 1$ $\therefore x^3 = 0$ $\therefore x = 0$

(6) $(\sqrt{8})^x = 64 = (\sqrt{8})^4$ $\therefore x = 4$

(7) $4^x = 8\sqrt{2}$ $\therefore 2^{2x} = 2^{\frac{7}{2}}$

$\therefore 2x = \frac{7}{2}$ $\therefore x = \frac{7}{4}$

(8) $(\sqrt{5})^{x^2} = 625\sqrt{5} = (\sqrt{5})^9$

$\therefore x^2 = 9$ $\therefore x = \pm 3$

7

(1) 1.1761 (2) 4.7549 (3) − 2.1893

8

(1) 1.7159 (2) 25.8226 (3) 0.5012

9

(1) $2x+1>0$ $\quad \therefore x>-\frac{1}{2}$

$\therefore$ The domain $= \left]-\frac{1}{2}, \infty\right[$

(2) $x>0$ $\quad \therefore$ The domain $=]0, \infty[$

(3) $x-2>0$ $\quad \therefore x>2$

$\therefore$ The domain $=]2, \infty[$

(4) The function is defined for every x

satisfying $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$

i.e. The domain $=]0, \infty[-\{1\}$

(5) The function is defined for every x

satisfying $\begin{cases} x>0 \\ x-2>0 \\ x-2\neq 1 \end{cases}$ *i.e.* $\begin{cases} x>0 \\ x>2 \\ x\neq 3 \end{cases}$

i.e. The domain $=]2, \infty[-\{3\}$

(6) The function is defined for every x

satisfying $\begin{cases} x>0 \\ 2-x>0 \\ 2-x\neq 1 \end{cases}$ *i.e.* $\begin{cases} x>0 \\ x<2 \\ x\neq 1 \end{cases}$

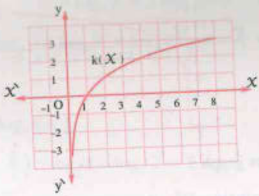
i.e. The domain $=]0, 2[-\{1\}$

10

(1) Taking the power of the number 2 as values of x:

$\{2^{-2}, 2^{-1}, 2^{0}, 2^{1}, 2^{2}\}$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
k(x)	−2	−1	0	1	2



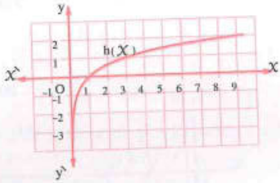
From the graph :

* The range $= \mathbb{R}$
* The function is increasing on its domain.

(2) Taking the power of the number 3 as values of x

$\{3^{-2}, 3^{-1}, 3^{0}, 3^{1}, 3^{2}\}$

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
h(x)	−2	−1	0	1	2



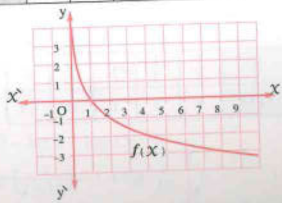
From the graph :

* The range $= \mathbb{R}$
* The function is increasing on its domain.

(3) Taking the power of the number $\frac{1}{2}$ as values of x

$\left(\frac{1}{2}\right)^{-2}, \left(\frac{1}{2}\right)^{-1}, \left(\frac{1}{2}\right)^{0}, \left(\frac{1}{2}\right)^{1}, \left(\frac{1}{2}\right)^{2}$

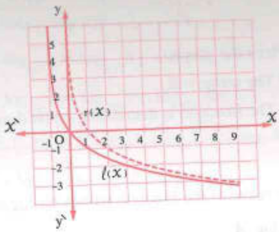
x	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
$f(x)$	−2	−1	0	1	2



From the graph :

* The range $= \mathbb{R}$
* The function is decreasing on its domain.

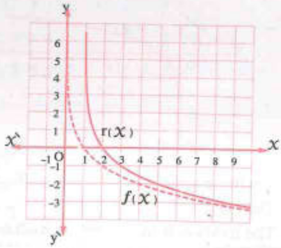
(4) The curve of the function ℓ is the same curve of the function f with horizontal translation 1 unit in the direction of $\overrightarrow{OX}$



The range $= \mathbb{R}$

, the function is decreasing on its domain.

(5) The curve of the function r is the same curve of the function f with horizontal translation 1 unit in the direction $\overrightarrow{OX}$



From the graph :

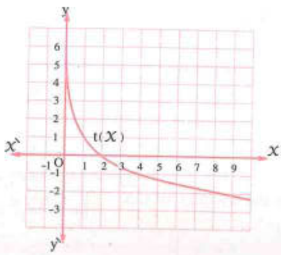
* The range = $\mathbb{R}$

* The function is decreasing on its domain.

(6) Taking the power of the number 2 as values of X

$\{2^{-2}, 2^{-1}, 2^{0}, 2^{1}, 2^{2}\}$

X	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
t (X)	3	2	1	0	−1



From the graph :

* The range = $\mathbb{R}$

* The function is decreasing on its domain.

11

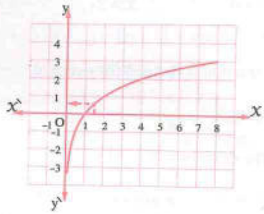
$\because$ The curve of the function passes through (4 , 2)

$\therefore 2 = \log_a 4$

$\therefore a^2 = 4$

$\therefore$ a = 2 (and the negative solution is refused)

X	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
f (X)	−3	−2	−1	0	1	2	3



From the graph :

* The range = $\mathbb{R}$

* The function is increasing on its domain.

* $\log_2 1.5 \simeq 0.6$

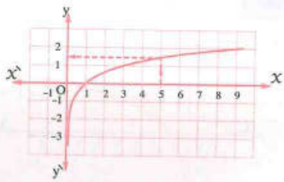
12

$\because$ The curve of the function passes through (81 , 4)

$\therefore 4 = \log_a 81$ $\quad\quad$ $\therefore a^4 = 81$

$\therefore$ a = 3 (and the negative solution is refused)

X	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
f (X)	−2	−1	0	1	2



From the graph :

* The domain = $\mathbb{R}^+$, the range = $\mathbb{R}$

* The function is increasing on its domain

* The intersection point with the X-axis is (1 , 0)

* $\log_3 5 \simeq 1.5$

Exercise 10

First Multiple choice questions

(1) a	(2) c	(3) b	(4) a	(5) a	(6) d
(7) c	(8) d	(9) c	(10) a	(11) b	(12) c
(13) c	(14) d	(15) c	(16) c	(17) b	(18) b
(19) d	(20) b	(21) a	(22) a	(23) b	(24) b
(25) c	(26) b	(27) c	(28) a	(29) a	(30) a
(31) a	(32) d				

Second Essay questions

1

(1) $\log 2 + \log 5 = \log (2 \times 5) = \log 10 = 1$

(2) $\log_3 2 + \log_3 \frac{1}{2} = \log_3 \left(2 \times \frac{1}{2}\right) = \log_3 1 = 0$

Another solution :

$\log_3 2 + \log_3 \frac{1}{2} = \log_3 2 + \log_3 2^{-1}$
$= \log_3 2 - \log_3 2 = 0$

(3) $\log_2 4 + \log_2 16 = 2 + 4 = 6$

Another solution :

$\log_2 4 + \log_2 16 = \log_2 (4 \times 16) = \log_2 64 = 6$

(4) $\log_5 15 - \log_5 3 = \log_5 \frac{15}{3} = \log_5 5 = 1$

(5) $\log_3 5 \times \log_5 3 = \frac{\log 5}{\log 3} \times \frac{\log 3}{\log 5} = 1$

(6) $\log_3 5^3 + \log_3 \frac{243}{125} = \log_3 \frac{5^3 \times 243}{125} = \log_3{}^{243}$
$= \log_3 3^5 = 5 \log_3 3 = 5$

(7) $\log_3 81 \times \log_9 3 = \log_9 3^4 \times \log_9 9^{\frac{1}{2}} = 4 \times \frac{1}{2} = 2$

(8) $\log_6 6 = 1$

(9) $\log_6 \frac{54}{9} = \log_6 6 = 1$

(10) $\log_2 \left(12 \times \frac{2}{3}\right) = \log_2 8 = \log_2 2^3 = 3 \log_2 2 = 3$

(11) $\log \frac{48 \times 125}{6} = \log 1000 = 3$

(12) $\log_2 \frac{15 \times 14}{105} = \log_2 2 = 1$

(13) $\log 54 - 3 \log 3 - \log 2$
$= \log 54 - \log 27 - \log 2$
$= \log \frac{54}{27 \times 2} = \log 1 =$ zero

(14) $\log_2 \frac{3}{25} + \log_2 3125 + \log_2 27 - \log_2 \frac{125}{12}$
$- \log_2 243$
$= \log_2 \frac{3 \times 3125 \times 27 \times 12}{25 \times 125 \times 243} = \log_2 4 = 2$

(15) $\log 0.009 - \log \frac{27}{16} + \log 15 \frac{5}{8} - \log \frac{1}{12}$
$= \log \left(0.009 \times \frac{16}{27} \times 15 \frac{5}{8} \times 12\right) = \log 1 = 0$

(16) $\log_{100} \frac{7}{170} - \log_{100} \frac{35}{18} + \log_{100} \frac{17}{36}$
$= \log_{100} \left(\frac{7}{170} \times \frac{18}{35} \times \frac{17}{36}\right)$
$= \log_{100} \frac{1}{100} = \log_{100} (100)^{-1} = -1$

(17) $\log_2 2^4 + \log_3 3^{\frac{1}{2}} + \log (10)^{-1}$
$= 4 + \frac{1}{2} - 1 = 3 \frac{1}{2}$

(18) $\frac{\log 10 - \log 2}{\log 5^3} = \frac{\log 5}{3 \log 5} = \frac{1}{3}$

(19) $\frac{\log 7^2 + 3 \log 7}{\log 7} = \frac{2 \log 7 + 3 \log 7}{\log 7}$
$= \frac{5 \log 7}{\log 7} = 5$

(20) $1 + \log 3 - \log 2 - \log 15$
$= 1 + \log \left(\frac{3}{2 \times 15}\right) = 1 + \log \frac{1}{10}$
$= 1 + \log (10)^{-1} = 1 - 1 = 0$

(21) $\log 25 + \frac{\log 8 \times \log 16}{\log 64}$
$= \log 25 + \frac{3 \log 2 \times 4 \log 2}{6 \log 2}$
$= \log 25 + 2 \log 2 = \log 25 + \log 4$
$= \log (25 \times 4) = \log 100 = 2$

(22) $\log_{xy} x + \log_{xy} y = \log_{xy} xy = 1$

(23) $\log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc} abc = 1$

(24) $\frac{1}{2} \log_3 a + \frac{1}{2} \log_3 b + 2 \log_3 c - \log_3 \sqrt{ab} - \log_3 3c^2$
$= \log_3 a^{\frac{1}{2}} + \log_3 b^{\frac{1}{2}} + \log_3 c^2 - \log_3 \sqrt{ab}$
$- (\log_3 3 + \log_3 c^2)$
$= \log_3 \sqrt{a} + \log_3 \sqrt{b} + \log_3 c^2 - \log_3 \sqrt{ab}$
$- \log_3 3 - \log_3 c^2$
$= \log_3 \sqrt{ab} - \log_3 \sqrt{ab} - 1 = -1$

(25) $\frac{1}{\log_2 12}+\frac{1}{\log_8 12}+\frac{1}{\log_9 12}$

$=\log_{12} 2+\log_{12} 8+\log_{12} 9$

$=\log_{12}(2\times 8\times 9)$

$=\log_{12} 144=\log_{12}(12)^2=2\log_{12} 12=2$

2

(1) L.H.S. $=\log_4(16\times 64)$

$=\log_4 1024=\log_4 4^5=5\log_4 4=5$

(2) L.H.S. $=\log_3 \frac{243}{9}=\log_3 27=\log_3 3^3=3\log_3 3=3$

(3) L.H.S. $=\log_5 5^3+\log 10+\log_3 3^3=3+1+3=7$

(4) L.H.S. $=\log_2 \frac{4\times 130\times 77}{11\times 7\times 65}=\log_2 8=3$

, R.H.S. $=\log_5 5^3=3$ $\quad\therefore$ L.H.S. = R.H.S.

(5) L.H.S. $=\log 12-\log\frac{16}{4}=\log 12-\log 4$

$=\log\frac{12}{4}=\log 3$

(6) L.H.S. $=(\log 10-\log 5)(\log 100-\log 25)$

$=\left(\log\frac{10}{5}\right)\left(\log\frac{100}{25}\right)=(\log 2)(\log 4)$

$=(\log 2)(2\log 2)=2(\log 2)^2$

(7) L.H.S. $=\frac{\log_2 3^5-\log_3 2^5}{\log_2 3^3-\log_3 2^3}$

$=\frac{5\log_2 3-5\log_3 2}{3\log_2 3-3\log_3 2}=\frac{5(\log_2 3-\log_3 2)}{3(\log_2 3-\log_3 2)}=\frac{5}{3}$

(8) L.H.S. $=\frac{(\log 5)^2-2\log 5}{\log 5-2}$

$=\frac{(\log 5)(\log 5-2)}{\log 5-2}=\log 5$

, R.H.S. $=\log 10-\log 2=\log\frac{10}{2}=\log 5$

$\therefore$ L.H.S. = R.H.S.

3

(1) $x\log 5=\log 13$

$\therefore x=\frac{\log 13}{\log 5}\simeq 1.59$

(2) $(x+1)\log 2=\log 8$

$\therefore x=\frac{\log 8-\log 2}{\log 2}=2$

(3) $(x+2)\log 3=\log 6$

$\therefore x=\frac{\log 6-2\log 3}{\log 3}\simeq -0.37$

(4) $(x-1)\log 5=\log 2$

$\therefore x=\frac{\log 2+\log 5}{\log 5}\simeq 1.43$

(5) $7^{x-2}=\frac{1}{4}$

$\therefore (x-2)\log 7=\log\frac{1}{4}$

$\therefore x=\frac{\log\frac{1}{4}+2\log 7}{\log 7}\simeq 1.29$

(6) $x\log\frac{2}{5}=\log 0.042$

$\therefore x=\frac{\log 0.042}{\log\frac{2}{5}}\simeq 3.46$

(7) $(x+1)\log 7=(x-2)\log 3$

$\therefore x(\log 7-\log 3)=-2\log 3-\log 7$

$\therefore x=\frac{-2\log 3-\log 7}{\log 7-\log 3}\simeq -4.89$

(8) $(2x-3)\log 3=(1-x)\log 11$

$\therefore x(2\log 3+\log 11)=3\log 3+\log 11$

$\therefore x=\frac{3\log 3+\log 11}{2\log 3+\log 11}\simeq 1.24$

(9) $(x-3)\log 2=(x+1)\log 3$

$\therefore x=\frac{\log 3+3\log 2}{\log 2-\log 3}\simeq -7.84$

(10) $x^{\frac{8}{5}}=94.5$

$\therefore \frac{8}{5}\log|x|=\log 94.5$

$\therefore \log|x|=\frac{\log 94.5}{\frac{8}{5}}$

$\therefore |x|=10^{\frac{5\log 94.5}{8}}\simeq 17.17$

$\therefore x=\pm 17.17$

(11) $7^{x-1}(7^2+1)=300$ $\quad\therefore 7^{x-1}=6$

$\therefore (x-1)\log 7=\log 6$

$\therefore x=\frac{\log 6+\log 7}{\log 7}\simeq 1.92$

(12) $5^{2x}-27\times 5^x+50=0$ $\quad\therefore (5^x-25)(5^x-2)=0$

$\therefore 5^x=5^2$ $\quad\therefore x=2$

or $5^x=2$ $\quad\therefore x\log 5=\log 2$

$\therefore x=\frac{\log 2}{\log 5}\simeq 0.43$ $\quad\therefore x=2$ or $x=0.43$

4

(1) $\log_2 14 = \log_2 (2 \times 7) = \log_2 7 + \log_2 2$

$= 2.807 + 1 = 3.807$

(2) $\log_2 56 = \log_2 (7 \times 8) = \log_2 7 + \log_2 8$

$= \log_2 7 + 3 \log_2 2$

$= 2.807 + 3 = 5.807$

(3) $\log_2 \frac{7}{4} = \log_2 7 - \log_2 4 = \log_2 7 - 2 \log_2 2$

$= 2.807 - 2 = 0.807$

5

(1) $\log 6 = \log 2 + \log 3 = 0.301 + 0.4771 = 0.7781$

(2) $\log 9 = \log 3^2 = 2 \log 3 = 2 \times 0.4771 = 0.9542$

(3) $\log 12 = \log 3 + \log 4 = \log 3 + 2 \log 2$

$= 0.4771 + 2 \times 0.301$

$= 1.0791$

6

(1) $\log x = \log (3 \times 10)$ $\quad \therefore x = 30$

$\therefore$ The S.S. $= \{30\}$

(2) $\log_5 \frac{x}{2} = 2$ $\quad \therefore \frac{x}{2} = 25$ $\quad \therefore x = 50$

$\therefore$ The S.S. $= \{50\}$

(3) $\log_3 (x + 6) = \log_3 x^2$ $\quad \therefore x + 6 = x^2$

$\therefore x^2 - x - 6 = 0$ $\quad \therefore (x + 2)(x - 3) = 0$

$\therefore x = -2$ (refused) or $x = 3$

$\therefore$ The S.S. $= \{3\}$

(4) $\log_2 x (x + 2) = 3$

$\therefore x (x + 2) = 2^3 = 8$ $\quad \therefore x^2 + 2x - 8 = 0$

$\therefore (x + 4)(x - 2) = 0$

$\therefore x = -4$ (refused) or $x = 2$

$\therefore$ The S.S. $= \{2\}$

(5) $\log x (x - 3) = 1$

$\therefore x (x - 3) = (10)^1$ $\quad \therefore x^2 - 3x - 10 = 0$

$\therefore (x - 5)(x + 2) = 0$

$\therefore x = 5$ or $x = -2$ (refused)

$\therefore$ The S.S. $= \{5\}$

(6) $\log \frac{x + 3}{3} = \log x$ $\quad \therefore \frac{x + 3}{3} = x$

$\therefore x + 3 = 3x$ $\quad \therefore 2x = 3$

$\therefore x = \frac{3}{2}$ $\quad \therefore$ The S.S. $= \left\{\frac{3}{2}\right\}$

(7) $\log_2 \frac{x - 1}{x - 2} = 2$ $\quad \therefore \frac{x - 1}{x - 2} = 4$

$\therefore x - 1 = 4x - 8$ $\quad \therefore 3x = 7$

$\therefore x = \frac{7}{3}$ $\quad \therefore$ The S.S. $= \left\{\frac{7}{3}\right\}$

(8) $\log_3 x^3 = 3$ $\quad \therefore 3 \log_3 x = 3$

$\therefore \log_3 x = 1$ $\quad \therefore x = 3$

$\therefore$ The S.S. $= \{3\}$

(9) $\log (x + 1)(x - 1) = \log (x + 5)$

$\therefore x^2 - 1 = x + 5$ $\quad \therefore x^2 - x - 6 = 0$

$\therefore (x - 3)(x + 2) = 0$

$\therefore x = 3$ or $x = -2$ (refused)

$\therefore$ The S.S. $= \{3\}$

(10) $\log \frac{x + 8}{x - 1} = 1$ $\quad \therefore \frac{x + 8}{x - 1} = 10$

$\therefore 10x - 10 = x + 8$ $\quad \therefore 9x = 18$

$\therefore x = 2$

$\therefore$ The S.S. $= \{2\}$

(11) $\log_5 2x^2 = \log_5 18$ $\quad \therefore 2x^2 = 18$

$\therefore x^2 = 9$ $\quad \therefore x = \pm 3$

$\therefore$ The S.S. $= \{3, -3\}$

(12) $\log_3 (7x^2 - 4) = \log_3 (x^2 \times 3)$

$\therefore 7x^2 - 4 = 3x^2$ $\quad \therefore 4x^2 = 4$

$\therefore x = 1$ or $x = -1$ (refused)

$\therefore$ The S.S. $= \{1\}$

(13) $\log_{11} \frac{3^4 \times 3}{18 \times x^3 \times 4} = 0$ $\quad \therefore \frac{81 \times 3}{18 \times 4 x^3} = 1$

$\therefore x^3 = \frac{27}{8}$ $\quad \therefore x = \frac{3}{2}$

$\therefore$ The S.S. $= \left\{\frac{3}{2}\right\}$

(14) $\log (x + 2)(x - 2) = \log 10 - \log 2$

$\therefore \log (x^2 - 4) = \log \frac{10}{2} = \log 5$

$\therefore x^2 - 4 = 5$ $\quad \therefore x^2 = 9$

$\therefore x = 3$ or $x = -3$ (refused)

$\therefore$ The S.S. $= \{3\}$

(15) $\frac{\log x}{\log 2} = \frac{\log 9}{\log 4}$ $\therefore \frac{\log x}{\log 2} = \frac{2 \log 3}{2 \log 2}$

$\therefore \log x = \log 3$ $\therefore x = 3$

$\therefore$ The S.S. $= \{3\}$

(16) $\frac{\log x}{\log 3} = \frac{\log 3}{\log x}$ $\therefore (\log x)^2 = (\log 3)^2$

$\therefore \log x = \log 3$ then $x = 3$

or $\log x = -\log 3 = \log 3^{-1}$ then $x = \frac{1}{3}$

$\therefore$ The S.S. $= \left\{3 , \frac{1}{3}\right\}$

(17) $\log x = \frac{(\log 3)^2 - 3 \log 3}{\log 3 - \log 1000} = \frac{\log 3 [\log 3 - 3]}{\log 3 - 3} = \log 3$

$\therefore x = 3$ $\therefore$ The S.S. $= \{3\}$

(18) $(\log x)^2 - \log x^2 - 3 = 0$

$\therefore (\log x)^2 - 2 \log x - 3 = 0$

$\therefore (\log x + 1)(\log x - 3) = 0$

$\therefore \log x = -1$ or $\log x = 3$

$\therefore x = 0.1$ or $x = 1000$

$\therefore$ The S.S. $= \{0.1 , 1000\}$

7

(1) $\log_x 2 + \log_x 3 = 2$ $\therefore \log_x 6 = 2$

$\therefore x^2 = 6$ $\therefore x = \pm\sqrt{6}$

The negative solution is refused

$\therefore x = \sqrt{6}$ $\therefore$ The S.S. $= \{\sqrt{6}\}$

(2) By multipling by $(\log x)$

$\therefore (\log x)^2 - 2(\log x) - 3 = 0$

$\therefore (\log x + 1)(\log x - 3) = 0$

$\therefore \log x = -1$ $\therefore x = 0.1$

or $\log x = 3$ $\therefore x = 1000$

$\therefore$ The S.S. $= \{0.1 , 1000\}$

(3) Multiplying by $2 \log x$

$\therefore 2(\log x)^2 - 3 \log x - 2 = 0$

$\therefore (2 \log x + 1)(\log x - 2) = 0$ $\therefore \log x = \frac{-1}{2}$

$\therefore x = \frac{1}{\sqrt{10}}$ or $\log x = 2$ $\therefore x = 100$

$\therefore$ The S.S. $= \left\{\frac{1}{\sqrt{10}} , 100\right\}$

(4) $\log 7 \times \log 3^6 = \log 7^2 \times \log x^3$

$\therefore 6 \log 7 \log 3 = 6 \log 7 \log x$

$\therefore \log 3 = \log x$ $\therefore x = 3$

$\therefore$ The S.S. $= \{3\}$

(5) Let $\log_2 x = k$ $\therefore \log_x 2 = \frac{1}{k}$

$\therefore k + \frac{1}{k} = 2$ $\therefore k^2 - 2k + 1 = 0$

$\therefore (k-1)^2 = 0$ $\therefore k = 1$

$\therefore \log_2 x = 1$ $\therefore x = 2$

$\therefore$ The S.S. $= \{2\}$

(6) $(\log x)^3 = 9 \log x$

$\therefore (\log x)^3 - 9 \log x = 0$

$\therefore \log x \left((\log x)^2 - 9\right) = 0$

$\therefore \log x (\log x - 3)(\log x + 3) = 0$

$\therefore \log x = 0$ $\therefore x = 1$

or $\log x = 3$ $\therefore x = 1000$

or $\log x = -3$ $\therefore x = 0.001$

$\therefore$ The S.S. $= \{1 , 1000 , 0.001\}$

(7) $\log_2 \frac{x^2 + 6x + 9}{x - 1} = \log_5 5^4 = 4$

$\therefore \frac{x^2 + 6x + 9}{x - 1} = 2^4 = 16$

$\therefore x^2 - 10x + 25 = 0$ $\therefore (x-5)^2 = 0$

$\therefore x = 5$ $\therefore$ The S.S. $= \{5\}$

(8) $\frac{\log x}{\log 2} + \frac{\log x}{\log 4} = \frac{-3}{2}$ $\therefore \frac{\log x}{\log 2} + \frac{\log x}{2 \log 2} = \frac{-3}{2}$

$\therefore \frac{3 \log x}{2 \log 2} = \frac{-3}{2}$ $\therefore \frac{\log x}{\log 2} = -1$

$\therefore \log x = \log 2^{-1}$ $\therefore x = \frac{1}{2}$

$\therefore$ The S.S. $= \left\{\frac{1}{2}\right\}$

8

(1) The perimeter $= 2(\log_6 8 + \log_6 27)$

$= 2\left(\log_6 (8 \times 27)\right) = 2 \log_6 216$

$= 2 \log_6 6^3 = 6$ cm.

(2) The perimeter $= \log_{\sqrt{10}} 5 + \log_{\sqrt{10}} 5 + \log_{\sqrt{10}} 4$

$= \log_{\sqrt{10}} (5 \times 5 \times 4) = \log_{\sqrt{10}} 100$

$= 4 \log_{\sqrt{10}} \sqrt{10} = 4$ cm.

9

L.H.S. $= \frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log d} \times \frac{\log d}{\log a}$

$= 1 =$ R.H.S.

, expression $= \frac{\log 3}{\log 2} \times \frac{\log 5}{\log 3} \times \frac{\log 16}{\log 5} = \frac{4 \log 2}{\log 2} = 4$

Third Higher skills

(1) (d) (2) (d) (3) (d) (4) (c)

Instructions to solve :

(1) $\because \log 1 + \log 2 + \log 3 =$ zero $+ \log (2 \times 3)$

$= \log 6$

, $\because \log (1 + 2 + 3) = \log 6$

$\therefore \log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

(2) $\log x = \frac{\log 36 \times \log 5}{\log 6} = \frac{2 \log 6 \times \log 5}{\log 6} = \log 25$

$\therefore x = 25$

, $\log y = \frac{\log 64 \times \log 6}{\log 36} = \frac{\log 64 \times \log 6}{2 \log 6} = \log 8$

$\therefore y = 8$ $\quad \therefore x + y = 25 + 8 = 33$

(3) The expression

$= \frac{1}{\log_b b + \log_b a + \log_b c} + \frac{1}{\log_c c + \log_c a + \log_c b}$

$+ \frac{1}{\log_a a + \log_a b + \log_a c}$

$= \frac{1}{\log_b abc} + \frac{1}{\log_c abc} + \frac{1}{\log_a abc}$

$= \log_{abc} b + \log_{abc} c + \log_{abc} a$

$= \log_{abc} abc = 1$

(4) $\because \frac{1}{\log_2 x} + \frac{1}{\log_4 x} + \frac{1}{\log_8 x} + \frac{1}{\log_{16} x} = 5$

$\therefore \log_x 2 + \log_x 4 + \log_x 8 + \log_x 16 = 5$

$= \log_x (2 \times 4 \times 8 \times 16) = 5$

$\therefore \log_x 1024 = 5$

$\therefore x^5 = 1024 = (4)^5$ $\quad \therefore x = 4$

Answers of Life Applications on Unit Two

1

a = L.E. 10000 , n = 3 years , c = L.E. 12597

$\therefore r = \left(\frac{12597}{10000}\right)^{\frac{1}{3}} - 1 \simeq 0.08$

$\therefore$ The percentage of the profit $= 0.08 \times 100\% = 8\%$

2

$\because n = 5$ months $\quad \therefore Z = 75 (4.22)^{\frac{5}{6}} \simeq 249$

$\therefore$ The number of rabbits expected over 5 months

= 249 rabbits

3

$V = 1331 \text{ cm}^3$. $\quad \therefore \ell = \sqrt[3]{1331} = 11$ cm.

4

The increase in the length of the radius

$= \left(\frac{3 \times 36 \times \pi}{4 \pi}\right)^{\frac{1}{3}} - \left(\frac{3 \times \frac{32}{3} \times \pi}{4 \pi}\right)^{\frac{1}{3}}$

$= (27)^{\frac{1}{3}} - (8)^{\frac{1}{3}} = 1$ length unit.

5

(1) After 4 weeks

The number of the organisms (y) $= 8192 \left(\frac{1}{2}\right)^{4-1}$

$= 8192 \left(\frac{1}{2}\right)^3$

= 1024 organisms

(2) $\because$ The number of organisms = 256 organism

$\therefore 8192 \left(\frac{1}{2}\right)^{n-1} = 256$

$\therefore \left(\frac{1}{2}\right)^{n-1} = \frac{256}{8192}$ $\quad \therefore \left(\frac{1}{2}\right)^{n-1} = \frac{1}{32}$

$\therefore \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^5$ $\quad \therefore n - 1 = 5$

$\therefore n = 6$

$\therefore$ The number of organisms will be 256 after 6 weeks.

6

The function of reproduction $f : y = 20000 (2)^n$

(1) number of cells after 5 hours (y) $= 20000 (2)^5$

= 640000 cells

(2) $\because$ The number of cells is 2560000 cells

$\therefore 20000\ (2)^n = 2560000$ $\therefore 2^n = 128$

$\therefore 2^n = 2^7$ $\therefore n = 7$

$\therefore$ The number of cells will be 2 million and 560 thousands of cells after 7 hours.

7

(1) $f\ (0) = 70 - 4 \log_2 1 = 70$ marks.

(2) $f\ (7) = 70 - 4 \log_2 8 = 58$ marks.

8

(1) $f\ (3600) = \frac{10}{100} \times 3600 = 360$ pounds.

(2) $f\ (8000) = \frac{10}{100} \times 8000 + 100 \log (8000 - 4999)$

$= 1147.7266$ pounds

9

(1) a is the initial number of the population

, n is the number of years

$\therefore$ The number of the population after n years

$= a \left(1 + \frac{7}{100}\right)^n = a\ (1.07)^n$

$\therefore$ The number of the population after 1 year $= a\ (1.07)$

(2) when the population is doubled

$\therefore 2\ a = a\ (1.07)^n$ $\therefore 2 = (1.07)^n$

taking the logarithms to the both sides.

$\therefore n \log 1.07 = \log 2$

$\therefore n \simeq 10$ years.

10

$N = 10^5\ (1.3)^{t - 2010}$

(1) In 2015 :

The number of the population $N = 10^5\ (1.3)^{2015 - 2010}$

$= 371293$ people.

(2) When the number of the population 1.4 million people

$\therefore 1.4 \times 10^6 = 10^5\ (1.3)^{t - 2010}$

$\therefore (1.3)^{t - 2010} = 14$

taking the logarithms to the both sides

$\therefore (t - 2010) \log 1.3 = \log 14$

$\therefore t - 2010 \simeq 10$ $\therefore t \simeq 2020$

11

$k = k_o\ (0.9)^n$ $\therefore \frac{40}{100} k_o = k_o\ (0 - 9)^n$

$\therefore 0.4 = (0.9)^n$

taking the logarithms to the both sides.

$\therefore n \log 0.9 = \log 0.4$ $\therefore n = \frac{\log 0.4}{\log 0.9} \simeq 9$ years.

SECOND

Answers of Calculus and Trigonometry



UNIT 3

Answers of "Unit Three"

Exercise 11

First Multiple choice questions

(1) c (2) a (3) d (4) c

(5) First : a Second : c Third : d Fourth : d

(6) First : d Second : d Third : b Fourth : a

(7) First : d Second : c Third : d Fourth : b

Fifth : d

Second Essay questions

1

x	1.9	1.99	1.999	⟶	2	⟵	2.001	2.01	2.1
$f(x)$	13.5	13.95	13.995	⟶	14	⟵	14.005	14.05	14.5

$\therefore \lim_{x \to 2} f(x) = 14$

2

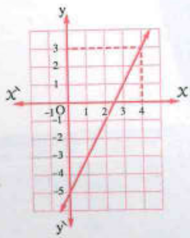
x	1.9	1.99	1.999	⟶	2	⟵	2.001	2.01	2.1
$f(x)$	0.256	0.251	0.25	⟶	0.25	⟵	0.2499	0.249	0.244

$\therefore \lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{1}{4}$

3

(1) Graphically :

$\therefore \lim_{x \to 4} (2x-5) = 3$



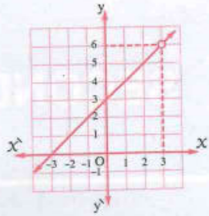
Numerically :

x	3.9	3.99	3.999	⟶	4	⟵	4.001	4.01	4.1
$f(x)$	2.8	2.98	2.998	⟶	3	⟵	3.002	3.02	3.2

$\therefore \lim_{x \to 4} (2x-5) = 3$

(2) Graphically :

$\therefore \lim_{x \to 3} \frac{x^2-9}{x-3} = 6$



Numerically :

x	2.9	2.99	2.999	⟶	3	⟵	3.001	3.01	3.1
$f(x)$	5.9	5.99	5.999	⟶	6	⟵	6.001	6.01	6.1

$\therefore \lim_{x \to 3} \frac{x^2-9}{x-3} = 6$

4

(1) zero (2) does not exist (3) −1

(4) 2 (5) 3 (6) 1

5

(1) zero (2) 3

(3) 2 (4) 2

6

(1) 1 (2) does not exist

7

(1) 2 (2) 1

(3) 1.5 (4) 1.5

8

(1) 1 (2) does not exist

9

(1) undefined (2) ∞

10

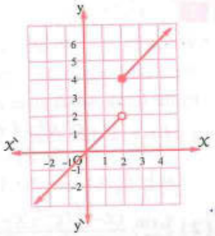
(1) undefined (2) does not exist

11

(1) 1 (2) does not exist

(3) undefined (4) 3

12 From the graph , we find that : $\lim\limits_{x \to 2} f(x)$ does not exist.



Third Higher skills

(1) (c) (2) (d) (3) (a) (4) (d)

Instructions to solve :

(1) Notice that each of the figures (a) , (b) and (d) contains a jump at $x = 3$ therefore the limit of the function at $x = 3$ is not exist
But at figure (c) there is an open dot at $x = 3$ therefore the limit of the function at $x = 3$ is exist.

(2) At $\theta \longrightarrow \frac{\pi}{2}$ $\therefore y \longrightarrow \sqrt{(10)^2 + (10)^2}$
i.e. $y \longrightarrow 10\sqrt{2}$

(3) $\because$ The curve intersects the x-axis at $x = 3$
$\therefore$ The curve passes through the point (3 , 0)
, $\because$ the function is polynomial
$\therefore \lim\limits_{x \to 3} f(x) = \text{zero}$

(4) $\because$ The curve intersects the y-axis at y = 3
$\therefore$ The curve passes through the point (0 , 3)
, $\because$ the function is polynomial
$\therefore \lim\limits_{x \to 0} f(x) = 3$

Exercise 12

First Multiple choice questions

(1) c	(2) b	(3) b	(4) c	(5) d	(6) b
(7) b	(8) b	(9) b	(10) b	(11) a	(12) d
(13) b	(14) d	(15) b	(16) c	(17) b	(18) c
(19) b	(20) d	(21) c	(22) a	(23) a	(24) c
(25) c	(26) c	(27) d	(28) d	(29) a	(30) a
(31) b					

Second Essay questions

1

(1) $\lim\limits_{x \to 5} \frac{(x-5)(x+5)}{x-5} = \lim\limits_{x \to 5} (x+5) = 10$

(2) $\lim\limits_{x \to 3} \frac{(x-3)(x-5)}{x-3} = \lim\limits_{x \to 3} (x-5) = -2$

(3) $\lim\limits_{x \to 4} \frac{(x-3)(x-4)}{x-4} = \lim\limits_{x \to 4} (x-3) = 1$

(4) $\lim\limits_{x \to 0} \frac{x^2}{x^2(3x-2)} = \lim\limits_{x \to 0} \frac{1}{3x-2} = -\frac{1}{2}$

(5) $\lim\limits_{x \to 2} \frac{5(x-2)}{4(x-2)} = \frac{5}{4}$

(6) $\lim\limits_{x \to 4} \frac{4(x-4)(x+4)}{x-4} = \lim\limits_{x \to 4} 4(x+4) = 32$

(7) $\lim\limits_{x \to 4} \frac{2(x-4)}{(x-4)(x+3)} = \lim\limits_{x \to 4} \frac{2}{x+3} = \frac{2}{7}$

(8) $\lim\limits_{x \to -1} \frac{(x-1)(x+1)}{x(x+1)} = \lim\limits_{x \to -1} \frac{x-1}{x} = 2$

(9) $\lim\limits_{x \to 5} \frac{x(x+5)(x-5)}{x-5} = \lim\limits_{x \to 5} x(x+5) = 50$

(10) $\lim\limits_{x \to -3} \frac{(x+3)(x+1)}{(x-3)(x+3)} = \lim\limits_{x \to -3} \frac{x+1}{x-3} = \frac{-2}{-6} = \frac{1}{3}$

(11) $\lim\limits_{x \to 2} \frac{x(x-2)}{(x-2)(x+1)} = \lim\limits_{x \to 2} \frac{x}{x+1} = \frac{2}{3}$

(12) $\lim\limits_{x \to -1} \frac{(x-4)(x+1)}{(x-2)(x+1)} = \lim\limits_{x \to -1} \frac{x-4}{x-2} = \frac{5}{3}$

(13) $\lim\limits_{x \to \frac{1}{2}} \frac{(2x-1)(x-2)}{2x-1} = \lim\limits_{x \to \frac{1}{2}} (x-2) = -\frac{3}{2}$

(14) $\lim\limits_{x \to \frac{3}{2}} \frac{(2x-3)(x+1)}{(2x-3)(2x+3)} = \lim\limits_{x \to \frac{3}{2}} \frac{x+1}{2x+3} = \frac{5}{12}$

(15) $\lim\limits_{x \to -3} \frac{(2x-1)(x+3)}{(x-2)(x+3)} = \lim\limits_{x \to -3} \frac{2x-1}{x-2} = \frac{7}{5}$

(16) $\lim\limits_{x \to 9} \frac{-(x-9)}{(x-9)(x+9)} = \lim\limits_{x \to 9} \frac{-1}{x+9} = -\frac{1}{18}$

2

(1) $\lim\limits_{x \to 0} \frac{(x+2-2)(x+2+2)}{x(x+1)} = \lim\limits_{x \to 0} \frac{x+4}{x+1} = 4$

(2) $\lim\limits_{x \to 0} \frac{4x^2-4x}{5x} = \lim\limits_{x \to 0} \frac{4x(x-1)}{5x}$
$= \lim\limits_{x \to 0} \frac{4(x-1)}{5} = \frac{-4}{5}$

(3) $\lim\limits_{x \to 2} \frac{(x-3-1)(x-3+1)}{(x-2)(2x+1)}$
$= \lim\limits_{x \to 2} \frac{x-4}{2x+1} = \frac{-2}{5}$

(4) $\lim\limits_{x \to -2} \frac{(x+5-3)(x+5+3)}{(x+2)(x-2)}$
$= \lim\limits_{x \to -2} \frac{x+8}{x-2} = \frac{-3}{2}$

(5) $\lim\limits_{x \to 2} \frac{(x^2-4)(x^2+5)}{x-2}$

$= \lim\limits_{x \to 2} \frac{(x-2)(x+2)(x^2+5)}{x-2}$

$= \lim\limits_{x \to 2} (x+2)(x^2+5) = 36$

(6) $\lim\limits_{x \to 2} \frac{(x-2)^2(x+2)^2}{x-2}$

$= \lim\limits_{x \to 2} (x-2)(x+2)^2 = \text{zero}$

(7) $\lim\limits_{x \to -2} \frac{x+2}{(x+2)(x-2)(x^2+4)}$

$= \lim\limits_{x \to -2} \frac{1}{(x-2)(x^2+4)} = -\frac{1}{32}$

(8) $\lim\limits_{x \to 1} \frac{x^{\frac{1}{2}}(x^3-1)}{x(x-1)}$

$= \lim\limits_{x \to 1} \frac{x^{\frac{1}{2}}(x-1)(x^2+x+1)}{x(x-1)}$

$= \lim\limits_{x \to 1} \frac{x^2+x+1}{x^{\frac{1}{2}}} = 3$

(9) $\lim\limits_{x \to 1} \frac{x^2(x-1)+2(x-1)}{x-1}$

$= \lim\limits_{x \to 1} \frac{(x-1)(x^2+2)}{x-1} = \lim\limits_{x \to 1} (x^2+2) = 3$

(10) $\lim\limits_{x \to -1} \frac{2x^3-2x-x^2+1}{x^3+1}$

$= \lim\limits_{x \to -1} \frac{2x(x^2-1)-(x^2-1)}{x^3+1}$

$= \lim\limits_{x \to -1} \frac{(x^2-1)(2x-1)}{x^3+1}$

$= \lim\limits_{x \to -1} \frac{(x-1)(x+1)(2x-1)}{(x+1)(x^2-x+1)}$

$= \lim\limits_{x \to -1} \frac{(x-1)(2x-1)}{x^2-x+1} = 2$

(11) $\lim\limits_{x \to 3} \frac{5}{x} + \lim\limits_{x \to 3} \frac{x(x-3)}{x-3} = \frac{5}{3} + 3 = \frac{14}{3}$

(12) $\lim\limits_{x \to -1} \frac{x^2-3x-4}{x^2-1} = \lim\limits_{x \to -1} \frac{(x+1)(x-4)}{(x+1)(x-1)}$

$= \lim\limits_{x \to -1} \frac{x-4}{x-1} = \frac{5}{2}$

(13) $\lim\limits_{x \to 1} \left(\frac{1}{x-1} - \frac{3}{(x-1)(x^2+x+1)}\right)$

$= \lim\limits_{x \to 1} \left(\frac{x^2+x+1-3}{(x-1)(x^2+x+1)}\right)$

$= \lim\limits_{x \to 1} \left(\frac{(x-1)(x+2)}{(x-1)(x^2+x+1)}\right)$

$= \lim\limits_{x \to 1} \frac{x+2}{x^2+x+1} = 1$

3

- We use the long division in each of the following. We divide each of the numerator and denominator by the factor which gives zero , we get :

(1) $\lim\limits_{x \to 4} \frac{(x-4)(x^2+4x+1)}{x-4}$

$= \lim\limits_{x \to 4} (x^2+4x+1) = 33$

(2) $\lim\limits_{x \to 1} \frac{(x-1)(x^2+x-1)}{(x-1)(x+2)}$

$= \lim\limits_{x \to 1} \frac{x^2+x-1}{x+2} = \frac{1}{3}$

(3) $\lim\limits_{x \to 4} \frac{x(x-4)(x^2+4x-5)}{x-4}$

$= \lim\limits_{x \to 4} x(x^2+4x-5) = 108$

(4) $\lim\limits_{x \to 2} \frac{(x-2)(x^2+x-3)}{x-2} = \lim\limits_{x \to 2} (x^2+x-3) = 3$

(5) $\lim\limits_{x \to -3} \frac{(x+3)(x^2-3x-1)}{(x+3)(x-1)}$

$= \lim\limits_{x \to -3} \frac{x^2-3x-1}{x-1} = -\frac{17}{4}$

(6) $\lim\limits_{x \to -2} \frac{(x+2)(2x^2-x+2)}{(x+2)(x^2-2x+4)}$

$= \lim\limits_{x \to -2} \frac{2x^2-x+2}{x^2-2x+4} = \frac{12}{12} = 1$

(7) $\lim\limits_{x \to -2} \frac{(x+2)^2}{(x-3)(x+2)^2} = \lim\limits_{x \to -2} \frac{1}{x-3} = -\frac{1}{5}$

4

(1) $\lim\limits_{x \to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)}$

$= \lim\limits_{x \to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim\limits_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$

(2) $\lim\limits_{x \to 5} \frac{(\sqrt{x-1}-2)(\sqrt{x-1}+2)}{(x-5)(\sqrt{x-1}+2)}$

$= \lim\limits_{x \to 5} \frac{x-1-4}{(x-5)(\sqrt{x-1}+2)}$

$= \lim\limits_{x \to 5} \frac{1}{\sqrt{x-1}+2} = \frac{1}{4}$

(3) $\lim\limits_{x \to -1} \frac{x+1}{\sqrt{x+5}-2} \times \frac{\sqrt{x+5}+2}{\sqrt{x+5}+2}$

$= \lim\limits_{x \to -1} \frac{(x+1)(\sqrt{x+5}+2)}{x+5-4}$

$= \lim\limits_{x \to -1} (\sqrt{x+5}+2) = 4$

(4) $\lim_{x \to 6} \frac{(x-6)(\sqrt{x-2}+2)}{(\sqrt{x-2}-2)(\sqrt{x-2}+2)}$

$= \lim_{x \to 6} \frac{(x-6)(\sqrt{x-2}+2)}{(x-2-4)}$

$= \lim_{x \to 6} (\sqrt{x-2}+2) = 4$

(5) $\lim_{x \to 1} \frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(\sqrt{x+3}+2)}$

$= \lim_{x \to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)}$

$= \lim_{x \to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}$

(6) $\lim_{x \to 3} \frac{(\sqrt{4x-3}-3)(\sqrt{4x-3}+3)}{(x-3)(\sqrt{4x-3}+3)}$

$= \lim_{x \to 3} \frac{4x-3-9}{(x-3)(\sqrt{4x-3}+3)}$

$= \lim_{x \to 3} \frac{4}{\sqrt{4x-3}+3} = \frac{2}{3}$

(7) $\lim_{x \to 0} \frac{(\sqrt{2x+9}-3)(\sqrt{2x+9}+3)}{x(x+1)(\sqrt{2x+9}+3)}$

$= \lim_{x \to 0} \frac{2x+9-9}{x(x+1)(\sqrt{2x+9}+3)}$

$= \lim_{x \to 0} \frac{2}{(x+1)(\sqrt{2x+9}+3)} = \frac{1}{3}$

(8) $\lim_{x \to 5} \frac{x(x-5)}{(\sqrt{x+4}-3)} \times \frac{(\sqrt{x+4}+3)}{(\sqrt{x+4}+3)}$

$= \lim_{x \to 5} \frac{x(x-5)(\sqrt{x+4}+3)}{x+4-9}$

$= \lim_{x \to 5} x(\sqrt{x+4}+3) = 30$

(9) $\lim_{x \to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(x+3)(\sqrt{x}+1)}$

$= \lim_{x \to 1} \frac{x-1}{(x-1)(x+3)(\sqrt{x}+1)}$

$= \lim_{x \to 1} \frac{1}{(x+3)(\sqrt{x}+1)} = \frac{1}{8}$

(10) $\lim_{x \to 3} \frac{(x-3)(x+2)(\sqrt{5x-6}+3)}{(\sqrt{5x-6}-3)(\sqrt{5x-6}+3)}$

$= \lim_{x \to 3} \frac{(x-3)(x+2)(\sqrt{5x-6}+3)}{5x-6-9}$

$= \lim_{x \to 3} \frac{(x-3)(x+2)(\sqrt{5x-6}+3)}{5(x-3)}$

$= \lim_{x \to 3} \frac{(x+2)(\sqrt{5x-6}+3)}{5} = 6$

(11) $\lim_{x \to 0} \frac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{2x(\sqrt{1+x}+\sqrt{1-x})}$

$= \lim_{x \to 0} \frac{(1+x)-(1-x)}{2x(\sqrt{1+x}+\sqrt{1-x})}$

$= \lim_{x \to 0} \frac{1}{\sqrt{1+x}+\sqrt{1-x}} = \frac{1}{2}$

(12) $\lim_{x \to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)(\sqrt{x-2}+1)}{(\sqrt{x-2}-1)(\sqrt{x+1}+2)(\sqrt{x-2}+1)}$

$= \lim_{x \to 3} \frac{(x+1-4)(\sqrt{x-2}+1)}{(x-2-1)(\sqrt{x+1}+2)}$

$= \lim_{x \to 3} \frac{\sqrt{x-2}+1}{\sqrt{x+1}+2} = \frac{2}{4} = \frac{1}{2}$

5

$\lim_{x \to 2} \frac{f(x)-5}{x-2} = 1$ (has an existance)

$\therefore f(2)-5=0$ $\quad\quad$ $\therefore f(2)=5$

$\therefore \lim_{x \to 2} f(x) = f(2) = 5$

6

$\lim_{x \to -1} \frac{(x+1)(x-a)}{x+1} = 4$ $\quad\quad$ $\therefore \lim_{x \to -1} (x-a) = 4$

$\therefore -1-a=4$ $\quad\quad$ $\therefore a=-5$

Third Higher skills

(1) (a) (2) (d) (3) (d)

Instructions to solve :

(1) $\because x(f(x)+1) = f(x)+x^2$

$\therefore x f(x) + x = f(x) + x^2$

$\therefore x f(x) - f(x) = x^2 - x$

$\therefore (x-1) f(x) = x(x-1)$

$\therefore f(x) = \frac{x(x-1)}{x-1} = x$ where $x \neq 1$

$\therefore \lim_{x \to 1} f(x) = 1$

(2) $\because \lim_{x \to 1} \frac{x^2 + ax + b}{x - 1}$ is exist and equals 5

, $\because$ the denominator = zero at $x = 1$

$\therefore$ The numerator = zero at $x = 1$

$\therefore 1 + a + b = 0 \qquad \therefore b = -a - 1$

$\therefore \lim_{x \to 1} \frac{x^2 + ax - a - 1}{x - 1} = 5$

$\therefore \lim_{x \to 1} \frac{(x^2 - 1) + (ax - a)}{x - 1} = 5$

$\therefore \lim_{x \to 1} \frac{(x-1)(x+1) + a(x-1)}{(x-1)} = 5$

$\therefore \lim_{x \to 1} \frac{(x-1)(x+1+a)}{(x-1)} = 5$

$\therefore 2 + a = 5 \qquad \therefore a = 3$ and hence $b = -4$

$\therefore a - b = 7$

(3) $\because 2 \lim_{x \to m} f(x) - 5 \lim_{x \to m} g(x) = 10 \qquad (1)$

, $\lim_{x \to m} f(x) + \lim_{x \to m} g(x) = 6 \qquad (2)$

, by multiplying (2) by 5

$\therefore 5 \lim_{x \to m} f(x) + 5 \lim_{x \to m} g(x) = 30 \qquad (3)$

, solving (1) and (3) : $\therefore 7 \lim_{x \to m} f(x) = 40$

$\therefore \lim_{x \to m} f(x) = \frac{40}{7}$

, substituting in (2) : $\lim_{x \to m} g(x) = 6 - \frac{40}{7} = \frac{2}{7}$

$\therefore \lim_{x \to m} \frac{f(x)}{g(x)} = \frac{\lim_{x \to m} f(x)}{\lim_{x \to m} g(x)} = \frac{\frac{40}{7}}{\frac{2}{7}} = 20$

Exercise 13

First Multiple choice questions

(1) d (2) d (3) d (4) a (5) a (6) d
(7) a (8) c (9) a (10) b (11) c (12) b
(13) d (14) c (15) b (16) c (17) d (18) d
(19) d (20) b (21) a (22) b (23) a (24) b
(25) c (26) d

Second Essay questions

1

(1) $\lim_{x \to 2} \frac{x^3 - 2^3}{x - 2} = 3(2)^2 = 12$

(2) $\lim_{x \to -5} \frac{x^4 - (-5)^4}{x - (-5)} = \frac{4}{1}(-5)^{4-1} = -500$

(3) $\lim_{x \to a} \frac{x^5 - a^5}{x - a} = 5a^4$

(4) $\lim_{x \to 2} \frac{x^5 - 2^5}{x^2 - 2^2} = \frac{5}{2}(2)^{5-2} = 20$

(5) $\lim_{x \to 2} \frac{x^7 - 2^7}{x^3 - 2^3} = \frac{7}{3}(2)^{7-3} = \frac{112}{3}$

(6) $\lim_{x \to \frac{1}{2}} \frac{x^3 - \left(\frac{1}{2}\right)^3}{x^2 - \left(\frac{1}{2}\right)^2} = \frac{3}{2}\left(\frac{1}{2}\right)^{3-2} = \frac{3}{4}$

(7) $\lim_{x \to 2} \frac{x^6 - (2)^6}{x^7 - (2)^7} = \frac{6}{7}(2)^{6-7} = \frac{3}{7}$

(8) $\lim_{x \to -3} \frac{x^5 - (-3)^5}{x - (-3)} = 5(-3)^4 = 405$

(9) $\lim_{x \to -3} \frac{x^4 - (-3)^4}{x - (-3)} = 4(-3)^3 = -108$

(10) $\lim_{x \to -3} \frac{x^4 - (-3)^4}{x^5 - (-3)^5} = \frac{4}{5}(-3)^{4-5} = -\frac{4}{15}$

(11) $\lim_{x \to -2} \frac{x^5 - (-2)^5}{x^3 - (-2)^3} = \frac{5}{3}(-2)^{5-3} = \frac{20}{3}$

(12) $\lim_{x \to 4} \frac{2(x^3 - 64)}{x^2 - 16} = 2 \lim_{x \to 4} \frac{x^3 - 4^3}{x^2 - 4^2}$

$= 2\left(\frac{3}{2}\right) \times 4 = 12$

(13) $\lim_{x \to -2} \frac{x^6 - 64}{3(x+2)} = \frac{1}{3} \lim_{x \to -2} \frac{x^6 - (-2)^6}{x - (-2)}$

$= \frac{1}{3} \times 6 \times (-2)^5 = -64$

(14) $\lim_{x \to -1} \frac{x(x^9 + 1)}{x(x^6 - 1)} = \lim_{x \to -1} \frac{x^9 - (-1)^9}{x^6 - (-1)^6}$

$= \frac{9}{6}(-1)^3 = \frac{-9}{6} = \frac{-3}{2}$

(15) $-1 \times \lim_{x \to 1} \frac{x^9 - 1}{x^7 - 1} = (-1) \times \frac{9}{7}(1)^{9-7} = \frac{-9}{7}$

(16) $\lim_{2x \to 1} \frac{(2x)^7 - (1)^7}{(2x)^5 - (1)^5} = \frac{7}{5}(1)^{7-5} = \frac{7}{5}$

(17) $\lim_{2x \to -1} \frac{(2x)^5 - (-1)^5}{(2x)^6 - (-1)^6} = \frac{5}{6}(-1)^{-1} = -\frac{5}{6}$

(18) $\lim_{3x \to -2} \frac{(3x)^5 - (-2)^5}{(3x)^3 - (-2)^3} = \frac{5}{3}(-2)^{5-3} = \frac{20}{3}$

2

(1) $\lim_{x \to 2} \frac{x^{-7} - (2)^{-7}}{x - 2} = -7(2)^{-8} = \frac{-7}{256}$

(2) $\lim\limits_{x\to -1} \frac{x^{-4}-(-1)^{-4}}{x^{-18}-(-1)^{-18}} = \frac{-4}{-18} = \frac{2}{9}$

(3) $\lim\limits_{x\to 2} \frac{x^{-5}-(2)^{-5}}{x^{-7}-(2)^{-7}} = \frac{-5}{-7}(2)^2 = \frac{20}{7}$

(4) $\lim\limits_{x\to 4} \frac{x^{-2}-4^{-2}}{x-4} = -2(4)^{-3} = -\frac{1}{32}$

(5) $\lim\limits_{x\to 2} \frac{x^{-8}-2^{-8}}{x-2} = -8\times 2^{-9} = \frac{-1}{64}$

(6) $\lim\limits_{x\to 1} \frac{x^{\frac{1}{7}}-1^{\frac{1}{7}}}{x-1} = \frac{1}{7}$

(7) $\lim\limits_{x\to 2} \frac{x^{\frac{1}{3}}-2^{\frac{1}{3}}}{x-2} = \frac{1}{3}(2)^{\frac{-2}{3}} = \frac{1}{3\sqrt[3]{4}}$

(8) $\lim\limits_{x\to 1} \frac{x^{\frac{1}{2}}(x^{10}-1)}{x^{\frac{2}{3}}(x^4-1)}$

$= \lim\limits_{x\to 1} x^{-\frac{1}{6}} \times \lim\limits_{x\to 1} \frac{x^{10}-(1)^{10}}{x^4-(1)^4} = \frac{5}{2}$

(9) $\lim\limits_{x\to 1} \frac{x^{17}-(1)^{17}}{(3x+5)(x-1)}$

$= \lim\limits_{x\to 1} \frac{1}{3x+5} \times \lim\limits_{x\to 1} \frac{x^{17}-(1)^{17}}{x-1}$

$= \frac{1}{8}\times 17(1)^{16} = \frac{17}{8}$

3

(1) $\lim\limits_{(1+x)\to 1} \frac{(1+x)^{10}-(1)^{10}}{(1+x)^7-(1)^7} = \frac{10}{7}(1)^3 = \frac{10}{7}$

(2) $\lim\limits_{(x-5)\to 1} \frac{(x-5)^7-(1)^7}{(x-5)-(1)} = 7(1)^6 = 7$

(3) $\lim\limits_{(x+3)\to 1} \frac{(x+3)^5-(1)^5}{(x+3)-(1)} = 5(1)^4 = 5$

(4) $\lim\limits_{x\to 0} \frac{(x+2)^5-2^5}{x} = 5(2)^4 = 80$

(5) $\frac{1}{6}\lim\limits_{h\to 0} \frac{(3+h)^4-3^4}{h} = \frac{1}{6}\times 4(3)^3 = 18$

(6) $4\lim\limits_{h\to 0} \frac{(1+4h)^8-(1)^8}{4h}$

$= 4\lim\limits_{(1+4h)\to 1} \frac{(1+4h)^8-(1)^8}{(1+4h)-(1)} = 4\times 8(1)^7 = 32$

(7) $\lim\limits_{(x+2)\to 3} \frac{(x+2)^4-3^4}{(x+2)-3} = 4\times 3^3 = 108$

(8) $\lim\limits_{(x-2)\to -1} \frac{(x-2)^4-(-1)^4}{(x-2)-(-1)} = 4(-1)^3 = -4$

(9) $\frac{-2}{5}\lim\limits_{x\to 0} \frac{(1-2x)^5-1^5}{-2x}$

$= \frac{-2}{5}\lim\limits_{(1-2x)\to 1} \frac{(1-2x)^5-1^5}{(1-2x)-1} = \frac{-2}{5}\times 5 = -2$

(10) $3\lim\limits_{h\to 0} \frac{(x+3h)^5-x^5}{3h}$

$= 3\lim\limits_{(x+3h)\to x} \frac{(x+3h)^5-x^5}{(x+3h)-x}$

$= 3\times 5x^4 = 15x^4$

(11) $\frac{3}{2}\lim\limits_{x\to 0} \frac{\sqrt[3]{1+3x}-1}{3x}$

$= \frac{3}{2}\lim\limits_{(1+3x)\to 1} \frac{(1+3x)^{\frac{1}{3}}-(1)^{\frac{1}{3}}}{(1+3x)-(1)}$

$= \frac{3}{2}\times\frac{1}{3}\times(1)^{-\frac{2}{3}} = \frac{1}{2}$

(12) $\lim\limits_{(x-4)\to -2} \frac{(x-4)^5-(-2)^5}{(x-4)-(-2)}$

$= (\frac{5}{1})\times(-2)^{5-1} = 80$

(13) $\lim\limits_{(x+3)\to 8} \frac{(x+3)^{\frac{1}{3}}-8^{\frac{1}{3}}}{(x+3)-8} = \frac{1}{3}(8)^{\frac{-2}{3}}$

$= \frac{1}{3}\times\frac{1}{4} = \frac{1}{12}$

(14) $3\lim\limits_{(3x+2)\to -1} \frac{(3x+2)^9-(-1)^9}{(3x+2)-(-1)} = 3\times 9(-1)^8 = 27$

(15) $\lim\limits_{x\to 1} \frac{x^{19}-1+x^8-1}{x-1}$

$= \lim\limits_{x\to 1} \frac{x^{19}-(1)^{19}}{x-1} + \lim\limits_{x\to 1} \frac{x^8-(1)^8}{x-1}$

$= 19(1)^{18} + 8(1)^7 = 27$

(16) $\lim\limits_{x\to -1} \frac{x^7+1+x^9+1}{x+1}$

$= \lim\limits_{x\to -1} \frac{x^7-(-1)^7}{x-(-1)} + \lim\limits_{x\to -1} \frac{x^9-(-1)^9}{x-(-1)}$

$= 7(-1)^6 + 9(-1)^8 = 16$

4

(1) $\lim\limits_{x\to 2} \frac{x^3-2^3}{x^5-2^5} + \lim\limits_{x\to 2} \frac{x^4-2^4}{x^7-2^7}$

$= \frac{3}{5}(2)^{-2} + \frac{4}{7}(2)^{-3} = \frac{31}{140}$

(2) $\lim\limits_{x\to -1} \left(\frac{(x-3)(x+1)}{x+1}\right) + \lim\limits_{x\to -1} \frac{x^3-(-1)^3}{x^5-(-1)^5}$

$= \lim\limits_{x\to -1}(x-3) + \frac{3}{5}(-1)^{-2} = -4 + \frac{3}{5} = -\frac{17}{5}$

(3) $\lim\limits_{x\to 3} \left(\frac{x-2}{x^2-4} \times \frac{x^5-243}{x-3}\right)$

$= \lim\limits_{x\to 3} \frac{1}{x+2} \times \lim\limits_{x\to 3} \frac{x^5-3^5}{x-3}$

$= \frac{1}{5}\times 5\times 3^4 = 81$

(4) $\left[\lim\limits_{x\to -3} \frac{x^4-(-3)^4}{x^3-(-3)^3}\right]^3 = \left[\frac{4}{3}(-3)\right]^3 = -64$

5 $\because \lim\limits_{x\to a} \frac{x^{12}-a^{12}}{x^{10}-a^{10}} = 30$ $\quad \therefore \frac{12}{10}\, a^2 = 30$

$\therefore a^2 = 25$ $\quad \therefore a = \pm 5$

6 $\lim\limits_{x\to -1} \frac{x^{15}-(-1)^{15}}{x-(-1)} = 15\,(-1)^{14} = 15$

$, \because \lim\limits_{x\to k} \frac{x^5-k^5}{x^3-k^3} = \frac{5}{3}\,(k)^2$

$\therefore \frac{5}{3}\,k^2 = 15$ $\quad \therefore k^2 = 9$ $\quad \therefore k = \pm 3$

7 $\because$ The limit exists $\quad \therefore 64 = 2^n$

$\therefore n = 6$

$\therefore$ The limit $= \frac{6}{1} \times 2^{6-1}$ $\quad \therefore \ell = 192$

Exercise 14

First Multiple choice questions

(1) d	(2) b	(3) d	(4) d	(5) a	(6) c
(7) d	(8) c	(9) a	(10) a	(11) c	(12) c
(13) c	(14) b	(15) d	(16) b	(17) a	(18) d
(19) a	(20) d	(21) b	(22) b	(23) d	(24) c
(25) c	(26) c	(27) a	(28) a	(29) b	(30) c

Second Essay questions

1

(1) By dividing both of numerator and denominator by x

$$\lim_{x\to\infty} \frac{2-\frac{5}{x}}{3+\frac{8}{x}} = \frac{2}{3}$$

(2) By dividing both of numerator and denominator by x^2

$$\lim_{x\to\infty} \frac{\frac{2}{x}-\frac{5}{x^2}}{3+\frac{8}{x^2}} = \text{zero}$$

(3) By dividing both of numerator and denominator by x

$$\lim_{x\to\infty} \frac{2x-\frac{5}{x^2}}{3+\frac{8}{x}} = \infty$$

2

(1) By dividing both of numerator and denominator by x, we get : $\lim\limits_{x\to\infty} \frac{5-\frac{4}{x}}{3-\frac{2}{x}} = \frac{5}{3}$

(2) By dividing both of numerator and denominator by x, we get : $\lim\limits_{x\to\infty} \frac{\frac{4}{x}-2}{3-\frac{1}{x}} = \frac{-2}{3}$

(3) By dividing both of numerator and denominator by x^2, we get : $\lim\limits_{x\to\infty} \frac{2+\frac{5}{x}+\frac{1}{x^2}}{3-\frac{7}{x^2}} = \frac{2}{3}$

(4) By dividing both of numerator and denominator by x^2, we get : $\lim\limits_{x\to\infty} \frac{\frac{5}{x^2}-\frac{6}{x}-3}{2+\frac{1}{x}+\frac{4}{x^2}} = -\frac{3}{2}$

(5) At $x \to \infty$ $\quad$, then $|x| \to x$

$\therefore \lim\limits_{x\to\infty} \frac{x^3-2}{x^3+1}$

By dividing both of numerator and denominator by x^3, we get : $\lim\limits_{x\to\infty} \frac{1-\frac{2}{x^3}}{1+\frac{1}{x^3}} = 1$

(6) By dividing both of numerator and denominator by x^3, we get : $\lim\limits_{x\to\infty} \frac{\frac{6}{x^2}-4}{\frac{2}{x^3}-7} = \frac{-4}{-7} = \frac{4}{7}$

(7) By dividing both of numerator and denominator by x^4, we get : $\lim\limits_{x\to\infty} \frac{2+\frac{2}{x^2}-\frac{1}{x^4}}{\frac{5}{x^4}-\frac{1}{x^2}-2} = -1$

(8) By dividing both of numerator and denominator by x^5, we get : $\lim\limits_{x\to\infty} \frac{4+\frac{5}{x^5}}{8+\frac{1}{x}-\frac{2}{x^5}} = \frac{4}{8} = \frac{1}{2}$

(9) By dividing both of numerator and denominator by x^3, we get : $\lim\limits_{x\to\infty} \frac{\frac{7}{x}+\frac{1}{x^3}}{4-\frac{8}{x^2}+\frac{1}{x^3}} = \text{zero}$

(10) By dividing both of numerator and denominator by x^4, we get : $\lim\limits_{x\to\infty} \frac{2x+\frac{3}{x^3}-\frac{2}{x^4}}{3+\frac{5}{x^3}-\frac{1}{x^4}} = \infty$

(11) By dividing both of numerator and denominator

by x^4, we get : $\lim\limits_{x \to \infty} \dfrac{5x^3 + \dfrac{2}{x^3} - \dfrac{1}{x^4}}{6 + \dfrac{13}{x^4}} = \infty$

(12) By dividing both of numerator and denominator

by x^{14}, we get : $\lim\limits_{x \to \infty} \dfrac{\dfrac{5}{x^{14}} - \dfrac{7}{x^6} + 3}{\dfrac{7}{x^{14}} - 6 + \dfrac{2}{x^8}} = -\dfrac{1}{2}$

(13) $\lim\limits_{x \to \infty} \left(\dfrac{7}{x^2} + \dfrac{2}{x} - 3\right) = -3$

(14) $\lim\limits_{x \to \infty} \dfrac{\dfrac{5}{x^3} + \dfrac{4}{x^2} - 3}{\dfrac{7}{x^3} - \dfrac{2}{x^2} + 8} = \dfrac{-3}{8}$

(15) At $x \longrightarrow \infty$, then $|2x|^3 \longrightarrow (2x)^3$

$\therefore \lim\limits_{x \to \infty} \dfrac{5x^3 - 4x^2 + 2}{7 - x + 8x^3}$

By dividing both of numerator and denominator

by x^3, we get : $\lim\limits_{x \to \infty} \dfrac{5 - \dfrac{4}{x} + \dfrac{2}{x^3}}{\dfrac{7}{x^3} - \dfrac{1}{x^2} + 8} = \dfrac{5}{8}$

(16) $\lim\limits_{x \to \infty} (x^3 + 5x^2 + 1) = \infty + \infty + 1 = \infty$

(17) $\lim\limits_{x \to \infty} (x^2 - x + 5) = \infty - \infty$

= unspecified quantity

$\therefore \lim\limits_{x \to \infty} x^2 \left(1 - \dfrac{1}{x} + \dfrac{5}{x^2}\right)$

$= \lim\limits_{x \to \infty} x^2 \times \lim\limits_{x \to \infty} \left(1 - \dfrac{1}{x} + \dfrac{5}{x^2}\right)$

$= \infty \times 1 = \infty$

3

(1) By dividing both of numerator and denominator

by x^2, we get : $\lim\limits_{x \to \infty} \dfrac{\left(3 + \dfrac{9}{x}\right)\left(2 + \dfrac{7}{x}\right)}{\left(1 + \dfrac{1}{x}\right)\left(5 + \dfrac{4}{x}\right)}$

$= \dfrac{3 \times 2}{1 \times 5} = \dfrac{6}{5}$

(2) By dividing both of numerator and denominator

by x^2, we get : $\lim\limits_{x \to \infty} \dfrac{3 - \dfrac{4}{x} + \dfrac{5}{x^2}}{\left(1 + \dfrac{2}{x}\right)^2} = \dfrac{3}{1} = 3$

(3) By dividing both of numerator and denominator

by x^2, we get : $\lim\limits_{x \to \infty} \dfrac{\left(2 + \dfrac{3}{x}\right)^2}{\left(\dfrac{5}{x^2} - \dfrac{3}{x} - 1\right)} = \dfrac{4}{-1}$

$= -4$

(4) By dividing both of numerator and denominator

by x^2, we get : $\lim\limits_{x \to \infty} \dfrac{6 - \dfrac{5}{x}}{\left(\dfrac{3}{x} - 1\right)\left(\dfrac{2}{x} + 1\right)} = \dfrac{6}{-1}$

$= -6$

(5) By dividing both of numerator and denominator

by x^2, we get : $\lim\limits_{x \to \infty} \dfrac{\left(1 + \dfrac{1}{x}\right)\left(5 - \dfrac{3}{x}\right)}{1 + \dfrac{3}{x^2}} = 5$

(6) By dividing both of numerator and denominator

by x^3, we get : $\lim\limits_{x \to \infty} \dfrac{8 - \dfrac{1}{x^2} + \dfrac{1}{x^3}}{\left(1 + \dfrac{1}{x}\right)\left(2 - \dfrac{3}{x^2}\right)} = \dfrac{8}{2} = 4$

(7) By dividing both of numerator and denominator

by x^3, we get : $\lim\limits_{x \to \infty} \dfrac{1 - \dfrac{4}{x^2} + \dfrac{5}{x^3}}{\left(2 - \dfrac{1}{x}\right)^3} = \dfrac{1}{8}$

(8) By dividing both of numerator and denominator

by x^3, we get : $\lim\limits_{x \to \infty} \dfrac{\left(2 + \dfrac{3}{x}\right)\left(4 - \dfrac{5}{x^2}\right)}{\left(3 - \dfrac{8}{x^2}\right)\left(5 - \dfrac{3}{x}\right)} = \dfrac{8}{15}$

(9) By dividing both of numerator and denominator by x^3,

we get : $\lim\limits_{x \to \infty} \dfrac{\left(2 + \dfrac{3}{x}\right)\left(5 - \dfrac{1}{x}\right)\left(1 - \dfrac{2}{x}\right)}{1 \times \left(1 + \dfrac{1}{x}\right)\left(3 - \dfrac{1}{x}\right)} = \dfrac{10}{3}$

(10) By dividing both of numerator and denominator

by $x = \sqrt{x^2}$

we get : $\lim\limits_{x \to \infty} \dfrac{\left(\dfrac{7}{\sqrt{x}} + 1\right)\left(\dfrac{3}{\sqrt{x}} + 1\right)}{4 - \dfrac{3}{x}} = \dfrac{1}{4}$

4

(1) By dividing both of numerator and denominator

by $x = \sqrt{x^2}$, we get : $\lim\limits_{x \to \infty} \dfrac{1 + \dfrac{2}{x}}{\sqrt{9 + \dfrac{25}{x^2}}} = \dfrac{1}{3}$

(2) By dividing both of numerator and denominator by $x = \sqrt{x^2}$, we get : $\lim_{x \to \infty} \frac{\sqrt{1+\frac{81}{x^2}}}{2-\frac{1}{x}} = \frac{1}{2}$

(3) By dividing both of numerator and denominator by $x = \sqrt{x^2}$, we get : $\lim_{x \to \infty} \frac{\sqrt{4+\frac{1}{x^2}}}{1-\frac{2}{x}} = 2$

(4) By dividing both of numerator and denominator by $x^2 = \sqrt{x^4}$, we get : $\lim_{x \to \infty} \frac{\frac{4}{x^2}-3}{\sqrt{1+\frac{5}{x^4}}} = -3$

(5) By dividing both of numerator and denominator by x, we get : $\lim_{x \to \infty} \frac{\sqrt{\frac{3}{x^2}+4}}{1} = \sqrt{4} = 2$

(6) By dividing both of numerator and denominator by $x = \sqrt{x^2}$, we get : $\lim_{x \to \infty} \frac{2+\frac{1}{x}}{\sqrt{4+\frac{3}{x}-\frac{4}{x^2}}} = \frac{2}{2} = 1$

(7) By dividing both of numerator and denominator by $x = \sqrt[3]{x^3}$, we get : $\lim_{x \to \infty} \frac{2-\frac{3}{x}}{\sqrt[3]{125+\frac{5}{x^3}}} = \frac{2}{5}$

(8) By dividing both of numerator and denominator by $x = \sqrt[3]{x^3}$, we get : $\lim_{x \to \infty} \frac{\sqrt[3]{8+\frac{5}{x^2}-\frac{2}{x^3}}}{3+\frac{2}{x}} = \frac{2}{3}$

(9) By dividing both of numerator and denominator by $x^3 = \sqrt{x^6}$, we get : $\lim_{x \to \infty} \frac{\frac{4}{x^3}-3}{\sqrt{1+\frac{9}{x^6}}} = -3$

(10) By dividing both of numerator and denominator by $x = \sqrt{x^2} = \sqrt[3]{x^3}$,

we get : $\lim_{x \to \infty} \frac{\sqrt{9-\frac{3}{x}+\frac{8}{x^2}}}{\sqrt[3]{\frac{3}{x}+125+\frac{2}{x^3}}} = \frac{3}{5}$

(11) By dividing both of numerator and denominator by $\sqrt{x^2} = \sqrt[4]{x^4}$,

we get : $\lim_{x \to \infty} \frac{\sqrt{1+\frac{1}{x^2}}}{\sqrt[4]{1+\frac{2}{x^4}}} = \frac{1}{1} = 1$

(12) By dividing both of numerator and denominator by $x = \sqrt{x^2}$,

we get : $\lim_{x \to \infty} \frac{\sqrt{4+\frac{7}{x^2}}+3}{2+\frac{9}{x}} = \frac{5}{2}$

5

(1) $\lim_{x \to \infty} \left(\frac{2}{x}\right) + \lim_{x \to \infty} \left(\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}\right) = \text{zero} + 1 = 1$

(2) $\lim_{x \to \infty} 7 + \lim_{x \to \infty} \frac{2x^2}{(x+3)^2} = 7 + \lim_{x \to \infty} \frac{2}{\left(1+\frac{3}{x}\right)^2}$

$= 7 + 2 = 9$

(3) $\lim_{x \to \infty} \frac{x}{2x+1} + \lim_{x \to \infty} \frac{3x^2}{(x-3)^2}$

$= \lim_{x \to \infty} \frac{1}{2+\frac{1}{x}} + \lim_{x \to \infty} \frac{3}{\left(1-\frac{3}{x}\right)^2} = \frac{1}{2} + 3 = \frac{7}{2}$

(4) $\lim_{x \to \infty} \frac{2}{3} - \lim_{x \to \infty} \frac{3x}{2x+7}$

$= \frac{2}{3} - \lim_{x \to \infty} \frac{3}{2+\frac{7}{x}} = \frac{2}{3} - \frac{3}{2} = -\frac{5}{6}$

(5) $\lim_{x \to \infty} \left(\frac{3}{x}\right) + \lim_{x \to \infty} \frac{2x^5+1}{x^5+2x^2}$

$= 0 + \lim_{x \to \infty} \frac{2+\frac{1}{x^5}}{1+\frac{2}{x^3}} = 2$

(6) $\lim_{x \to \infty} \frac{2x^3-2x^3-x}{2x^2+1} = \lim_{x \to \infty} \frac{-x}{2x^2+1} = 0$

(7) $\lim_{x \to \infty} \frac{(x^2-1)(x-2)-(x^2+1)(x+2)}{x^2-4}$

$= \lim_{x \to \infty} \frac{x^3-2x^2-x+2-x^3-2x^2-x-2}{x^2-4}$

$= \lim_{x \to \infty} \frac{-4x^2-2x}{x^2-4} = \lim_{x \to \infty} \frac{-4-\frac{2}{x}}{1-\frac{4}{x^2}} = -4$

(8) $\lim\limits_{x \to \infty} \frac{\left(\sqrt{x^2-2}-\sqrt{x^2+x}\right)\left(\sqrt{x^2-2}+\sqrt{x^2+x}\right)}{\sqrt{x^2-2}+\sqrt{x^2+x}}$

$= \lim\limits_{x \to \infty} \frac{x^2-2-x^2-x}{\sqrt{x^2-2}+\sqrt{x^2+x}}$

$= \lim\limits_{x \to \infty} \frac{-2-x}{\sqrt{x^2-2}+\sqrt{x^2+x}}$

$= \lim\limits_{x \to \infty} \frac{\frac{-2}{x}-1}{\sqrt{1-\frac{2}{x^2}}+\sqrt{1+\frac{1}{x}}} = -\frac{1}{2}$

(9) $\lim\limits_{x \to \infty} \frac{\left(\sqrt{x^2+x-1}-\sqrt{x^2-x+1}\right)\left(\sqrt{x^2+x-1}+\sqrt{x^2-x+1}\right)}{\left(\sqrt{x^2+x-1}+\sqrt{x^2-x+1}\right)}$

$= \lim\limits_{x \to \infty} \frac{x^2+x-1-x^2+x-1}{\sqrt{x^2+x-1}+\sqrt{x^2-x+1}}$

$= \lim\limits_{x \to \infty} \frac{2x-2}{\sqrt{x^2+x-1}+\sqrt{x^2-x+1}}$

$= \lim\limits_{x \to \infty} \frac{2-\frac{2}{x}}{\sqrt{1+\frac{1}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}$

$= \frac{2}{1+1} = 1$

(10) $\lim\limits_{x \to \infty} \frac{\sqrt{4x^2+1}-\sqrt{x^2+1}}{x}$

$= \lim\limits_{x \to \infty} \frac{\sqrt{4+\frac{1}{x^2}}-\sqrt{1+\frac{1}{x^2}}}{1} = \frac{2-1}{1} = 1$

(11) $\lim\limits_{x \to \infty} \frac{\left(\sqrt{x^2+5x}-x\right)\left(\sqrt{x^2+5x}+x\right)}{\left(\sqrt{x^2+5x}+x\right)}$

$= \lim\limits_{x \to \infty} \frac{x^2+5x-x^2}{\sqrt{x^2+5x}+x} = \lim\limits_{x \to \infty} \frac{5x}{\sqrt{x^2+5x}+x}$

$= \lim\limits_{x \to \infty} \frac{5}{\sqrt{1+\frac{5}{x}}+1} = \frac{5}{1+1} = \frac{5}{2}$

(12) $\lim\limits_{x \to \infty} \frac{x\left(\sqrt{4x^2+1}-2x\right)\left(\sqrt{4x^2+1}+2x\right)}{\left(\sqrt{4x^2+1}+2x\right)}$

$= \lim\limits_{x \to \infty} \frac{x(4x^2+1-4x^2)}{\left(\sqrt{4x^2+1}+2x\right)}$

$= \lim\limits_{x \to \infty} \frac{1}{\sqrt{4+\frac{1}{x^2}}+2} = \frac{1}{4}$

6

$\because$ The limit exists and equals 3

$\therefore$ The degree of numerator = the degree of denominator

$\therefore n = 2$ $\qquad \therefore \lim\limits_{x \to \infty} \frac{4ax^2-4x+5}{3-9x+8x^2} = 3$

By dividing both of numerator and denominator by x^2, we get :

$\lim\limits_{x \to \infty} \frac{4a-\frac{4}{x}+\frac{5}{x^2}}{\frac{3}{x^2}-\frac{9}{x}+8} = 3 \quad \therefore \frac{4a}{8} = 3 \quad \therefore a = 6$

7

$\lim\limits_{x \to \infty} \frac{\sqrt[3]{a+\frac{3}{x^3}}}{\sqrt{4+\frac{7}{x^2}}} = -1$

$\therefore \frac{\sqrt[3]{a}}{2} = -1 \qquad \therefore a = -8$

Answers of "Unit Four"

Exercise 15

First Multiple choice questions

(1) c	(2) c	(3) a	(4) a	(5) d	(6) a
(7) c	(8) a	(9) b	(10) a	(11) b	(12) c
(13) a	(14) b	(15) b	(16) b	(17) c	(18) a
(19) c	(20) d	(21) a	(22) b	(23) c	(24) b
(25) c	(26) d	(27) a	(28) b	(29) d	(30) d
(31) c	(32) b	(33) d	(34) b	(35) d	(36) a
(37) d	(38) a	(39) c	(40) c	(41) d	

Second Essay questions

1

$\because m(\angle Z) = 180° - (80° + 60°) = 40°$

$\therefore \frac{x}{\sin 80°} = \frac{y}{\sin 60°} = \frac{10}{\sin 40°}$

$\therefore x = \frac{10 \sin 80°}{\sin 40°} \simeq 15$ cm. , $y = \frac{10 \sin 60°}{\sin 40°} \simeq 13$ cm.

2

$\because m(\angle C) = 180° - (112° + 33°) = 35°$

$\therefore \frac{b}{\sin 33°} = \frac{19}{\sin 35°}$

$\therefore b = \frac{19 \sin 33°}{\sin 35°} \simeq 18.04$ cm. , $2r = \frac{19}{\sin 35°}$

$\therefore r = \frac{19}{2 \sin 35°} \simeq 16.56$ cm.

3

(1) $\because m(\angle X) = 180° - (100° + 40°) = 40°$

$\therefore \frac{x}{\sin 40°} = \frac{68.4}{\sin 100°}$

$\therefore x = \frac{68.4 \sin 40°}{\sin 100°} \simeq 44.64$ cm.

(2) $2r = \frac{68.4}{\sin 100°}$ $\therefore r \simeq 34.73$ cm.

(3) Area of $\Delta XYZ = \frac{1}{2} x y \sin Z$

$= \frac{1}{2} \times 44.64 \times 68.4 \sin 40°$

$\simeq 981.34 \text{ cm}^2.$

4

$m(\angle B) = 180° - (40° + 80°) = 60°$

$\because \angle C$ is the greatest angle in measure

$\therefore$ c is the length of the greatest side

$\therefore \frac{c}{\sin 80°} = \frac{10}{\sin 60°}$ $\therefore c = \frac{10 \sin 80°}{\sin 60°} \simeq 11$ cm.

5

$\because m(\angle B) = 180° - (100° + 15°) = 65°$

$\therefore \angle B$ is the smallest angle in measure and hence

$\therefore$ b is the length of the smallest side

$\therefore \frac{b}{\sin 15°} = \frac{4.5}{\sin 65°}$ $\therefore b = \frac{4.5 \sin 15°}{\sin 65°} \simeq 1.3$ cm.

6

(1) $\because \frac{x}{\sin 48°} = \frac{10}{\sin 93°}$ $\therefore x = \frac{10 \sin 48°}{\sin 93°} \simeq 7.4$ cm.

(2) $\because m(\angle C) = 180° - (21° + 48°) = 111°$

$\therefore \frac{x}{\sin 111°} = \frac{7.3}{\sin 48°}$ $\therefore x = \frac{7.3 \sin 111°}{\sin 48°} \simeq 9.2$ cm.

7

$r = \frac{7\sqrt{3}}{2 \sin 60°} = 7$ cm.

$\therefore$ The area of the circle $= \frac{22}{7} \times 7^2 = 154 \text{ cm}^2.$

The circumference of the circle $= 2 \times \frac{22}{7} \times 7 = 44$ cm.

8

$\because r = \frac{13}{2 \sin 53° 8'} \simeq 8.1$ cm. , $\frac{13}{\sin 53° 8'} = \frac{15}{\sin C}$

$\therefore \sin C = \frac{15 \sin 53° 8'}{13}$

$\therefore m(\angle C) = 67° 23' 9''$ or $112° 36' 51''$

9

$\because \frac{8}{\sin 35°} = \frac{6}{\sin B}$ $\therefore \sin B = \frac{6 \sin 35°}{8}$

$\therefore m(\angle B) \simeq 25° 28' 45''$

and the another solution is refused.

10

$\because m(\angle B) = 180° - (67° 22' + 44° 33') = 68° 5'$

$\therefore \frac{a}{\sin 67° 22'} = \frac{100}{\sin 68° 5'} = \frac{c}{\sin 44° 33'}$

$\therefore a = \frac{100 \sin 67° 22'}{\sin 68° 5'} \simeq 99$ cm.

$, c = \frac{100 \sin 44° 33'}{\sin 68° 5'} \simeq 76$ cm.

$\therefore$ The perimeter of the triangle $= 100 + 99 + 76 = 275$ cm.

, the area of the triangle $= \frac{1}{2} \times 100 \times 99 \sin 44° 33' \simeq 3473 \text{ cm}^2$.

11

$\because m(\angle A) = 180° - (35° + 70°) = 75°$

$\therefore \frac{a}{\sin 75°} = \frac{b}{\sin 35°} = \frac{c}{\sin 70°} = 32$

$\therefore a = 32 \sin 75° \simeq 31$ cm.

$, b = 32 \sin 35° \simeq 18$ cm. $, c = 32 \sin 70° \simeq 30$ cm.

$\therefore$ The area of the triangle $= \frac{1}{2} \times 31 \times 18 \times \sin 70° \simeq 262 \text{ cm}^2$.

, the perimeter of the triangle $= 31 + 18 + 30 \simeq 79$ cm.

12

$\because \Delta ABC$ is isosceles $\quad \therefore m(\angle B) = m(\angle C) = 30°$

$\therefore \frac{c}{\sin C} = 2r \quad \therefore \frac{c}{\sin 30°} = 24$

$\therefore c = 24 \sin 30° = 12$ cm. $\therefore b = 12$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 12 \times 12 \times \sin 120° \simeq 62.4 \text{ cm}^2$.

13

$m(\angle C) = 180° - (15° + 15°) = 150°$

$\therefore \frac{a}{\sin 15°} = \frac{b}{\sin 15°} = \frac{c}{\sin 150°}$

$\therefore$ One of the ratios $= \frac{a + b + c}{\sin 15° + \sin 15° + \sin 150°}$

$= \frac{25}{\sin 15° + \sin 15° + \sin 150°} \simeq 24.57$

$\therefore 2r \simeq 24.57 \quad \therefore r = 12.285$ cm.

The area of the circle $= \pi (12.285)^2 \simeq 474 \text{ cm}^2$.

14

$\because m(\angle C) = 180° - (44° + 66°) = 70°$

$\therefore \frac{a}{\sin 44°} = \frac{b}{\sin 66°} = \frac{c}{\sin 70°}$

$= \frac{40}{\sin 44° + \sin 66° + \sin 70°}$

$\therefore a \simeq 10.9$ cm. $, b \simeq 14.3$ cm. $c \simeq 14.8$ cm.

15

$m(\angle A) = 60° \div 3 = 20°$

$m(\angle C) = 180° - (20° + 60°) = 100°$

$\therefore \frac{a}{\sin 20°} = \frac{12}{\sin 100°} \quad \therefore a = \frac{12 \sin 20°}{\sin 100°} \simeq 4.2$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 4.2 \times 12 \sin 60° \simeq 22 \text{ cm}^2$.

16

$\because m(\angle A) = 180° - (82° + 56°) = 42°$

$, \because$ the area of $\Delta ABC = \frac{1}{2}$ a b sin C

$\therefore 450 = \frac{1}{2} a b \sin 56° \quad \therefore b = \frac{900}{a \sin 56°}$

$, \because \frac{a}{\sin 42°} = \frac{b}{\sin 82°} \quad \therefore \frac{a}{\sin 42°} = \frac{900}{a \sin 56° \sin 82°}$

$\therefore a^2 = \frac{900 \sin 42°}{\sin 56° \sin 82°} \quad \therefore a \simeq 27$ cm.

17

$\because 43.2 = \frac{1}{2} AB \times 12 \times 0.6 \quad \therefore AB = 12$ cm.

$\because m(\angle A) \approx 36° 52'$ (and the other solution is refused because the triangle is acute-angled triangle)

$\therefore m(\angle B) = m(\angle C) = \frac{180° - 36° 52'}{2} = 71° 34'$

$\therefore \frac{BC}{\sin 36° 52'} = \frac{12}{\sin 71° 34'}$

$\therefore BC = \frac{12 \times \sin 36° 52'}{\sin 71° 34'} \simeq 7.6$ cm.

18

$\because \frac{7}{\sin 60°} = \frac{8}{\sin B} \quad \therefore m(\angle B) \simeq 81° 47' 12''$

(and the other solution is refused because the triangle is acute-angled triangle)

$\therefore m(\angle C) = 180° - (60° + 81° 47' 12'') = 38° 12' 48''$

$\therefore \frac{7}{\sin 60°} = \frac{\text{Perimeter of } \Delta ABC}{\sin 60° + \sin 81° 47' 12'' + \sin 38° 12' 48''}$

$\therefore$ Perimeter of $\Delta ABC \simeq 20$ cm.

19

(1) $2r = \frac{a}{\sin A} = \frac{21}{\sin 75°} \simeq 21.7$ cm.

(2) $2r = \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{c - b}{\sin C - \sin B}$

$= \frac{6}{\sin 65° - \sin 50°} \simeq 42.8$ cm.

20

$\because \tan C = \frac{4}{3} \qquad \therefore m(\angle C) \simeq 53^\circ\, 7'\, 48''$

$\therefore m(\angle A) = 180^\circ - (30^\circ + 53^\circ\, 7'\, 48'')$

$= 96^\circ\, 52'\, 12''$

$\therefore \frac{a}{\sin 96^\circ\, 52'\, 12''} = \frac{5}{\sin 30^\circ} = \frac{c}{\sin 53^\circ\, 7'\, 48''}$

$\therefore a = \frac{5 \sin 96^\circ\, 52'\, 12''}{\sin 30^\circ} \simeq 10$ cm.

$, c = \frac{5 \sin 53^\circ\, 7'\, 48''}{\sin 30^\circ} \simeq 8$ cm.

, area of $\Delta ABC = \frac{1}{2} a\, c \sin B$

$= \frac{1}{2} \times 10 \times 8 \times \sin 30^\circ = 20\ cm^2.$

21

$\frac{X + y + z}{\sin X + \sin Y + \sin Z} = 2r \quad \therefore r = \frac{56.88}{2 \times 2.37} = 12$ cm.

22

$\sin A : \sin B : \sin C = 2 : 4 : 5 \quad \therefore a : b : c = 2 : 4 : 5$

$\therefore a = 2m , b = 4m$ and $c = 5m$

$\because c - b = 5m - 4m = m \qquad \therefore m = 3$

$\therefore a = 6$ cm. and $b = 12$ cm.

23

$\because m(\angle A) : m(\angle B) : m(\angle C) = 3 : 4 : 3$

$\therefore m(\angle A) = 3k , m(\angle B) = 4k , m(\angle C) = 3k$

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$

$\therefore 3k + 4k + 3k = 180^\circ \qquad \therefore k = 18^\circ$

$\therefore \frac{5}{\sin 54^\circ} = \frac{b}{\sin 72^\circ} = \frac{c}{\sin 54^\circ}$

$= \frac{\text{Perimeter of } \Delta ABC}{\sin 54^\circ + \sin 72^\circ + \sin 54^\circ}$

$\therefore$ Perimeter of $\Delta ABC \simeq 15.9$ cm.

24

$\because m(\angle A) + \frac{3}{2} m(\angle A) + 2m(\angle A) = 180^\circ$

$\therefore m(\angle A) = 40^\circ , m(\angle B) = 60^\circ , m(\angle C) = 80^\circ$

$, \because \frac{a}{\sin 40^\circ} = \frac{b}{\sin 60^\circ} = \frac{c}{\sin 80^\circ} = 20$

$\therefore a = 20 \sin 40^\circ , b = 20 \sin 60^\circ$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times 20 \sin 40^\circ \times 20 \sin 60^\circ \times \sin 80^\circ$

$\simeq 110\ cm^2.$

25

$\because 6 \sin A = 4 \sin B = 3 \sin C$

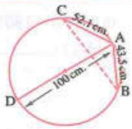
$\therefore \frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{4}$

Put $a = 2m , b = 3m , c = 4m$

$\therefore 2m + 3m + 4m = 45 \qquad \therefore m = 5$

$\therefore a = 10$ cm. and $c = 20$ cm.

26



In ΔABC :

$\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$

$\therefore \frac{a}{\sin A} = \frac{52.1}{\sin B} = \frac{43.5}{\sin C} = 100$

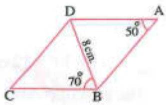
$\therefore \sin B = \frac{52.1}{100} \qquad \therefore m(\angle B) \simeq 31^\circ\, 24'$

$, \sin C = \frac{43.5}{100} \qquad \therefore m(\angle C) \simeq 25^\circ\, 47'$

$\therefore m(\angle BAC) = 180^\circ - (31^\circ\, 24' + 25^\circ\, 47') = 122^\circ\, 49'$

$, \because \frac{a}{\sin 122^\circ\, 49'} = 100 \quad \therefore a = 100 \sin 122^\circ\, 49' \simeq 84$ cm.

27



$\because ABCD$ is a parallelogram

$\therefore m(\angle C) = 50^\circ$

In ΔBDC :

$m(\angle BDC) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$

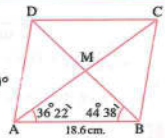
$\therefore \frac{8}{\sin 50^\circ} = \frac{BC}{\sin 60^\circ} = \frac{DC}{\sin 70^\circ}$

$\therefore BC \simeq 9$ cm. $, DC \simeq 9.8$ cm.

$\therefore$ The perimeter of the parallelogram $= 2(BC + CD)$

$\simeq 38$ cm.

28



In $\Delta ABM : m(\angle AMB)$

$= 180^\circ - (44^\circ\, 38' + 36^\circ\, 22') = 99^\circ$

$\therefore \frac{18.6}{\sin 99^\circ} = \frac{AM}{\sin 44^\circ\, 38'}$

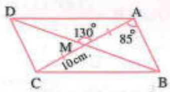
$\therefore AM = \frac{18.6 \times \sin 44^\circ\, 38'}{\sin 99^\circ}$

$\therefore AC = 2(AM) \simeq 26.46$ cm.

$\therefore$ The area of ▱ ABCD

$= 2 \times \frac{1}{2} \times 18.6 \times 26.46 \sin 36^\circ\, 22' \simeq 292\ cm^2.$

29



In Δ ABM :

$m(\angle M) = 50°$

, $m(\angle B) = 180° - (85° + 50°) = 45°$

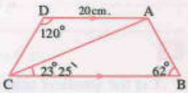
$\therefore \frac{10}{\sin 45°} = \frac{BM}{\sin 85°}$ $\quad \therefore BM = \frac{10 \sin 85°}{\sin 45°} \simeq 14.1$ cm.

$\therefore BD = 2\,BM = 28.2$ cm.

, area of ▱ ABCD

$= 4 \times \frac{1}{2} \times 10 \times 14.1 \sin 50° \simeq 216\ cm^2$

30



$\because \overline{AD} \parallel \overline{BC}$

$\therefore m(\angle ADC) +$

$m(\angle DCB) = 180°$

$\therefore m(\angle DCB) = 180° - 120° = 60°$

$\therefore m(\angle DCA) = 60° - 23°\,25' = 36°\,35'$

In Δ ADC : $\frac{20}{\sin 36°\,35'} = \frac{AC}{\sin 120°}$

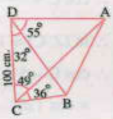
$\therefore AC = \frac{20 \sin 120°}{\sin 36°\,35'} \simeq 29$ cm.

In Δ ABC : $m(\angle A) = 180° - (62° + 23°\,25') = 94°\,35'$

$\therefore \frac{29}{\sin 62°} = \frac{BC}{\sin 94°\,35'}$

$\therefore BC = \frac{29 \sin 94°\,35'}{\sin 62°} \simeq 33$ cm.

31



In Δ BDC :

$m(\angle DBC) = 180° - (32° + 85°) = 63°$

$\therefore \frac{BD}{\sin 85°} = \frac{100}{\sin 63°}$

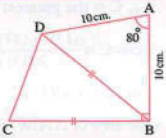
$\therefore BD = \frac{100 \sin 85°}{\sin 63°} = 112$ cm.

, in Δ ADC :

$m(\angle DAC) = 180° - (49° + 87°) = 44°$

$\therefore \frac{AC}{\sin 87°} = \frac{100}{\sin 44°}$ $\quad \therefore AC = \frac{100 \sin 87°}{\sin 44°} \simeq 144$ cm.

32



In Δ ABD :

$\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB)$

$= \frac{180° - 80°}{2} = 50°$

$\therefore \frac{10}{\sin 50°} = \frac{BD}{\sin 80°}$ $\quad \therefore BD = \frac{10 \sin 80°}{\sin 50°}$

, $\because m(\angle ABC) = 90°$

$\therefore m(\angle DBC) = 90° - 50° = 40°$

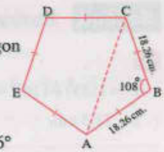
$\therefore$ Area of $\Delta ABD = \frac{1}{2} \times 10 \times 10 \times \sin 80° \simeq 49\ cm^2$.

, area of $\Delta DBC = \frac{1}{2} \times \frac{10 \sin 80°}{\sin 50°} \times \frac{10 \sin 80°}{\sin 50°} \times \sin 40°$

$\simeq 53\ cm^2$.

$\therefore$ Area of ABCD $= 49 + 53 = 102\ cm^2$.

33



$\because$ The figure is a regular pentagon

$\therefore m(\angle B) = 108°$

, $m(\angle BAC) = m(\angle BCA)$

$= \frac{180° - 108°}{2} = 36°$

$\therefore \frac{AC}{\sin 108°} = \frac{18.26}{\sin 36°}$ $\quad \therefore AC \simeq 29.5$ cm.

Third Higher skills

(1) (d) (2) (c) (3) (c) (4) (b)

Instructions to solve :

(1) $\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r = 6$

$\therefore \frac{a}{\sin A} \times \frac{b}{\sin B} \times \frac{c}{\sin C} = 6 \times 6 \times 6$

$\therefore \frac{abc}{\sin A \sin B \sin C} = 216$

(2) $\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$

$\therefore a \csc A = b \csc B = c \csc C = 2r$

$\therefore a \csc A + b \csc B + \csc C = 6r$

(3) $\because \frac{a + b + c}{\sin A + \sin B + \sin C} = 2r$

$\therefore \frac{\sin B + \sin C + \sin A}{\sin A + \sin B + \sin C} = 2r$

$\therefore 2r = 1$ $\quad \therefore r = \frac{1}{2}$

$\therefore$ The circumference of the circumcircle of $\Delta ABC = 2\pi r = 2\pi \times \frac{1}{2} = \pi$ length unit

(4) $\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2r}$

Multiply up and down the first ratio by a , the second ratio by b and third ratio by c and by adding the antecedents and consequents

$\therefore \frac{a \sin A + b \sin B + c \sin C}{a^2 + b^2 + c^2} = \frac{1}{2r}$

Exercise 16

First Multiple choice questions

(1) c	(2) a	(3) b	(4) d	(5) a	(6) b
(7) d	(8) d	(9) a	(10) a	(11) c	(12) a
(13) b	(14) b	(15) d	(16) d	(17) d	(18) b
(19) c	(20) b	(21) c	(22) b	(23) c	(24) d
(25) d	(26) b	(27) b	(28) b	(29) b	(30) c

Second Essay questions

1

$\because z^2 = (13)^2 + (16)^2 - 2 \times 13 \times 16 \cos 95° \simeq 461.26$

$\therefore z \simeq 21.5$ cm.

2

$\because b^2 = (3)^2 + (5)^2 - 2 \times 3 \times 5 \cos 36° 21' \simeq 9.8$

$\therefore b \simeq 3$ cm.

3

(1) $\because \cos C = \frac{3^2 + 5^2 - (\sqrt{19})^2}{2 \times 3 \times 5} = \frac{1}{2}$

$\therefore m(\angle C) = 60°$

(2) The area of the triangle $= \frac{1}{2} \times 3 \times 5 \times \sin 60°$

$= \frac{15\sqrt{3}}{4}$ cm^2.

4

(The length of the third side)^2

$= (\sqrt{10} + 2)^2 + (\sqrt{10} - 2)^2$

$- 2(\sqrt{10} + 2)(\sqrt{10} - 2) \cos 60° = 22$

$\therefore$ The length of the third side $= \sqrt{22}$

5

$\because c^2 = (4)^2 + (6)^2 - 2 \times 4 \times 6 \cos 57° \simeq 25.9$

$\therefore c \simeq 5$ cm.

$\therefore$ The perimeter of $\Delta ABC \simeq 4 + 6 + 5 = 15$ cm.

6

$\because \cos A = \frac{(5.8)^2 + (3.4)^2 - (7.6)^2}{2(5.8)(3.4)}$

$\therefore m(\angle A) \simeq 108° 34'$

, $\cos B = \frac{(7.6)^2 + (3.4)^2 - (5.8)^2}{2(7.6)(3.4)}$

$\therefore m(\angle B) \simeq 46° 20'$

$\therefore m(\angle C) = 180° - (108° 34' + 46° 20') = 25° 6'$

7

$\because \cos B = \frac{(13)^2 + (15)^2 - (14)^2}{2(13)(15)}$

$\therefore m(\angle B) = 59° 29'$

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 13 \times 15 \times \sin 59° 29'$

$\simeq 84$ cm^2.

8

$\because X$ is the smallest side in length

$\therefore \angle X$ is the smallest angle in measure

$\therefore \cos X = \frac{(27)^2 + (24)^2 - (18)^2}{2 \times 27 \times 24}$ $\quad \therefore m(\angle X) \simeq 40° 48'$

$\because r = \frac{18}{2 \sin 40° 48'}$

$\therefore$ The area of the circle $= \pi \left(\frac{18}{2 \sin 40° 48'}\right)^2$

$\simeq 596$ cm^2.

9

$\because c$ is the greatest side in length

$\therefore \angle C$ is the greatest angle in measure

$\therefore \cos C = \frac{(9)^2 + (15)^2 - (21)^2}{2 \times 9 \times 15} = -\frac{1}{2}$

$\therefore m(\angle C) = 120°$

$\therefore \cos C - 5\sqrt{3} \sin C + 8$

$= \cos 120° - 5\sqrt{3} \sin 120° + 8 =$ zero

10

$c = 52 - (13 + 17) = 22$ cm.

$\because c$ is the greatest side in length

$\therefore \angle C$ is the greatest angle in measure

$\therefore \cos C = \frac{(13)^2 + (17)^2 - (22)^2}{2(13)(17)}$

$\therefore m(\angle C) \simeq 93° 22'$

, the area of $\Delta ABC = \frac{1}{2} \times 13 \times 17 \times \sin 93° 22'$

$\simeq 110$ cm^2.

11

$\because X$ is the greatest side in length

$\therefore \angle X$ is the greatest angle in measure

$\therefore \cos X = \frac{(18)^2 + (10)^2 - (24.5)^2}{2 \times 18 \times 10}$

$\therefore m(\angle X) \simeq 119° 19'$

$\because r = \frac{24.5}{2 \sin 119° 19'} \simeq 14$ cm.

$\therefore$ The circumference of the circumcircle of ΔXYZ

$= 2 \times \frac{22}{7} \times 14 = 88$ cm.

12

Let $X = 4$ m , $y = 5$ m and $z = 6$ m

$\because X$ is the smallest side in length.

$\therefore \angle X$ is the smallest angle in measure

$\therefore \cos X = \frac{(5\text{ m})^2 + (6\text{ m})^2 - (4\text{ m})^2}{2 \times 5\text{ m} \times 6\text{ m}} = \frac{3}{4}$

$\therefore m(\angle X) \simeq 41° 25'$

13

$\because X : y : z = \sin X : \sin Y : \sin Z = 7 : 8 : 12$

, let $X = 7$ m. , $y = 8$ m. , $z = 12$ m.

, $\because$ z is the greatest side in length

$\therefore \angle Z$ is the greatest angle in measure

$\therefore \cos Z = \frac{(7\text{ m})^2 + (8\text{ m})^2 - (12\text{ m})^2}{2 \times 7\text{ m} \times 8\text{ m}} = \frac{-31}{112}$

$\therefore m(\angle Z) \simeq 106° 4'$

14

$\because c^2 = (4)^2 + (5)^2 - 2 \times 4 \times 5 \times -\frac{1}{2} = 61$

$\therefore c \simeq 7.8$ cm.

$\because \cos C = -\frac{1}{2} \qquad \therefore m(\angle C) = 120°$

$\therefore$ The area of the triangle $= \frac{1}{2} \times 4 \times 5 \times \sin 120°$

$= 5\sqrt{3}$ cm^2.

15

$\because 2 \sin A = 3 \sin B = 4 \sin C \quad \therefore \frac{\sin A}{6} = \frac{\sin B}{4} = \frac{\sin C}{3}$

$\therefore a : b : c = 6 : 4 : 3$

Let $a = 6$ m , $b = 4$ m and $c = 3$ m

$\because$ c is the smallest side in length

$\therefore \angle C$ is the smallest angle in measure.

$\therefore \cos C = \frac{(6\text{ m})^2 + (4\text{ m})^2 - (3\text{ m})^2}{2 \times 6\text{ m} \times 4\text{ m}} = \frac{43}{48}$

$\therefore m(\angle C) \simeq 26° 23'$

16

$\because a : b : c = 3 : 4 : 5$

Let $a = 3$ k , $b = 4$ k , $c = 5$ k

$\therefore \cos C = \frac{(3\text{ k})^2 + (4\text{ k})^2 - (5\text{ k})^2}{2 \times 3\text{ k} \times 4\text{ k}} =$ zero

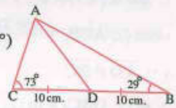
$\therefore m(\angle C) = 90°$

$\because 3\text{ k} + 4\text{ k} + 5\text{ k} = 24 \qquad \therefore k = 2$

$\therefore a = 6$ cm. , $b = 8$ cm. , $c = 10$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 6 \times 8 = 24$ cm^2.

17



In ΔABC :

$\because m(\angle A) = 180° - (29° + 73°)$

$= 78°$

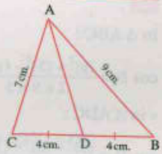
$\therefore \frac{20}{\sin 78°} = \frac{AB}{\sin 73°}$

$\therefore AB = \frac{20 \sin 73°}{\sin 78°} \simeq 19.55$ cm.

, $(AD)^2 = (19.55)^2 + (10)^2 - 2 \times 19.55 \times 10 \cos 29°$

$\therefore AD = 11.84$ cm.

18



In ΔABC :

$\because \cos B = \frac{(8)^2 + (9)^2 - (7)^2}{2 \times 8 \times 9} = \frac{2}{3}$

$\therefore m(\angle B) \simeq 48° 11'$

In ΔABD :

$(AD)^2 = (9)^2 + (4)^2 - 2 \times 9 \times 4 \cos 48° 11' = 49$

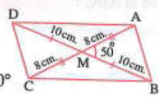
$\therefore AD = 7$ cm.

In ΔABC : $r = \frac{7}{2 \times \sin 48° 11'} = 4.7$ cm.

19

In ΔAMB : $\because (AB)^2 = (8)^2 + (10)^2 - 2 \times 8 \times 10 \cos 50°$

$\therefore AB \simeq 8$ cm.



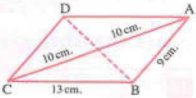
In ΔAMD :

$\therefore m(\angle AMD) = 180° - 50° = 130°$

$\therefore (AD)^2 = (8)^2 + (10)^2 - 2 \times 8 \times 10 \cos 130°$

$\therefore AD \simeq 16$ cm.

20



In Δ ABC :

$\because \cos B = \frac{(9)^2 + (13)^2 - (20)^2}{2\,(9)\,(13)}$

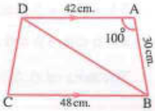
$\therefore m(\angle B) \simeq 129^\circ\ 52'$

$\therefore m(\angle BAD) = 180^\circ - 129^\circ\ 52' = 50^\circ\ 8'$

In Δ ABD : $(BD)^2 = (9)^2 + (13)^2 - 2\,(9)\,(13) \cos 50^\circ\ 8'$

$\therefore BD \simeq 10$ cm.

21



In Δ ABD :

$\because (BD)^2 = (30)^2 + (42)^2 - 2 \times 30 \times 42 \cos 100^\circ$

$\therefore BD \simeq 55.7$ cm.

, $\cos(\angle ADB) = \frac{(55.7)^2 + (42)^2 - (30)^2}{2 \times 55.7 \times 42} \simeq 0.85$

, $\because \overline{AD} \,/\!/\, \overline{BC}$

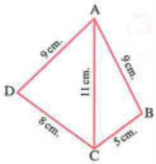
$\therefore m(\angle DBC) = m(\angle ADB)$ «Alternate angles»

$\therefore \cos(\angle DBC) = 0.85$

$\therefore (CD)^2 = (55.7)^2 + (48)^2 - 2 \times 55.7 \times 48 \times 0.85$

$\therefore CD \simeq 29.3$ cm.

22



In Δ ABC :

$\cos B = \frac{(9)^2 + (5)^2 - (11)^2}{2 \times 9 \times 5} = -\frac{1}{6}$

, in Δ ADC :

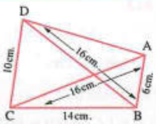
$\cos D = \frac{(9)^2 + (8)^2 - (11)^2}{2 \times 9 \times 8} = \frac{1}{6}$

$\therefore \cos B = -\cos D$

$\therefore m(\angle B) + m(\angle D) = 180^\circ$

$\therefore$ The figure is a cyclic quadrilateral.

23



In Δ ABC :

$\cos(\angle BAC) = \frac{(6)^2 + (16)^2 - (14)^2}{2 \times 6 \times 16} = \frac{1}{2}$

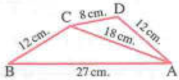
In Δ BCD :

$\cos(\angle BDC) = \frac{(10)^2 + (16)^2 - (14)^2}{2 \times 10 \times 16} = \frac{1}{2}$

$\therefore m(\angle BAC) = m(\angle BDC)$ and they are drawn on the same base.

$\therefore$ ABCD is a cyclic quadrilateral.

24



In Δ ADC :

$\cos(\angle DAC) = \frac{(12)^2 + (18)^2 - (8)^2}{2 \times 12 \times 18} = \frac{101}{108}$

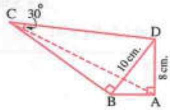
, in Δ CAB :

$\cos(\angle CAB) = \frac{(27)^2 + (18)^2 - (12)^2}{2\,(27)\,(18)} = \frac{101}{108}$

$\therefore m(\angle DAC) = m(\angle CAB)$

$\therefore \overrightarrow{AC}$ bisects $\angle BAD$

25



In Δ ADB :

$AB = \sqrt{(10)^2 - (8)^2} = 6$ cm.

, $\cos B = \frac{3}{5}$

$\therefore m(\angle B) \simeq 53^\circ\ 8'$

In Δ DBC : $DB = \frac{1}{2}$ DC

$\therefore DC = 20$ cm. , $BC = 10\sqrt{3}$ cm.

In Δ ABC : $m(\angle ABC) = 90^\circ + 53^\circ\ 8' = 143^\circ\ 8'$

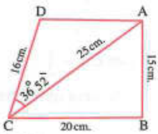
$\therefore (AC)^2 = (6)^2 + \left(10\sqrt{3}\right)^2 - 2\,(6)\left(10\sqrt{3}\right) \cos 143^\circ\ 8'$

$\therefore AC \simeq 22$ cm.

26

$\because (AD)^2 = (25)^2 + (16)^2 - 2 \times 25 \times 16 \cos 36^\circ\ 52'$

$\therefore AD \simeq 16$ cm.



, $\because$ in Δ ABC :

$(25)^2 = (15)^2 + (20)^2$

$\therefore m(\angle B) = 90^\circ$

$\therefore$ The area of the quadrilateral ABCD

= the area of Δ ABC + the area of Δ ACD

$= \frac{1}{2} \times 20 \times 15 + \frac{1}{2} \times 25 \times 16 \times \sin 36^\circ\ 52' \simeq 270\ cm^2$.

27

$\because c^2 = (3\,b)^2 + b^2 - 2 \times 3\,b \times b \cos 60^\circ$

$= 10\,b^2 - 6\,b^2 \times \frac{1}{2} = 7\,b^2$

$\therefore c = \sqrt{7}\, b$

$\therefore \cos B = \frac{7 b^2 + 9 b^2 - b^2}{2 \times \sqrt{7}\, b \times 3 b} = \frac{15}{6\sqrt{7}} = \frac{5}{2\sqrt{7}}$

$\therefore m(\angle B) \simeq 19^\circ\ 6'$

$\therefore m(\angle A) = 180^\circ - (19^\circ\ 6' + 60^\circ) = 100^\circ\ 54'$

28

$\because 10\sqrt{3} = \frac{1}{2} \times 5\, c \times \sin 120^\circ \qquad \therefore c = 8$ cm.

$\therefore b^2 = (5)^2 + (8)^2 - 2 \times 5 \times 8 \cos 120^\circ = 129$

$\therefore b \simeq 11.36$ cm.

$\therefore \cos A = \frac{(11.36)^2 + (8)^2 - (5)^2}{2 \times 11.36 \times 8} \quad \therefore m(\angle A) \simeq 22^\circ\ 24'$

29

$\because P - a = 8$ cm. , $P - b = 6$ cm. , $P - c = 4$ cm.

(by adding)

$\therefore 3P - (a + b + c) = 18 \qquad \therefore 3P - 2P = 18$

$\therefore P = 18 \qquad \therefore a = 10$ cm.

, $b = 12$ cm. , $c = 14$ cm.

$\because$ The length of the longest side in the triangle is c

$\therefore \cos C = \frac{10^2 + 12^2 - 14^2}{2 \times 10 \times 12} = \frac{1}{5} \quad \therefore m(\angle C) \simeq 78^\circ\ 28'$

30

$\because P - a = 26$, $P + a = 98$ (by subtracting)

$\therefore 2a = 72 \qquad \therefore a = 36$ cm. $\qquad \therefore P = 62$ cm.

$\therefore$ The perimeter $= 124$ cm.

, $\because b = 28$ cm. $\qquad \therefore c = 124 - (36 + 28) = 60$ cm.

, $\because$ the length of the shortest side in the triangle is b

$\therefore \cos B = \frac{36^2 + 60^2 - 28^2}{2 \times 36 \times 60} \quad \therefore m(\angle B) \simeq 17^\circ\ 51'$

31

$\because b : c = 3 : 4$, let $b = 3\,m$, $c = 4\,m$

, $\because$ the area of $\Delta ABC = 64\ cm^2$

$\therefore \frac{1}{2} \times 3\,m \times 4\,m \times \sin 30^\circ = 64 \qquad \therefore m = \frac{8}{\sqrt{3}}$

$\therefore b = \frac{24}{\sqrt{3}}$ cm. , $c = \frac{32}{\sqrt{3}}$

$\therefore a^2 = \left(\frac{32}{\sqrt{3}}\right)^2 + \left(\frac{24}{\sqrt{3}}\right)^2 - 2 \times \frac{32}{\sqrt{3}} \times \frac{24}{\sqrt{3}} \cos 30^\circ$

$\therefore a \simeq 9.5$ cm.

$\therefore$ The perimeter of $\Delta ABC \simeq 41.8$ cm.

32

$\because \sin A : \sin B : \sin C = 3 : 5 : 7$

$\therefore a : b : c = 3 : 5 : 7$

and let : $a = 3\,m$, $b = 5\,m$ and $c = 7\,m$

$\therefore \cos A : \cos B : \cos C$

$= \frac{(5\,m)^2 + (7\,m)^2 - (3\,m)^2}{2 \times 5\,m \times 7\,m} : \frac{(3\,m)^2 + (7\,m)^2 - (5\,m)^2}{2 \times 3\,m \times 7\,m}$

$: \frac{(3\,m)^2 + (5\,m)^2 - (7\,m)^2}{2 \times 3\,m \times 5\,m} = \frac{13}{14} : \frac{11}{14} : -\frac{1}{2} = 13 : 11 : -7$

33

$\because 34 = 12 + (6 + c) + c \qquad \therefore c = 8$ cm. , $b = 14$ cm.

$\because$ The smallest angle is opposite to the smallest side

$\therefore \cos C = \frac{(12)^2 + (14)^2 - (8)^2}{2 \times 12 \times 14}$

$\therefore m(\angle C) = 34^\circ\ 46'\ 19''$

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 12 \times 14 \sin 34^\circ\ 46'\ 19''$

$\simeq 47.9\ cm^2$

34

$\because y^2 = z^2 - 2zX + X^2 + zX$

$\therefore y^2 = z^2 + X^2 - zX$

, $\because y^2 = z^2 + X^2 - 2Xz \cos Y$

$\therefore zX = 2zX \cos Y$

$\therefore \cos Y = \frac{1}{2} \qquad \therefore m(\angle Y) = 60^\circ$

35

The answer of Ziad is wrong because the sine rule makes the sine of the acute or obtuse angle always positive , although the angle is obtuse , and using the cosine rule in Karim's answer confirm that.

Third Higher skills

(1) (c) (2) (d) (3) (b) (4) (b)

Instructions to solve :

(1) $\because \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\therefore b^2 + c^2 - a^2 = 2bc \cos A$

$\therefore (b^2 + c^2 - a^2) \tan A = 2\, bc \cancel{\cos A} \times \frac{\sin A}{\cancel{\cos A}}$

$= 2\, bc \sin A$

$= 4 \times \left(\frac{1}{2}\, b\, c \sin A\right)$

$= 4 \times 12 = 48$

(2) $\left(1 + \frac{a}{c} + \frac{b}{c}\right)\left(1 + \frac{c}{b} - \frac{a}{b}\right)$

$= \left(\frac{c + a + b}{c}\right)\left(\frac{b + c - a}{b}\right)$

$= \frac{(b + c)^2 - a^2}{bc} = \frac{b^2 + c^2 - a^2 + 2\, bc}{bc}$

$= \frac{b^2 + c^2 - a^2}{bc} + 2$

$= 2 \cos A + 2 = 2\left(\frac{1}{2}\right) + 2 = 3$

(3) $\because \frac{a^3 + b^3 + c^3}{a + b + c} = a^2 \quad \therefore a^3 + b^3 + c^3 = a^3 + ba^2 + ca^2$

$\therefore b^3 + c^3 = ba^2 + ca^2$

$\therefore (b + c)(b^2 - bc + c^2) = a^2 (b + c)$

$\therefore a^2 = b^2 - bc + c^2 \quad \therefore b^2 + c^2 - a^2 = bc$

$\therefore \frac{b^2 + c^2 - a^2}{2\, b\, c} = \frac{1}{2} \quad \therefore \cos A = \frac{1}{2}$

$\therefore m(\angle A) = 60^\circ$

(4) Let the side length of the small square = L

$\therefore$ The side length of the big square = 3 L

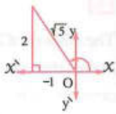
$\therefore XE = \sqrt{L^2 + L^2} = \sqrt{2}\, L$

, $BX = \sqrt{L^2 + (3L)^2} = \sqrt{10}\, L$

, $BE = 4\, L$

$\therefore \cos(\angle BXE) = \frac{\left(\sqrt{2}\, L\right)^2 + \left(\sqrt{10}\, L\right)^2 - (4\, L)^2}{2 \times \sqrt{2}\, L \times \sqrt{10}\, L}$

$= \frac{-4\, L^2}{4\sqrt{5}\, L^2} = \frac{-1}{\sqrt{5}}$



$\therefore \sin(\angle BXE) = \frac{2}{\sqrt{5}}$

Exercise 17

First Multiple choice questions

(1) d (2) c (3) c

(4) First : a Second : d (5) a (6) b

(7) a (8) a (9) a (10) b (11) d

Second Essay questions

Exercise on the first case

1

$m(\angle B) = 180^\circ - (67^\circ + 46^\circ) = 67^\circ$

$\therefore \frac{a}{\sin 67^\circ} = \frac{11}{\sin 67^\circ} = \frac{c}{\sin 46^\circ}$

$\therefore$ a = 11 cm. and c ≃ 8.6 cm.

2

$m(\angle C) = 180^\circ - (60^\circ + 40^\circ) = 80^\circ$

$\therefore \frac{8}{\sin 60^\circ} = \frac{b}{\sin 40^\circ} = \frac{c}{\sin 80^\circ}$

$\therefore$ b ≃ 5.94 cm. , c ≃ 9.1 cm.

3

$m(\angle C) = 180^\circ - (49^\circ\, 11' + 67^\circ\, 17') = 63^\circ\, 32'$

$\therefore \frac{a}{\sin 49^\circ\, 11'} = \frac{b}{\sin 67^\circ\, 17'} = \frac{11.22}{\sin 63^\circ\, 32'}$

$\therefore$ a = 9.5 cm. , b = 11.6 cm.

4

$m(\angle A) = 80^\circ$, $m(\angle B) = 40^\circ$, $m(\angle C) = 60^\circ$

$\therefore \frac{BC}{\sin 80^\circ} = \frac{AC}{\sin 40^\circ} = \frac{9}{\sin 60^\circ}$

$\therefore$ BC ≃ 10.2 cm. , AC ≃ 6.7 cm. ,

the area of $\Delta ABC = \frac{1}{2}\, BC \times CA \times \sin 60^\circ \simeq 30\ cm^2$.

5

$m(\angle Z) = 180^\circ - (75^\circ\, 12' + 48^\circ\, 15') = 56^\circ\, 33'$

$\therefore \frac{YZ}{\sin 75^\circ\, 12'} = \frac{XZ}{\sin 48^\circ\, 15'} = \frac{40}{\sin 56^\circ\, 33'}$

$\therefore$ YZ ≃ 46.4 cm. , XZ ≃ 35.8 cm.

The height = YZ × sin Y ≃ 34.6 cm.

Exercise on the second case

6

$(XY)^2 = (13)^2 + (16)^2 - 2 \times 13 \times 16 \cos 60^\circ$

$\therefore XY = \sqrt{217} \simeq 14.7$ cm.

$\therefore \cos X = \frac{(16)^2 + 217 - (13)^2}{2 \times 16 \times \sqrt{217}}$

$\therefore m(\angle X) \simeq 49° 51'$ $\therefore m(\angle Y) \simeq 70° 9'$

7

$c^2 = (5)^2 + (7)^2 - 2(5)(7)\cos 65°$ $\therefore c \simeq 6.7$ cm.

$\therefore \cos B = \frac{(5)^2 + (6.7)^2 - (7)^2}{2 \times 5 \times 6.7}$ $\therefore m(\angle B) \simeq 71° 50'$

$\therefore m(\angle A) = 180° - (71° 50' + 65°) \simeq 43° 10'$

8

$a^2 = (6)^2 + (6)^2 - 2 \times 6 \times \cos 153° 12'$

$\therefore a \simeq 11.67$ cm.

, $m(\angle B) = m(\angle C) = \frac{180° - 153° 12'}{2} = 13° 24'$

9

$m(\angle M) = 1.2^{rad} \simeq 68° 45'$

$\therefore m^2 = (12.5)^2 + (7.25)^2 - 2 \times 12.5 \times 7.25 \cos 68° 45'$

$\therefore m \simeq 11.96$ cm.

$\therefore \cos L = \frac{(11.96)^2 + (7.25)^2 - (12.5)^2}{2 \times 11.96 \times 7.25}$

$\therefore m(\angle L) \simeq 76° 53'$ $\therefore m(\angle N) = 34° 22'$

10

$(LN)^2 = (48.5)^2 + (46)^2 - 2 \times 48.5 \times 46 \times (-0.6)$

$\therefore LN \simeq 84.53$ cm. and $\cos L = \frac{(48.5)^2 + (84.53)^2 - (46)^2}{2 \times 48.5 \times 84.53}$

$\therefore m(\angle L) \simeq 25° 48'$, $m(\angle M) \simeq 126° 52'$

$\therefore m(\angle N) = 27° 20'$

Exercise on the third case

11

(1) $\cos A = \frac{(27)^2 + (24)^2 - (15)^2}{2 \times 27 \times 24}$ $\therefore m(\angle A) \simeq 33° 33'$

, $\cos B = \frac{(15)^2 + (27)^2 - (24)^2}{2 \times 15 \times 27}$

$\therefore m(\angle B) \simeq 62° 11'$

, $m(\angle C) = 180° - (62° 11' + 33° 33') \simeq 84° 16'$

(2) $\cos A = \frac{(35)^2 + (17)^2 - (28)^2}{2 \times 35 \times 17}$

$\therefore m(\angle A) \simeq 52° 10'$

, $\cos B = \frac{(28)^2 + (17)^2 - (35)^2}{2 \times 28 \times 17}$

$\therefore m(\angle B) \simeq 99° 11'$, $m(\angle C) = 28° 39'$

12

$\cos A = \frac{(14)^2 + (15)^2 - (13)^2}{2 \times 14 \times 15}$ $\therefore m(\angle A) \simeq 53° 8'$

, $\cos B = \frac{(15)^2 + (13)^2 - (14)^2}{2 \times 15 \times 13}$ $\therefore m(\angle B) \simeq 59° 29'$

$\therefore m(\angle C) = 67° 23'$

13

$\cos A = \frac{(8)^2 + (4)^2 - (5)^2}{2(8)(4)}$ $\therefore m(\angle A) \simeq 30° 45'$

, $\cos B = \frac{(5)^2 + (4)^2 - (8)^2}{2(5)(4)}$ $\therefore m(\angle B) \simeq 125° 6'$

$\therefore m(\angle C) = 180° - (30° 45' + 125° 6') \simeq 24° 9'$

14

$\cos A = \frac{\left(4\sqrt{2}\right)^2 + \left(2\sqrt{5}\right)^2 - (2)^2}{2 \times 4\sqrt{2} \times 2\sqrt{5}}$

$\therefore m(\angle A) \simeq 18° 26'$

$\cos B = \frac{(2)^2 + \left(2\sqrt{5}\right)^2 - \left(4\sqrt{2}\right)^2}{2 \times 2 \times 2\sqrt{5}}$

$\therefore m(\angle B) \simeq 116° 34'$ $\therefore m(\angle C) = 45°$

15

$\cos X = \frac{(15)^2 + (30)^2 - (25)^2}{2 \times 15 \times 30}$ $\therefore m(\angle X) \simeq 56° 15'$

$\cos Y = \frac{(15)^2 + (25)^2 - (30)^2}{2 \times 15 \times 25}$

$\therefore m(\angle Y) \simeq 93° 49'$, $m(\angle Z) \simeq 29° 56'$

Exercise on the activity

16

$\frac{10}{\sin A} = \frac{9}{\sin 57°} = \frac{c}{\sin C}$ $\therefore \sin A = \frac{10 \sin 57°}{9}$

$\therefore m(\angle A) = 68° 44'$ or $m(\angle A) = 111° 16'$

$\therefore m(\angle C) = 54° 16'$ or $m(\angle C) = 11° 44'$

, $\because \frac{9}{\sin 57°} = \frac{c}{\sin C}$ $\therefore c = 8.7$ cm. or $c = 2.2$ cm.

17

$\frac{4}{\sin 50°} = \frac{3}{\sin B} = \frac{c}{\sin C}$ $\therefore \sin B = \frac{3 \sin 50°}{4}$

$\therefore m(\angle B) = 35^\circ\ 4'$ or $m(\angle B) = 144^\circ\ 56'$ (refused)

$\therefore m(\angle C) = 94^\circ\ 56'$

$\therefore \frac{c}{\sin 94^\circ\ 56'} = \frac{4}{\sin 50^\circ}$ $\qquad \therefore c \simeq 5.2$ cm.

18

$\frac{12}{\sin 116^\circ} = \frac{10}{\sin A} = \frac{b}{\sin B}$

$\therefore \sin A = \frac{10 \sin 116^\circ}{12}$

$\therefore m(\angle A) = 48^\circ\ 30'$ or $m(\angle A) \simeq 131^\circ\ 30'$ (refused)

, $m(\angle B) \simeq 180^\circ - (48^\circ\ 30' + 116^\circ) \simeq 15^\circ\ 30'$

$\therefore \frac{b}{\sin 15^\circ\ 30'} = \frac{12}{\sin 116^\circ}$ $\qquad \therefore b \simeq 3.6$ cm.

19

(1) $\because \angle A$ is obtuse , $a > b$

$\therefore$ There is a unique solution

$\therefore \frac{15}{\sin 120^\circ} = \frac{10}{\sin B}$ $\qquad \therefore \sin B = \frac{10 \sin 120^\circ}{15}$

$\therefore m(\angle B) \simeq 35^\circ$

, $m(\angle C) = 180^\circ - (35^\circ + 120^\circ) = 25^\circ$

$\therefore \frac{c}{\sin 25^\circ} = \frac{15}{\sin 120^\circ}$ $\qquad \therefore c \simeq 7.3$ cm.

(2) $\because \angle A$ is obtuse , $a < b$

$\therefore$ The conditions don't satisfy the existence of any triangle at all.

(3) $\because \angle A$ is acute , $h = 28 \sin 42^\circ \simeq 18.7$ cm.

, $18.7 < 20 < 28$

$\therefore$ There are two solutions to the triangle

$\therefore \frac{20}{\sin 42^\circ} = \frac{28}{\sin B}$ $\qquad \therefore \sin B = \frac{28 \sin 42^\circ}{20}$

$\therefore m(\angle B) = 70^\circ$ or $m(\angle B) = 110^\circ$

$\therefore m(\angle C) = 180^\circ - (70^\circ + 42^\circ) = 68^\circ$

or $m(\angle C) = 180^\circ - (110^\circ + 42^\circ) = 28^\circ$

, $\because \frac{c}{\sin C} = \frac{20}{\sin 42^\circ}$

$\therefore c \simeq 27.7$ cm. or $c \simeq 14$ cm.

(4) $\because \angle A$ is acute , $h = 7 \sin 60^\circ \simeq 6.1$ cm.

$\therefore a < h$

$\therefore$ The conditions don't satisfy the existence of any triangle at all.

(5) $\because \angle A$ is acute , $a > c$

$\therefore$ There is a unique solution

$\therefore \frac{12}{\sin 27^\circ} = \frac{7}{\sin C}$ $\qquad \therefore m(\angle C) \simeq 15^\circ$

$\therefore m(\angle B) = 180^\circ - (15^\circ + 27^\circ) = 138^\circ$

$\therefore \frac{b}{\sin 138^\circ} = \frac{12}{\sin 27^\circ}$ $\qquad \therefore b \simeq 17.7$ cm.

Miscellaneous exercise

20

$\because m(\angle A) = 110^\circ$

$\therefore m(\angle B) = m(\angle C) = \frac{180^\circ - 110^\circ}{2} = 35^\circ$

$\therefore \frac{8}{\sin 110^\circ} = \frac{b}{\sin 35^\circ} = \frac{c}{\sin 35^\circ}$ $\qquad \therefore b = c \simeq 4.9$ cm.

21

$\because m(\angle B) \simeq 53^\circ\ 8'$ and $m(\angle C) \simeq 22^\circ\ 37'$

$\therefore m(\angle A) = 104^\circ\ 15'$

$\therefore \frac{21}{\sin 104^\circ\ 15'} = \frac{b}{\sin 53^\circ\ 8'} = \frac{c}{\sin 22^\circ\ 37'}$

$\therefore b \simeq 17.3$ cm. and $c \simeq 8.3$ cm.

22

The area of $\Delta ABC = \frac{1}{2} a\, c \sin B$

$\therefore 10\sqrt{3} = \frac{1}{2} \times 5\, c \sin 120^\circ$ $\qquad \therefore c = 8$ cm.

$\therefore b^2 = (5)^2 + (8)^2 - 2 \times 5 \times 8 \cos 120^\circ$

$\therefore b = 11.36$ cm.

$\therefore \cos A = \frac{(11.36)^2 + (8)^2 - (5)^2}{2 \times 11.36 \times 8}$

$\therefore m(\angle A) \simeq 22^\circ\ 24'$ $\qquad \therefore m(\angle C) = 37^\circ\ 36'$

23

$m(\angle A) = \frac{4}{15} \times 180^\circ = 48^\circ$

$m(\angle B) = \frac{5}{15} \times 180^\circ = 60^\circ$

$m(\angle C) = \frac{6}{15} \times 180^\circ = 72^\circ$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{a + b + c}{\sin A + \sin B + \sin C}$

$\therefore \frac{a}{\sin 48°} = \frac{b}{\sin 60°} = \frac{c}{\sin 72°} = \frac{50}{2.56}$

$\therefore a \simeq 14.5$ cm. , $b \simeq 16.9$ cm. , $c \simeq 18.6$ cm.

24

Let $\sin A = 3$ m , $\sin B = 4$ m and $\sin C = 6$ m

$\therefore \frac{a}{3\text{ m}} = \frac{b}{4\text{ m}} = \frac{c}{6\text{ m}} = \frac{a+b+c}{13\text{ m}}$

$\therefore \frac{a}{3} = \frac{b}{4} = \frac{c}{6} = \frac{52}{13}$

$\therefore a = 12$ cm. , $b = 16$ cm. and $c = 24$ cm.

$\therefore \cos A = \frac{(16)^2 + (24)^2 - (12)^2}{2 \times 16 \times 24}$

$\therefore m(\angle A) \simeq 26° 23'$ $\quad \therefore \cos B = \frac{(12)^2 + (24)^2 - (16)^2}{2 \times 12 \times 24}$

$\therefore m(\angle B) \simeq 36° 20'$ $\quad \therefore m(\angle C) = 117° 17'$

25

$\frac{21}{\sin A} = \frac{25}{\sin B} = \frac{c}{\sin C} = 28$

$\therefore m(\angle A) \simeq 48° 35'$, $m(\angle B) \simeq 63° 14'$

$\therefore m(\angle C) \simeq 68° 11'$ $\quad \therefore c = 28 \sin 68° 11' \simeq 26$ cm.

Answers of Life Applications on Unit Four

1

$\because m(\angle A) = 180° - (33° + 52°) = 95°$

$\therefore \frac{AC}{\sin 52°} = \frac{20}{\sin 33°} = \frac{BC}{\sin 95°}$ $\quad \therefore AC \simeq 29$ km.

i.e. The distance between the ship and the lighthouse (A) $\simeq$ 29 km.

, $BC \simeq 36.6$ km.

i.e. The distance between the ship and the lighthouse (B) $\simeq$ 36.6 km.

2

$\because m(\angle BAC) = 42° - 20° 35' = 21° 25'$

$\therefore$ In ΔABC : $\frac{AB}{\sin 20° 35'} = \frac{50}{\sin 21° 25'}$

$\therefore AB \simeq 48.1$ m.

, in ΔABD : $\frac{AD}{\sin 42°} = \frac{48.1}{\sin 90°}$

$\therefore AD \simeq 32.2$ m.

$\therefore$ The height of the minaret $\simeq$ 32.2 m.

3

$\because m(\angle Y) = 180° - (40° + 105°) = 35°$

$\therefore \frac{XZ}{\sin 35°} = \frac{9}{\sin 105°}$ $\quad \therefore XZ = \frac{9 \sin 35°}{\sin 105°} \simeq 5.3$ km.

$\therefore$ The distance between position X and position Z $\simeq$ 5.3 km.

$\therefore$ The area of $\Delta XYZ = \frac{1}{2} \times 9 \times 5.3 \sin 40° \simeq 15$ km^2.

4

$\because$ (The magnitude of displacement)^2

$= 8^2 + 9^2 - 2 \times 8 \times 9 \cos 80°$

$\therefore$ The magnitude of displacement $\simeq$ 11 km.

5

$\because$ (The distance)^2

$= (15)^2 + (17)^2 - 2 \times 15 \times 17 \cos 45°$

$\therefore$ The distance = 12.4 m.

6

$\because \cos C = \frac{(210)^2 + (140)^2 - (300)^2}{2 \times 210 \times 140}$

$\therefore m(\angle C) \simeq 116° 34'$

$\therefore$ The area of the land $= \frac{1}{2} \times 210 \times 140 \times \sin 116° 34'$

$\simeq 13148$ m^2.

7

$\because (AC)^2 = (2.15)^2 + (2.5)^2 - 2\,(2.15)\,(2.5) \cos 130°$

$\therefore AC \simeq 4.2$ km.

, $\because$ time $= \frac{\text{distance}}{\text{speed}}$ $\quad \therefore t_1 = \frac{2.5}{50} \times 60 = 3$ minutes

, $t_2 = \frac{2.15}{50} \times 60 = 2.58$ minutes

, $t_3 = \frac{4.2}{60} \times 60 = 4.2$ minutes

$\therefore$ The total time $= 3 + 2.58 + 4.2 \simeq 9.8$ minutes